1. Mars orbits the Sun at an average distance of 2.28×10^{11} m and has a radius of 3.39×10^{6} m. The Sun has a luminosity of 3.828×10^{26} W. How much solar energy falls on the surface of Mars each second? Ignore any effects of Mars' thin atmosphere. (Answer: 2.2×10^{16} W)

Solution: At the distance of Mars' orbit, the Sun's energy output is spread over a sphere with an area of

$$A = 4\pi r^2 = 4\pi(2.28 \times 10^{11}\ \text{m})^2 = 6.53 \times 10^{23}\ \text{m}^2.$$

Dividing the luminosity of the Sun by this area gives,

$$\frac{L_\odot}{A} = \frac{3.828 \times 10^{26}\ \text{W}}{6.53 \times 10^{23}\ \text{m}^2} = 586\ \frac{\text{W}}{\text{m}^2}.$$

Mars presents a circular area to the Sun of

$$A_{\text{Mars}} = \pi r_{\text{Mars}}^2 = \pi(3.39 \times 10^{6}\ \text{m})^2 = 3.61 \times 10^{13}\ \text{m}^2.$$

Therefore, the total energy that falls on the surface of Mars will be

$$\frac{L_\odot}{A} A_{\text{Mars}} = \left(586\ \frac{\text{W}}{\text{m}^2}\right)(3.61 \times 10^{13}\ \text{m}^2) = 2.12 \times 10^{16}\ \text{W}.$$

2. Suppose that at some time in the very recent past all the hydrogen and helium in the universe had been instantly fused into iron in stars, and the released energy thermalized into black body radiation.
Consider: baryon density $\rho_b = 4.2 \times 10^{-31}\text{gcm}^3$, about 75% hydrogen (=1 baryon) by mass and 25% helium (=4 baryons) by mass.
Note that the binding energy per nucleon of ${}^{56}_{26}Fe$ is 8.8 MeV and that of ${}^{4}_{2}He$ is 7.1 MeV.)
(Answers a) 4.40 k b) 0.00065841 m)

c) What is the current temperature of this black body radiation? (2 points)
d) Determine what wavelength the blackbody spectrum would peak at. What region of the electromagnetic spectrum would this be in? (1 point)

Solution: Answer: $T = 4.40K$.

Without considering intermediary products, we have these two reactions:

$$56\ {}^{1}_{1}\text{H} \rightarrow {}^{56}_{26}\text{Fe}$$

$$14\ {}^{4}_{2}\text{He} \rightarrow {}^{56}_{26}\text{Fe}$$

We can calculate the energy released in each of these reactions by computing the binding energy on each side. There is no binding energy for a hydrogen atom, so the energy released in the first reaction is 56×8.8 MeV $= 492.8$ MeV per Fe. For the second reaction, the binding energy will by (56×8.8 MeV) - ($14 \times 4 \times 7.1$ MeV) $= 95.2$ MeV per Fe.

To find how much iron is actually produced, we need to determine the number densities of hydrogen and helium using the mass fraction given in the question:

$$n_H = \frac{0.75\rho_b}{m_H}$$

$$n_{He} = \frac{0.25\rho_b}{m_{He}}$$

where m_H and m_{He} are the masses of hydrogen and helium respectively. Then , the total energy density released is:

$$u = \frac{492.8MeV \times n_H}{56} + \frac{95.2MeV \times n_{He}}{14}$$

To get the temperature, we use:

$$u = aT^4$$

where a is the radiation constant. This gives us a temperature of 4.40 K.

Solution: Answer: $\lambda = 6.59 \times 10^{-2}$ cm, microwave.

Using Wien's displacement law:

$$\lambda = \frac{b}{T}$$

where $b = 2.898 \times 10^{-3}$ m K, we get a wavelength of 6.59×10^{-2} cm, which is in the microwave region of the electomagnetic spectrum.