Primordiale Schwarze Löcher im inflationären Universum
(Primordial Black Holes in the Inflationary Universe)

Diploma Thesis

Torsten Bringmann, Department of Physics, University of Freiburg
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Summary

The present paradigm of cosmology makes use of an inflationary stage of expansion in the very early universe. This does not only provide an elegant solution to some of the initial value problems of the old big-bang cosmology, i.e. the horizon and flatness problems in particular, it also gives a causal explanation for the observed large-scale structures (LSS) in the universe, like the distribution of galaxies or patterns in the cosmic microwave background: It is believed that that they all have their origin in tiny primordial density perturbations which in turn resulted from vacuum fluctuations produced during inflation. Each inflationary model makes precise predictions about the form of these primordial fluctuations and can thereby be constrained by observations. Most inflationary models produce a scale invariant spectrum with Gaussian statistics, i.e. a fluctuation spectrum which is also supported by LSS observations.

It was realized already in the beginning of the 1970s that density perturbations in the early universe can lead the formation of primordial black holes (PBHs) by gravitational collapse. These PBHs might in principle be arbitrarily small and their number density can be computed from the statistics of the primordial fluctuations at the time of their formation, more precisely from the mass variance at horizon entry. Although there is little evidence so far for any directly observed PBHs, there are many constraints on their allowed number density, basically coming from the evaporation of black holes due to the Hawking effect and from the requirement that PBHs must not overclose the present universe. Of course, these limitations lead to constraints on the primordial fluctuations as well – which in turn can be used as constraints on inflationary models. The a priori most interesting aspect in this respect is that PBHs probe these fluctuations on very small scales that are not accessible by any other means (like for example direct LSS observations).

In this thesis, I review the process of structure formation in inflationary cosmology and the basic formalism of PBH formation. It is discussed in detail how the expressions for the mass variance used earlier in the lite-

rature have to be corrected: In order to obtain accurate numbers for PBH number densities one has to take into account the effects of a transfer function that encodes the behaviour of fluctuations on sub-horizon scales. The constraints on the PBH abundance are then used to derive constraints on the spectral index n of primordial fluctuations with a scale free (power-law) spectrum. Next, a primordial fluctuation spectrum with a characteristic scale is considered by artificially superimposing a step on such a power-law spectrum. Finally, a spectrum is studied that results from so-called broken scale-invariant (BSI) inflationary models with a jump in the first derivative of the inflaton potential. This spectrum looks like a power-law spectrum with a step and additional oscillations superimposed. In these scale-dependent spectra the constraint on n depends on the height of the step; thus one can obtain constraints on the allowed height of the step from the existing limits on n. If, for example, the scale-invariant case $n \approx 1$ turns out to be true there may be up to 10^4 times more power on small scales than what is observed on large scales.

The main results of this thesis may also be found in

[1] T. Bringmann, C. Kiefer and D. Polarski, *Phys. Rev. D* **65**, 024008 (2002).

[2] D. Blais, T. Bringmann, C. Kiefer and D. Polarski, *Phys. Rev. D* **67**, 024024 (2003).