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The unfolding of continuum γ-ray spectra

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Abstract

A new method for unfolding γ-ray spectra is described. The method is based on the fact that the contribution of Compton scattered γ-rays in the spectra is a slowly varying function of energy. Thus, the Compton part can be smoothed and subtracted from the original observed spectrum giving an unfolded spectrum with the same statistical fluctuations as the observed spectrum. In particular, we show that the method works very well for poor statistics continuum γ-ray spectra.

1. Introduction

The statistical behaviour of nuclear matter is of increasing interest. The level density at high excitation energy is so large that the γ-transitions between the individual levels cannot be resolved experimentally. The continuum γ-ray spectra display a broad energy distribution without outstanding γ-lines. In order to extract useful information a careful unfolding (deconvolution) into a true full energy γ-ray spectrum has to be performed.

The response function of a γ-ray detector depends on the various ways photons interact with matter. The photoelectric effect, Compton scattering and pair production are all processes which contribute to a measured γ-ray spectrum. The full energy peak is the result of one photoelectric effect or a full energy absorption through multiple processes. If the incident photon is scattered by an electron with a partial energy loss, the event appears in the energy region of the Compton background. The single and double escape peaks are due to pair production within the detector, and the 511 and ~200 keV peaks originate from annihilation radiation and backscattering in the surroundings, respectively.

Events with energy less than the full energy occur rather frequently. The peak-to-total ratio is typically less than 50%. In addition, the various interactions have different energy dependence. To introduce such a dramatic correction of the observed spectrum is therefore a great challenge.

Many unfolding techniques for γ-ray spectra have been proposed in the literature. The *inverse matrix method* [1,2] represents perhaps the most straightforward method. The *stripping method* [3–6] is often applied for Ge detectors and is based on a successive subtraction of Compton background from higher to lower channels. The *folding iteration method* [7–11] is based on successive foldings of better and better trial functions. Recently, a *neural network method* [12] has also been introduced.

In our case we apply the folding iteration method to estimate the shape of a spectrum which contains the Compton background, the single and double escape and the annihilation peaks. With proper smoothing and normalization treatments, this component of the spectrum is subtracted from the observed spectrum in order to obtain the true full energy γ-ray spectrum.

The method is demonstrated for NaI(Tl) detectors and found particularly appropriate for high energy continuum γ-ray spectra. However, the method should work as well using Ge, BGO, CsI or other detector materials provided that proper response functions are available.

2. The response function

A successful unfolding depends on a proper set of experimentally measured response functions [4,13]. In principle, the detector response function $R(E, E_\gamma)$, where E is the amount of energy deposited in the detector, should be established for all possible incident photon energies E_γ. Unfortunately, the response function can only be measured for a few monoenergetic γ-lines. Thus, it is important to find an appropriate procedure to interpolate between the observed spectra in order to obtain response functions for all energies E_γ. In this section we describe shortly our way to build a response matrix dedicated for the 28 collimated and shielded 5 in. × 5 in. NaI detectors in the CACTUS multi-detector array at the Oslo Cyclotron Laboratory [14].

Our measured response functions are recorded from radioactive γ-sources and from in-beam experiments. In

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Table 1
Intensities and FWHMs deduced for the NaI response functions

E_γ [MeV]	FWHM [keV][a]	ϵ_{tot}[b]	p_f	p_c	p_s	p_d	p_a
0.1	14	1.88	0.741	0.259	0.000	0.000	0.000
0.2	24	1.72	0.659	0.341	0.000	0.000	0.000
0.4	40	1.47	0.567	0.433	0.000	0.000	0.000
0.6	51	1.29	0.502	0.498	0.000	0.000	0.000
0.8	56	1.15	0.478	0.522	0.000	0.000	0.000
1.0	64	1.06	0.440	0.560	0.000	0.000	0.000
1.5	87	0.97	0.360	0.636	0.003	0.000	0.002
2.0	101	0.88	0.321	0.662	0.012	0.000	0.005
3.0	130	0.86	0.276	0.687	0.028	0.003	0.007
4.0	144	0.83	0.228	0.713	0.046	0.005	0.008
6.0	198	0.81	0.182	0.748	0.053	0.009	0.008
8.0	239	0.81	0.150	0.780	0.049	0.012	0.008
10.0	268	0.82	0.114	0.815	0.046	0.016	0.008

[a] Normalized to 79.9 keV (6%) at 1.33 MeV.
[b] Normalized to 1 at 1.33 MeV. This efficiency has to be multiplied with the discriminator efficiency for the specific experimental set-up (see text).

the cases where two γ-rays are emitted in coincidence, we have used a Ge detector to gate on one γ-line so that the other γ-ray hits the NaI detectors. Totally, ten spectra of monoenergetic γ-lines have been measured: 122, 245, 344, 662, 1173, 1333, 1836, 4439, 6130 and 15 110 keV. The full energy (f), single escape (s), double escape (d) and annihilation (a) peaks have been removed from these spectra. Thus, the remaining part of the observed response spectrum is the Compton[1] events (c). The reason why we separate out the peaks are threefold:

– It simplifies the interpolations between response functions since the positions and intensities have different energy dependence.

– In cases where the actual experimental energy resolution deviates from the resolution of the observed response functions, one can easily create a new response matrix with the appropriate energy resolution.

– In the new Compton subtraction method described in Section 4, one needs to smooth the components of the spectrum with different energy resolutions.

The probabilities p that an event belongs to one of the five components of the spectrum, are normalized by

$$p_f + p_c + p_s + p_d + p_a = 1\,, \qquad (1)$$

and are listed in Table 1 together with the full-width-half-maximum (FWHM) and total γ-ray efficiency for the CACTUS multi-detector. In our set-up the lower detection threshold is around 100 keV. Thus, the normalization of Eq. (1) is only valid if we extrapolate the Compton spectrum to zero energy.

[1] It would be more correct to denote this spectrum quasicompton, since backscattering, pile-up and other minor effects are included as well.

An interpolation between the peak structures to obtain a new response function for an intermediate full energy E_γ can easily be performed by adding a Gaussian distribution at the interpolated peak position with proper intensity and energy resolution.

The interpolation of the Compton background is more complicated, as illustrated in Fig. 1. The observed Compton background response functions c_1 and c_2 have different length according to their respective full energy values. Therefore, the interpolation should operate along a set of curves forming a fan, connecting the same channels in the

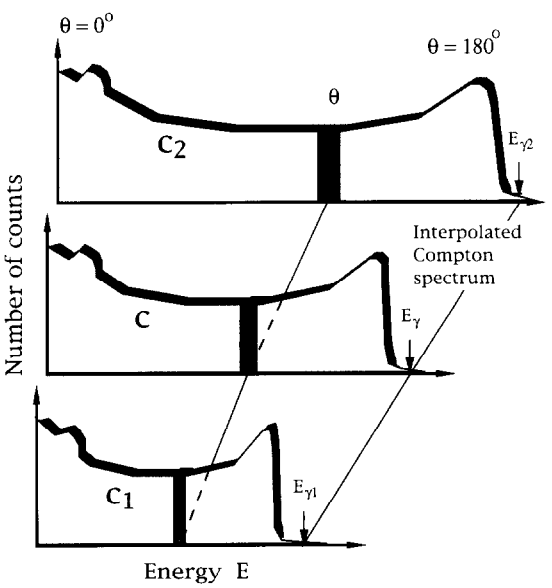

Fig. 1. The interpolation of the Compton part of the measured response functions c_1 and c_2. The figure illustrates that $\Delta\theta$ increases with the full energy E_γ.

lower end and the highest channels in the upper end of the spectra.

As a reasonable approach the Compton background is interpolated between channels corresponding to the same γ-ray scattering angle θ. A photon which is scattered an angle θ transfers an energy to the electron given by

$$E = E_\gamma - \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)}. \tag{2}$$

The fact that the electron energy E (which is the energy deposited in the detector) strongly depends on θ and E_γ has to be taken into account for the normalization of the interpolated spectrum

$$c(E) = \left(\frac{dE}{d\theta}\right)^{-1}_{E_\gamma}\left[c_1(E_1)\left(\frac{dE}{d\theta}\right)_{E_{\gamma 1}} + \frac{E_\gamma - E_{\gamma 1}}{E_{\gamma 2} - E_{\gamma 1}} \times \left(c_2(E_2)\left(\frac{dE}{d\theta}\right)_{E_{\gamma 2}} - c_1(E_1)\left(\frac{dE}{d\theta}\right)_{E_{\gamma 1}}\right)\right].$$

Here, $E_\gamma, E_{\gamma 1}$ and $E_{\gamma 2}$ are the full energies for the interpolated spectrum and the reference spectra, respectively. The quantities E, E_1 and E_2 are the corresponding energies that the electron deposits in the detector due to Compton scattering into an angle θ (see Fig. 1).

The Compton spectrum also has counts above the energy of the Compton edge at $E(\theta = 180°)$. From here we continue successively adding one and one channel and interpolating.

In the lower part of the spectra (~200 keV) we find the backscattering peak. This part of the spectrum is to a large extent dominated by backscattering, X-rays and other background events. Thus, from zero energy up to the backscattering energy, defined by

$$E_{\text{b.sc.}} = E_\gamma / \left(1 + \frac{2E_\gamma}{m_e c^2}\right), \tag{4}$$

we interpolate between the same channel numbers.

Fig. 2 shows an example of a response matrix for incident energies between 0 and 5 MeV (y-axis). The matrix has dimensions 512×512 channels and an energy resolution of 6% at 1332 keV.

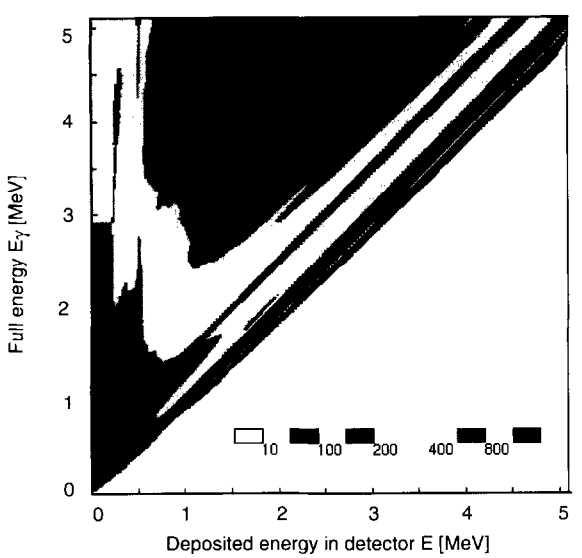

Fig. 2. Example of a response matrix **R** for collimated and shielded 5 in. × 5 in. NaI γ-detectors.

3. The folding iteration method

Our unfolding procedure, called the Compton subtraction method, utilizes an unfolded spectrum as a starting point, where the general shape is believed to be correct. The folding iteration method [7–11] is a safe technique, although the counts of the spectra are strongly fluctuating from channel to channel. This method is to our knowledge only described in various less available institute reports. Therefore an outline is given here.

The folding iteration method takes advantage of the fact that folding is a fast and simple task. We define R_{ij} as the response in channel i when the detector is hit by γ-rays with an energy corresponding to channel j. Each response function j is normalized by $\Sigma_i R_{i,j} = 1$. The folding can then be expressed by

$$\begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{pmatrix}, \tag{5}$$

or, in a compact form;

$$f = \mathbf{R}u, \tag{6}$$

where f and u represent the folded and unfolded spectra, respectively. Thus, if we can obtain better and better trial spectra u, we can fold them and compare them with the observed spectrum r. The folding iteration method is performed in four steps:

1) As the first trial function for the unfolded spectrum u^0 we use:

$$u^0 = r. \tag{7}$$

2) The first folded spectrum is now calculated:

$$f^0 = \mathbf{R}u^0. \tag{8}$$

3) The next trial function is obtained by adding the difference spectrum $r - f^0$ to the original trial function:

$$u^1 = u^0 + (r - f^0). \tag{9}$$

4) Again we fold and get a new f^1, which again is used to obtain the next trial function:

$$u^2 = u^1 + (r - f^1)\,, \tag{10}$$

and so on until $f^i \sim r$, where i is the iteration index.

As pointed out in Ref. [15] a response matrix with the experimental energy resolution gives artificial undershoots on both sides of pronounced peaks in the unfolded spectrum. They suggest that the best unfolding result is obtained with a smaller FWHM value than experimentally observed, namely $\mathrm{FWHM}^{\mathrm{resp}} = 0.5\,\mathrm{FWHM}^{\mathrm{exp}}$. This feature has also been verified by us and is taken into account in the procedure.

A few iteration steps of the difference approach applied to a NaI γ-ray spectrum from the excitation region $E_x = 0 - 8$ MeV in ^{162}Dy are shown in Fig. 3. The spectrum shows the γ-ray energy distribution detected in coincidence with α-particles, using the ^{163}Dy(^{3}He, $\alpha\gamma$)^{162}Dy reaction at a beam energy of 45 MeV (see Refs. [16,17]). The oscillations from channel to channel increase as a function of the number of iterations. For higher iterations the solution is almost identical with the exact matrix inversion ($u = \mathbf{R}^{-1}r$, see Eq. (6)) which exhibits strong oscillations. Thus, it is important to terminate the iteration when the folded spectrum agrees with the observed spectrum within the experimental uncertainties. In our case this would be for the iteration around $i = 10$.

The unfolding procedure described above is the *difference approach* of the folding iteration method, since the corrections are performed using the difference according to

$$u^{i+1} = u^i + (r - f^i)\,. \tag{11}$$

The *ratio approach* modifies the trial function by multiplying with a ratio

$$u^{i+1} = u^i(r/f^i)\,. \tag{12}$$

The ratio approach behaves much like the difference approach. However, decisions have to be made if f^i happens to be zero for some channels (see Eq. (12)). Furthermore, if the observed spectrum and the elements of the response matrix have only positive counts the ratio method fails to produce negative counts. This is an unwanted limitation and gives poorer reproduction of the observed spectrum.

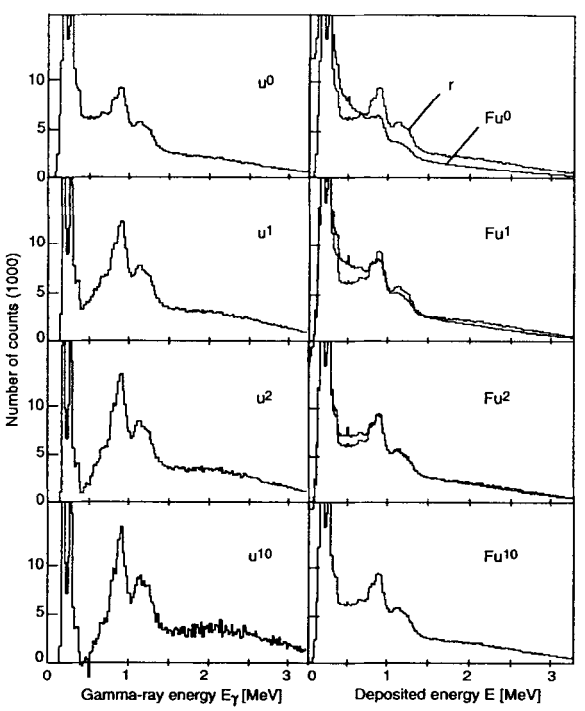

Fig. 3. The folding iteration method. Each iteration gives unfolded spectra (u^i) that by folding (Fu^i) give better and better reproduction of the observed spectrum (r).

4. The Compton subtraction method

Our starting point for the Compton subtraction method is the unfolded spectrum from the folding iteration method, typically the u spectrum in Fig. 3 obtained after $i = 10$ iterations. We call this oscillating spectrum u_0, and we will present a new method to produce a much less fluctuating unfolded spectrum. The starting spectrum u_0 could also be estimated applying some of the other methods mentioned in the introduction.

First we define a new spectrum, representing the observed spectrum minus the Compton contribution, given by

$$v(i) = p_f(i)u_0(i) + w(i)\,, \tag{13}$$

where $p_f u_0$ is the full-energy contribution and $w = u_s + u_d + u_a$ is the contribution from structures due to the single and double escape and annihilation processes. Here, we calculate

$$u_s(i - i_{511}) = p_s(i)u_0(i)\,, \tag{14}$$

$$u_d(i - i_{1022}) = p_d(i)u_0(i)\,, \tag{15}$$

and

$$u_a(i_{511}) = \sum_i p_a(i)u_0(i)\,, \tag{16}$$

where i_{511} and i_{1022} are the channels having energies 511 and 1022 keV, respectively. The probabilities p_f, p_s, p_d and p_a are taken from Table 1.

The u_a spectrum has all its counts in channel i_{511} and has to be smoothed with a resolution of 1.0 FWHM to obtain the energy resolution of the experimentally observed spectrum. The u_f, u_s and u_d spectra have an energy resolution determined by the observed spectrum (1.0 FWHM) and the response matrix (0.5 FWHM) giving $\sqrt{1.0^2 - 0.5^2}$ FWHM $= 0.87$ FWHM. Thus, we smooth $p_f u_0$, u_s and u_d with an additional 0.5 FWHM in order to

obtain a spectrum with the observed resolution of 1.0 FWHM. Then we extract the so-called Compton background spectrum by

$$c(i) = r(i) - v(i) . \quad (17)$$

This spectrum shows strong oscillations from channel to channel since it depends on the original u_0 spectrum (compare with Fig. 3). The idea is that this spectrum should be a *slowly varying function of energy*. Thus, one might use rather strong smoothing, e.g. a resolution of 1.0 FWHM, on this spectrum.

The next step is to subtract the smoothed Compton part c and the structures w from the observed spectrum r and correct for the full energy probability p_f

$$u(i) = [r(i) - c(i) - w(i)]/p_f(i) . \quad (18)$$

As a final step we calculate the true γ-ray energy distribution by correcting for the energy dependent total γ-ray detection efficiency,

$$U(i) = u(i)/\epsilon_{tot}(i) , \quad (19)$$

where the total γ-ray efficiency ϵ_{tot} is taken from Table 1. In addition, ϵ_{tot} should be multiplied with the efficiency due to the discriminator level of the experimental set-up. This efficiency depends e.g. on the timing and ADC thresholds and the intensity reduction due to absorbing material in front of the γ-ray detector. Typically, the experimental energy threshold is located around 100–200 keV with a width of about 100 keV. It is interesting to see that the total γ-ray efficiency is not playing any part in the extraction techniques for u.

5. Application to NaI γ-ray spectra

In Fig. 4 are shown examples of the spectra r, c and $u = r - c - w$ discussed in the preceding section. As test spectra we use a ^{152}Eu source spectrum and the observed spectrum of Fig. 3. The ^{152}Eu spectrum (left) exhibits discrete γ-lines without continuum background. The ^{162}Dy spectrum (right) is essentially a continuum spectrum. In both cases we find that the spectrum c deduced from Eq. (17) is a continuum distribution which might be smoothed without changing its major shape.

In the upper part of Fig. 5 (r) are shown two continuum spectra from the excitation regions 3.0 and 8.0 MeV in ^{162}Dy. Both spectra, which are taken from a 0.12 MeV wide excitation energy bin, are of the continuum type and possess low statistics. Below these spectra are shown the results of unfolding with the conventional folding iteration method (u_0). These spectra are seen to fluctuate strongly from channel to channel. Our new Compton subtraction method (u) gives a much smoother spectrum with the same fluctuations as the observed spectrum (r).

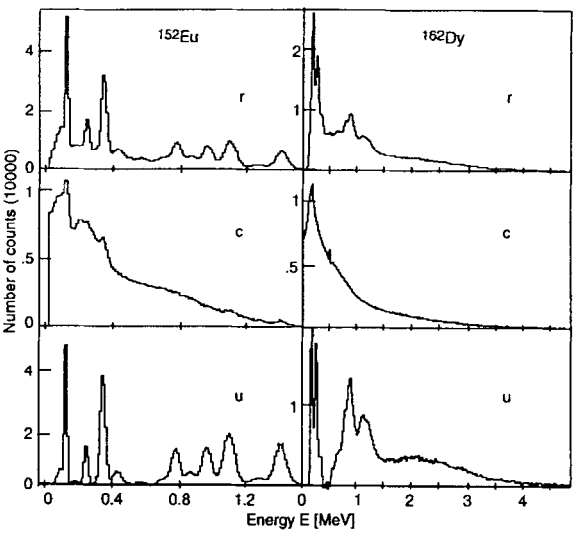

Fig. 4. The observed spectrum (r), the Compton-like spectrum (c) and the resulting unfolded spectrum (u) obtained with the Compton subtraction method (see text).

The case of a smoothed u_0 solution is displayed in the lower part of Fig. 5. The result is far from satisfactory compared to our new method (u). In particular, one can see here that smoothed spikes (one-channel peaks) look like

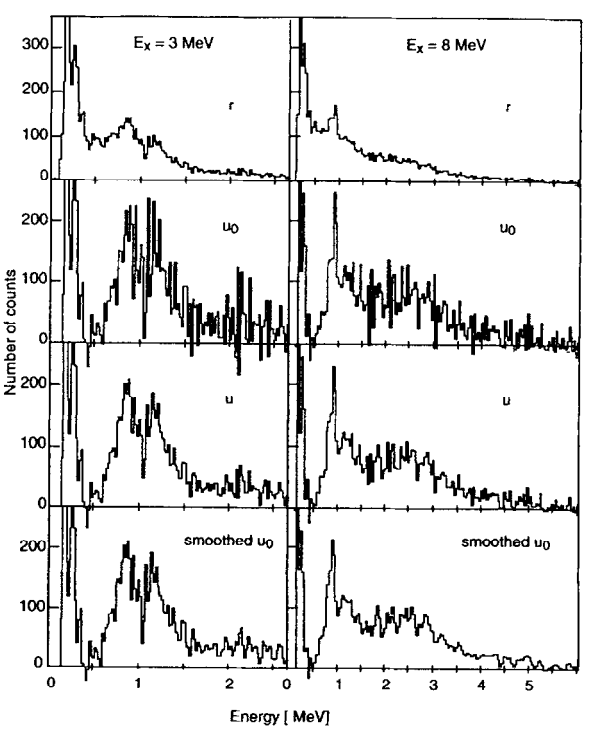

Fig. 5. The result from the Compton subtraction method (u) compared with the result from the folding iteration method (u_0) and the smoothing of that solution (smoothed u_0).

significant peak structures. This is very misleading and such a procedure of reducing the fluctuations of the spectrum is not recommended.

6. Conclusions

In the present work we have described a method to achieve reliable unfolded spectra with the right overall shape combined with small fluctuations. Previous methods like the *inverse matrix method* or the *folding iteration method* give realistic overall shapes, but large oscillations from channel to channel. The *stripping method* gives smooth spectra but might easily introduce a wrong overall shape of the unfolded spectrum due to systematic errors.

The Compton subtraction method takes an unfolded spectrum with the correct shape as a starting point. Then a Compton-like spectrum is extracted, which can be strongly smoothed, since it has no abrupt peak structures. This Compton part together with the single and double escape and annihilation peaks are then subtracted from the observed spectrum.

The method has been carefully tested on discrete and continuum γ-ray spectra. It can successfully extract unfolded γ-ray spectra with statistical fluctuations comparable with the observed spectrum.

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References

[1] N. Starfelt and H.W. Koch, Phys. Rev. 102 (1956) 1598.
[2] P.C. Fisher and L.B. Engle, Phys. Rev. 134 (1964) B 796.
[3] W. Trautmann, J.F. Sharpey-Schafer, H.R. Andrews, B. Haas, O. Hausser, P. Taras and D. Ward, Nucl. Phys. A 378 (1982) 141.
[4] D.C. Radford, I. Ahmad, R. Holzmann, R.V.F. Janssens and T.L. Khoo, Nucl. Instr. and Meth. A 258 (1987) 111.
[5] D.J.G. Love and A.H. Nelson, Nucl. Instr. and Meth. A 274 (1989) 541.
[6] J.C. Waddington, Nucl. Instr. and Meth. A 274 (1989) 608.
[7] N.E. Scofield, U.S. Naval Radiological Laboratory Report, USNRDL-TR 447 (1960) unpublished.
[8] J.F. Mollenauer, Lawrence Radiation Laboratory Report, UCRL-9748 (1961) unpublished.
[9] J.F. Mollenauer, Phys. Rev. 127 (1962) 867.
[10] W.J. O'Connell, Lawrence Livermore Laboratory Report, UCID-30079 (1973) unpublished.
[11] D. Sam, L.R. Bunney and D.C. Heater, Nucl. Instr. and Meth. 64 (1968) 148.
[12] R. Koohi-Fayegh, S. Green, N.M.J. Crout, G.C. Taylor and M.C. Scott, Nucl. Instr. and Meth. A 329 (1993) 269.
[13] R.L. Heath, R.G. Helmer, L.A. Schmittroth and G.A. Cazier, Nucl. Instr. and Meth. 47 (1967) 281.
[14] M. Guttormsen, B. Bjerke, J. Kownacki, S. Messelt, E.A. Olsen, T. Ramsøy, J. Rekstad, T.S. Tveter and J.C. Wikne, Department of Physics Report, University of Oslo, 89-14 (1989).
[15] Cs. Sükösd, B. Galster, I. Licot and M.P. Simonart, Nucl. Instr. and Meth. A 355 (1995) 552.
[16] M. Guttormsen, A. Atac, G. Løvhøiden, S. Messelt, T. Ramsøy, J. Rekstad, T.F. Thorsteinsen, T.S. Tveter and Z. Zelazny, Phys. Scripta T32 (1990) 54.
[17] L. Henden, L. Bergholt, M. Guttormsen, J. Rekstad and T.S. Tveter, Nucl. Phys. A 589 (1995) 249.