# Double Phase Transition in the Classical $J_1$-$J_2$-$J_3$ Heisenberg Model on the Triangular Lattice

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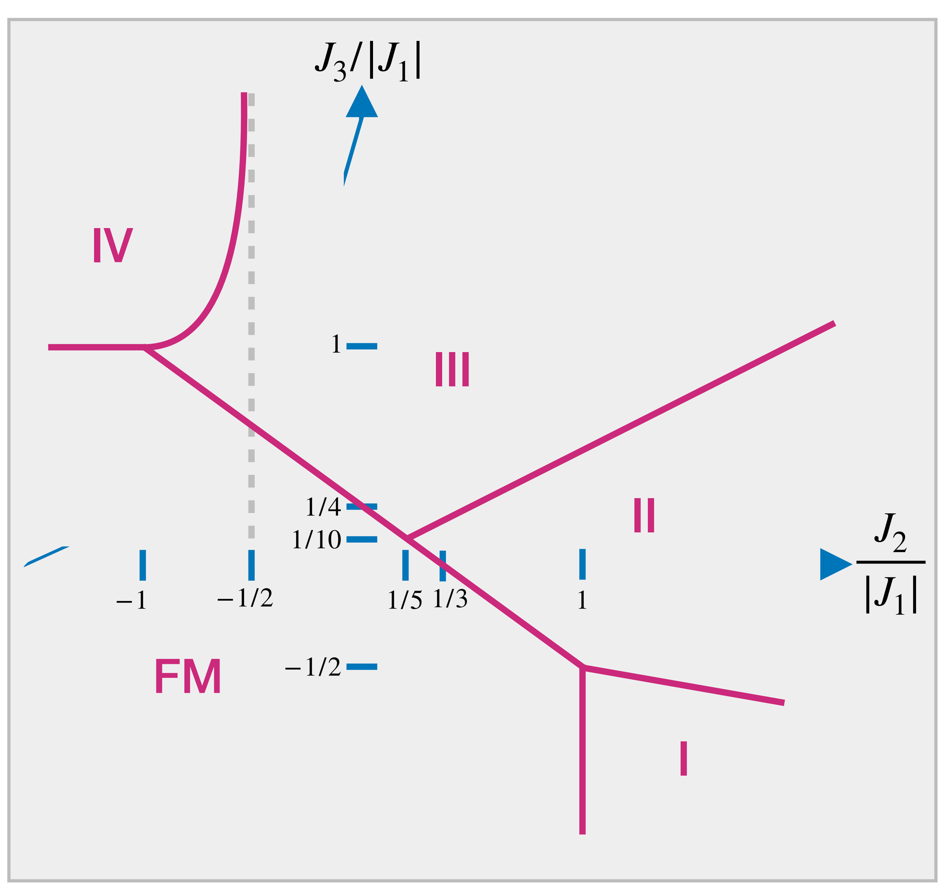
*We study the classical $J_1$-$J_2$-$J_3$ Heisenberg model on the triangular lattice using a field-theoretic framework[1]. Critical temperatures for lattice symmetry breaking phase transitions are extracted for the whole $J_2$-$J_3$ space with $J_1$ ferromagnetic. We find that in region III in the vicinity of the region II-III border, the system exhibits two phase transitions when lowering the temperature, breaking different mirror symmetries.*

## 1 INTRODUCTION

We use a newly developed field-theoretic framework[1] to study phase transitions in frustrated magnets on the triangular lattice, using the classical $J_1$-$J_2$-$J_3$ Heisenberg model with $J_1$ ferromagnetic(FM).

This framework uses diagrammatic techniques from statistical field theory to arrive at a set of self-consistent equations which are solved numerically by iteration. By solving the equations, one obtains information about phases, phase transitions and critical temperatures. This method captures also phases that break lattice symmetries[2], which are not prohibited by the Mermin-Wagner theorem.




*Regions of different classes of wave vectors minimizing $J_{\vec q}$ for FM nearest neighbour interactions on the triangular lattice.*

## 2 METHOD

### HEISENBERG MODEL ON TRIANGULAR LATTICE

The Hamiltonian for the classical Heisenberg model is

$$H = \frac{1}{2}\sum_{\vec R,\vec R'} J_{\vec R,\vec R'}\vec S_{\vec R}\cdot\vec S_{\vec R'},$$

where the spins are of unit length. In reciprocal space this can be rewritten as

$$H = \sum_{\vec q} J_{\vec q}\vec S_{\vec q}\cdot\vec S_{\vec q}.$$

The momentum vectors minimizing $J_{\vec q}$ depend on the couplings in real space. For the $J_1$-$J_2$-$J_3$ model on the triangular lattice with $J_1$ FM, we find five distinct regions of minima. There are only three of these regions where it is possible to break the lattice symmetry: Region I, II and III. The border between region II and III is at $J_3 = 1/2\, J_2$, where the energy minima form a ring-like structure.

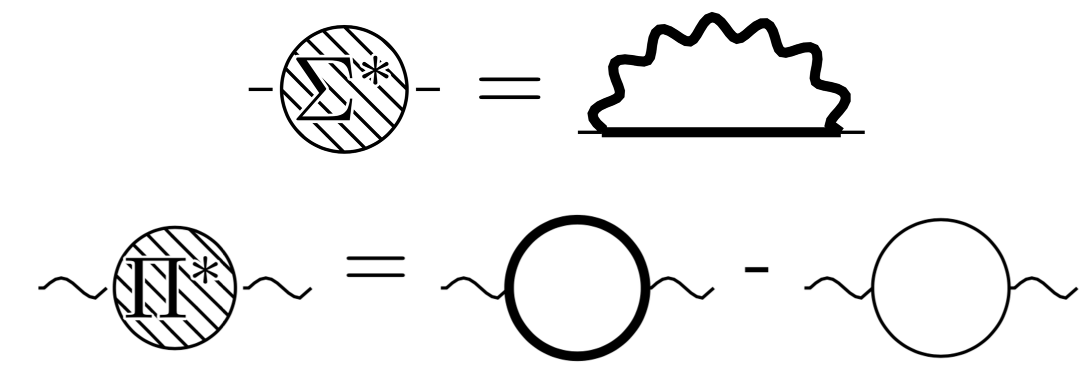
### FIELD-THEORETIC FRAMEWORK

The field-theoretic framework[1] is developed from the path integral formulation of the partition function

$$Z = \int D\vec S D\lambda e^{-\beta H + i\sum_i \lambda_i(|\vec S_i|^2 - 1)},$$

where the unit spin constraint is enforced as delta functions, giving rise to a constraint field $\lambda$. This enables us to integrate over the spins.

We do a saddle point approximation on the constraint field. Then we employ diagrammatic perturbation theory to integrate over the spatial fluctuations about the saddle point. We do an expansion in $1/N$, where $N$ is the number of spin components. This gives rise to two coupled self-consistent equations for the self-energy and polarization




The equations can be solved numerically by iteration, given a saddle point value and an initial guess for the self-energy. The saddle point value then determines the temperature.

### FREE ENERGY

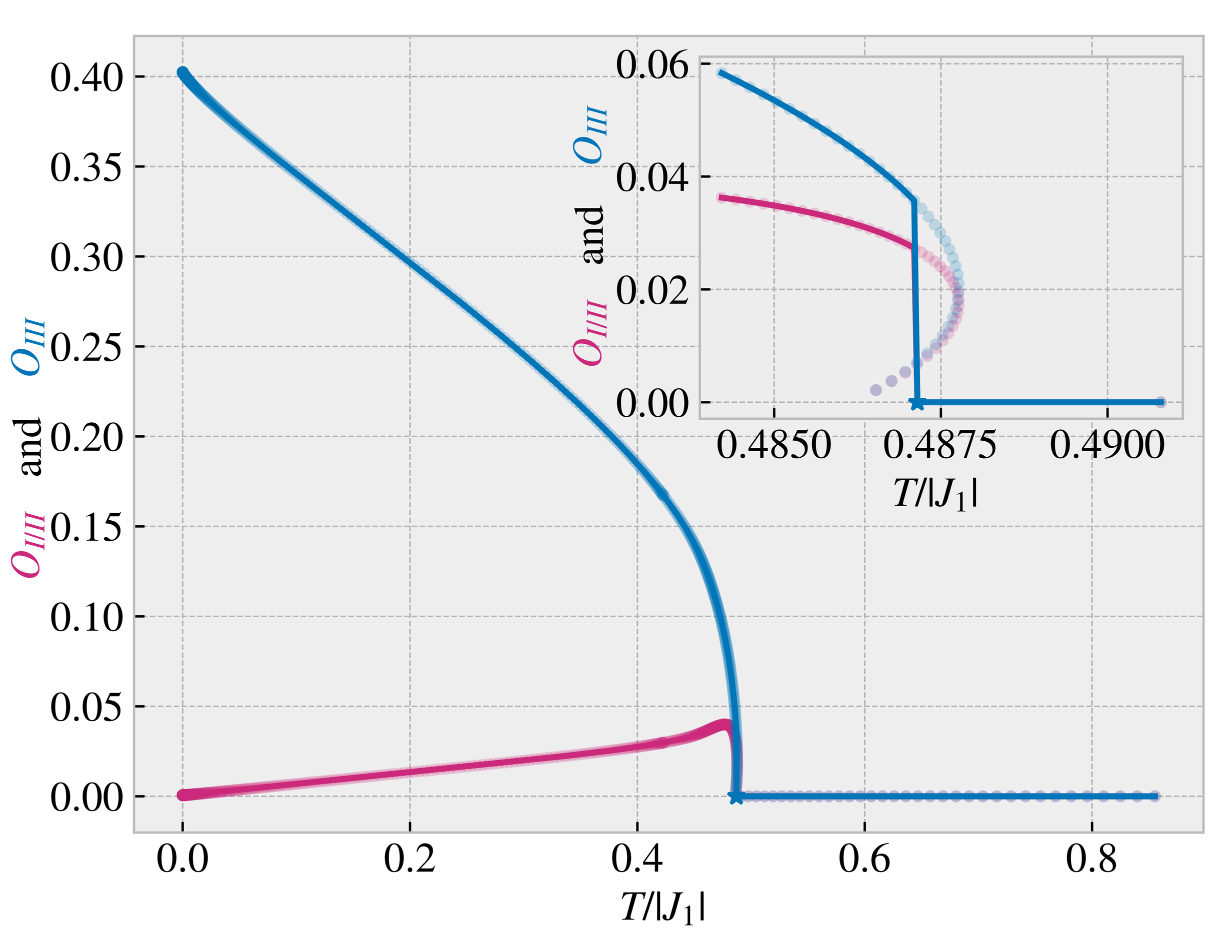
The free energy is calculated numerically, to the same approximation as the self-consistent equations. This helps us determine $T_c$ and the order of the phase transitions.

### ORDER PARAMETERS

We have constructed two order parameters:

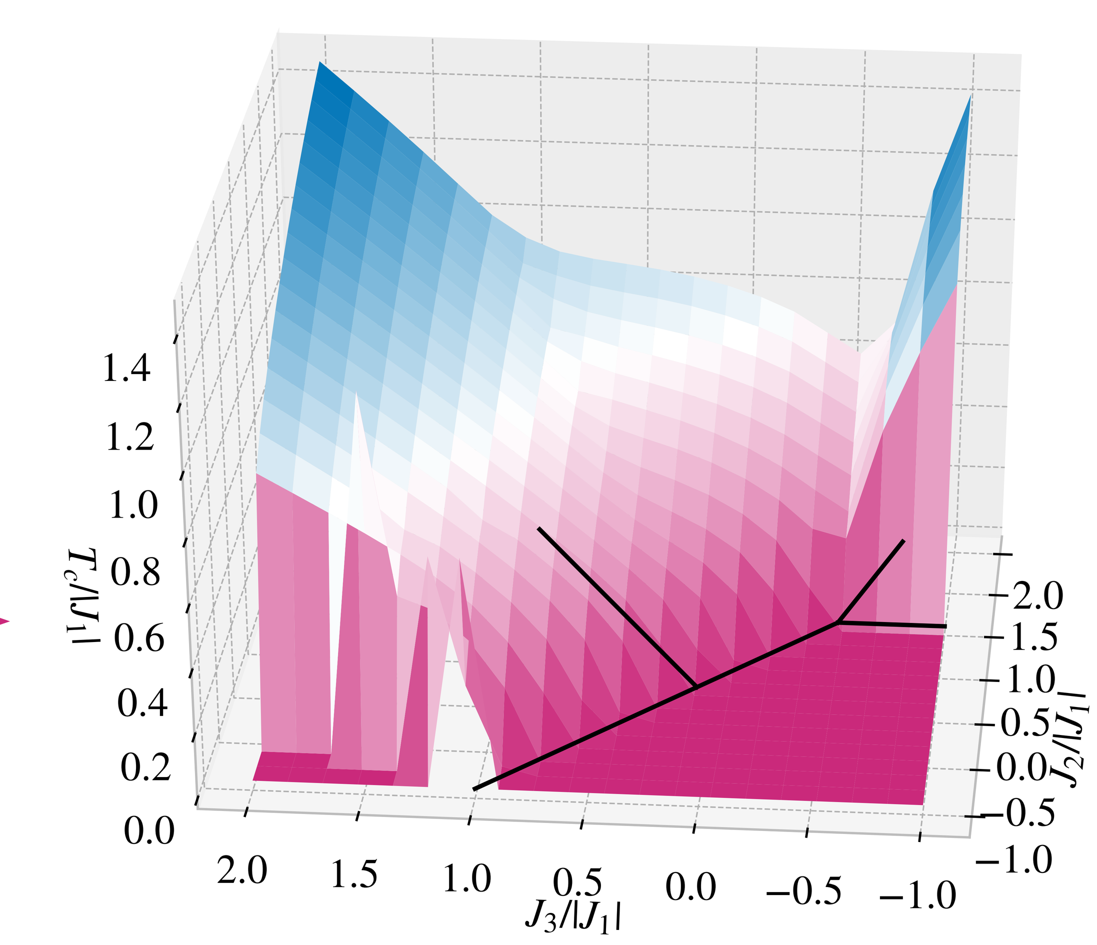
- $O_{I/III}$ breaks rotational symmetry, keeping mirror symmetry about one lattice diagonal.
- $O_{III}$ breaks rotational symmetry, keeping mirror symmetry about one coordinate axis.

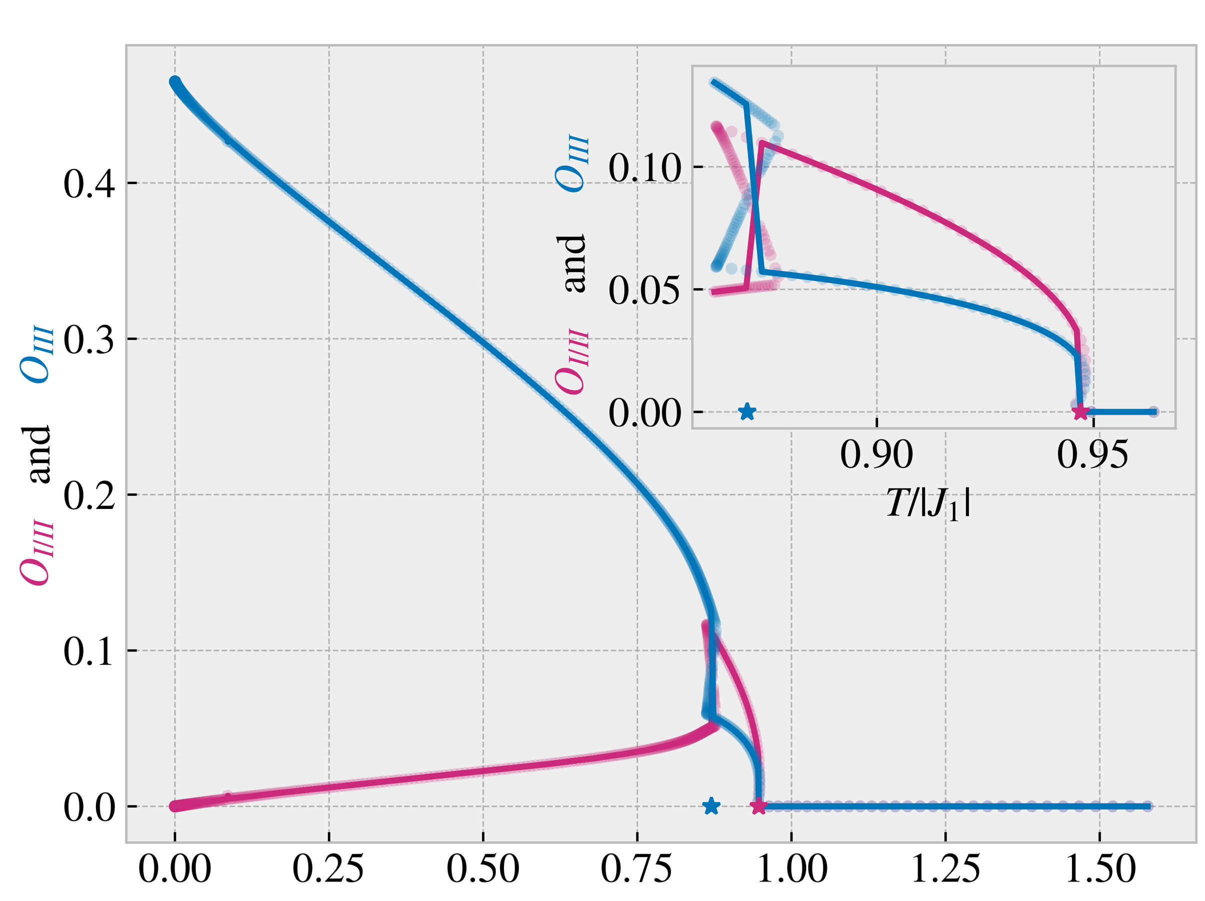
## 3 RESULTS




Order parameters for a 200x200 triangular lattice with $J_1 = -1$, $J_2 = 0.7$, $J_3 = 0.5$ (far away from the region II-III border). The system has one first order phase transition at the temperature marked by the star. The line shows the stable states. Close to a first order phase transition, we get unstable states with a higher free energy. These are shown as the dots that are outside the line.

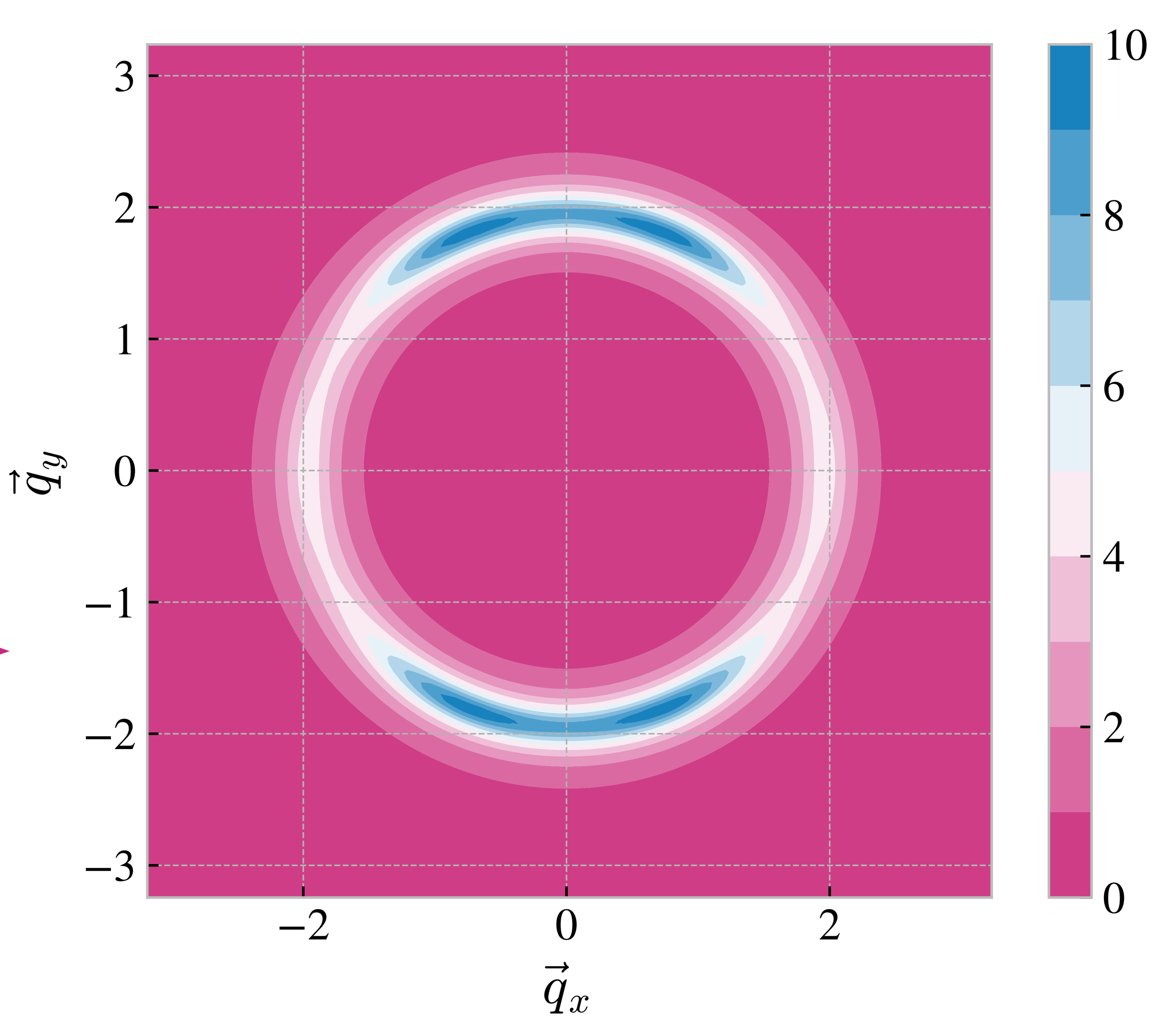
Critical temperatures calculated for the whole exchange coupling space. First order phase transition to a state with broken lattice symmetry[3] is generic in regions I, II and III. $T_c \to 0$ when approaching the FM region.




Close to the region II-III border $J_3 \gtrsim 1/2\, J_2$ the system exhibits a double phase transition on lowering the temperature. First, the rotational symmetry is broken, keeping mirror symmetry about one lattice diagonal. As the temperature is further lowered the system goes over to a single-$\vec q$ phase, which keeps mirror symmetry about one coordinate axis. Here we have used $J_1 = -1$, $J_2 = 2.0$ and $J_3 = 1.05$.

Colour plot of the $\vec q$-space spin correlation function $\langle \vec S_{\vec q}\cdot\vec S_{-\vec q}\rangle$ in the intermediate phase. Rotational symmetry is broken and mirror symmetry is conserved about one lattice diagonal.




### REFERENCES

[1] M. Schecter, O. F. Syljuåsen, and J. Paaske, Phys. Rev. Lett. **119**, 157202 (2017).
[2] C. L. Henley, Phys. Rev. Lett. **62**, 2056 (1989).
[3] R. Tamura and N. Kawashima, J. Phys. Soc. Jpn. **77**, 103002 (2008).

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