Scaling of crater formation - numerical modeling of impact processes on continental targets

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museum für naturkunde

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How do the observed shape and size of an impact crater depend on properties of the impactor...

- *size*
- *composition*
- *velocity*
- *angle*

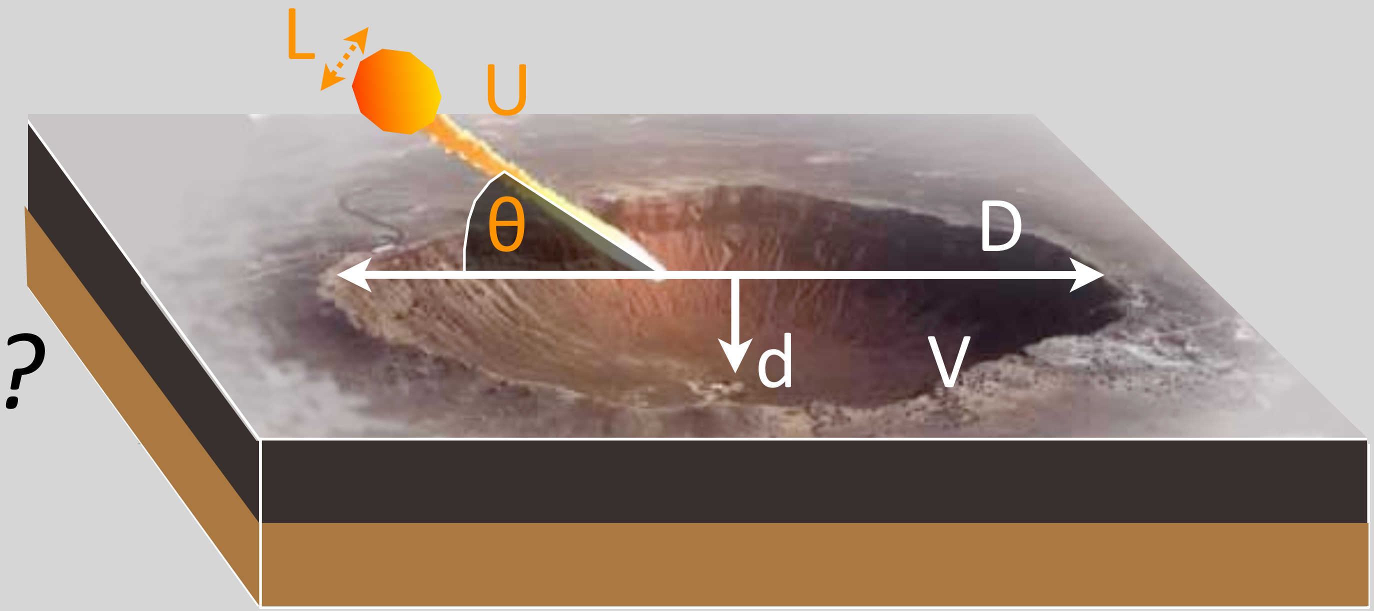


Image from:http://www.meteorcrater.com

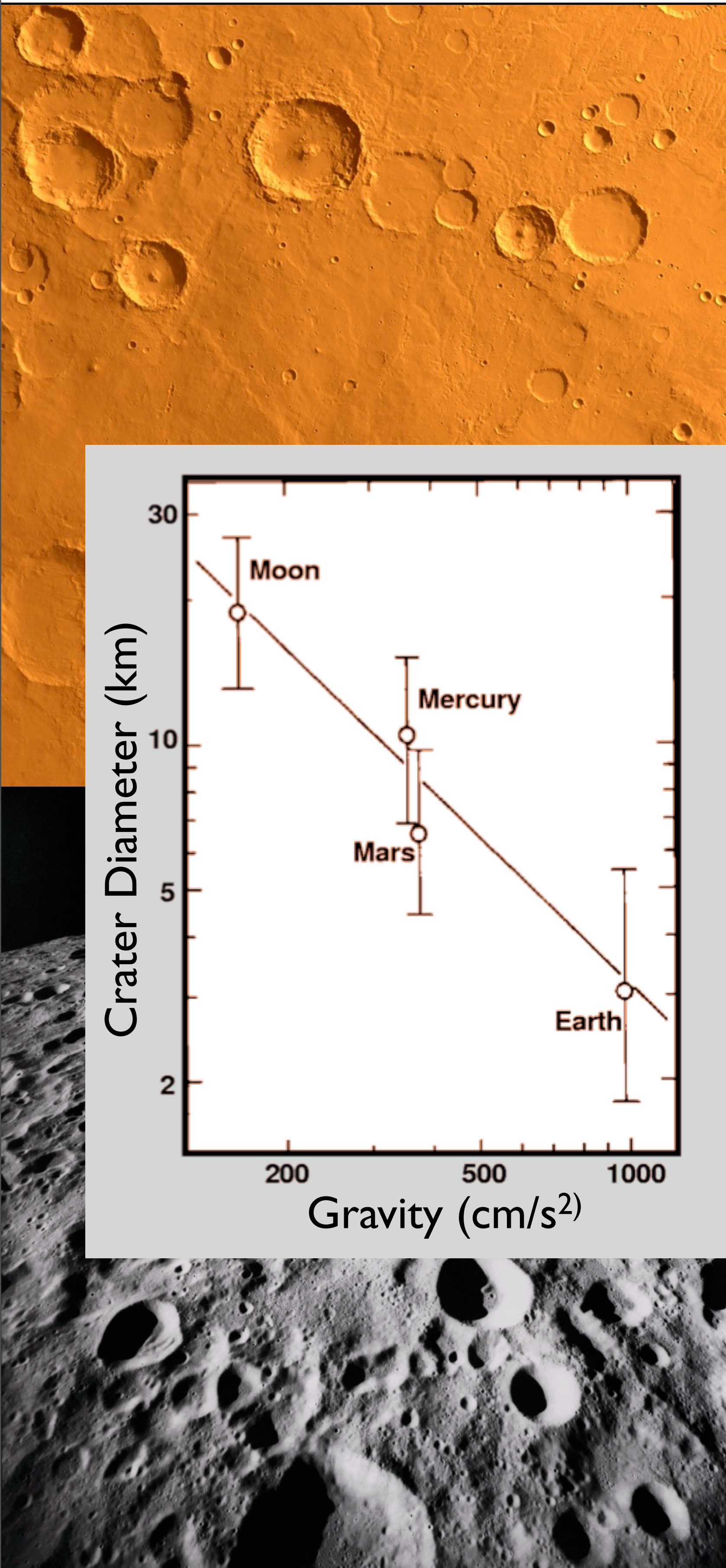
...and the underlying geology?

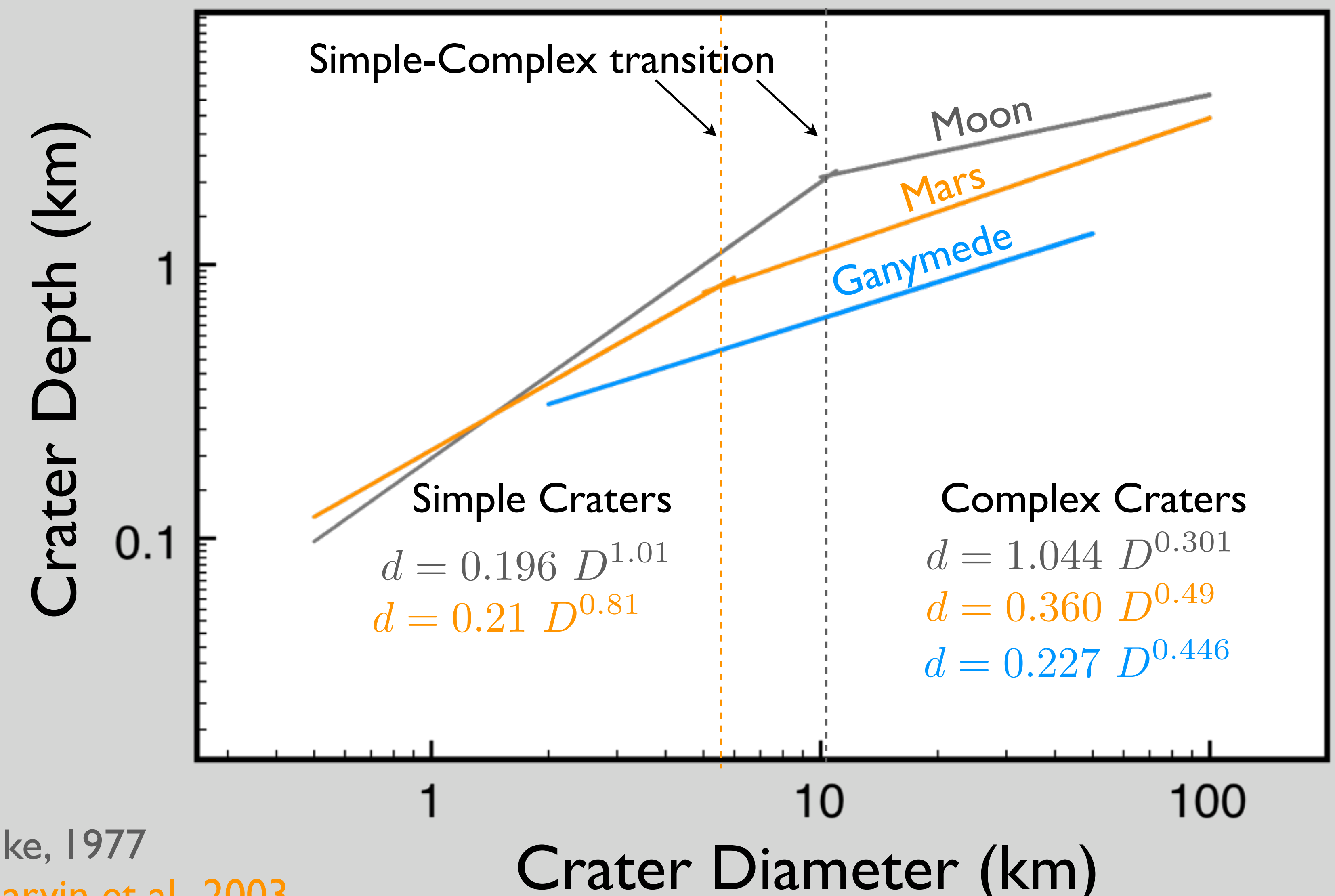
- *lithology*
- *strength*
- *layering*
- *gravity*

→ ***Scaling Laws***

Crater morphometry and morphology varies on different planetary surfaces as a function of size

→ *effect of target properties (e.g. strength) and gravity on crater morphometry*





Pike, 1977
Garvin et al., 2003
Schenk, 1991,2002

Simple question:

What is the size of a crater resulting from the impact of a projectile of a given size, mass, velocity, and angle?

The question can only be answered by

- ➡ *Impact and explosion experiments*
- ➡ *Theoretical solutions*
- ➡ *Numerical modeling*

➡ ***Scaling Laws***

Impactor properties:

m=mass θ=angle
U=velocity L=Diameter
δ=density

Transient crater parameter:

D=diameter
d=depth
V=volume

Target properties:

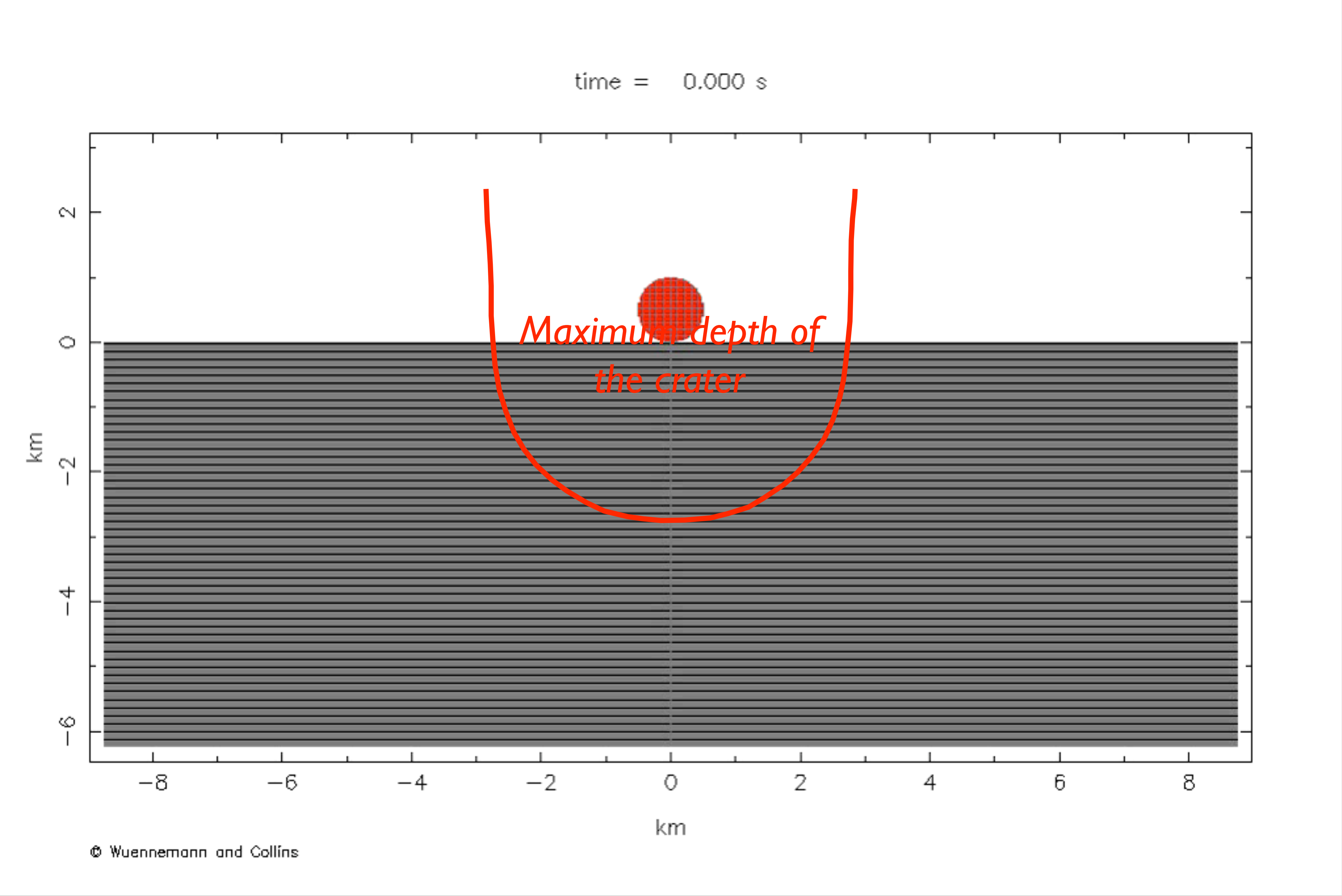
ρ Y =f (Φ,Y_c) α g
density, strength (=friction+cohesion), porosity, gravity

Scaling law:

crater size = ***F [*** *{impactor properties}*, *{target properties}* ***]***

{D,d,V} {δ,m,r,U,θ} {ρ,Y=f(Φ,Y_c),α,g}

*The size of the so-called „**transient crater**“ is the best measure of the energy that was released by an impact event!*



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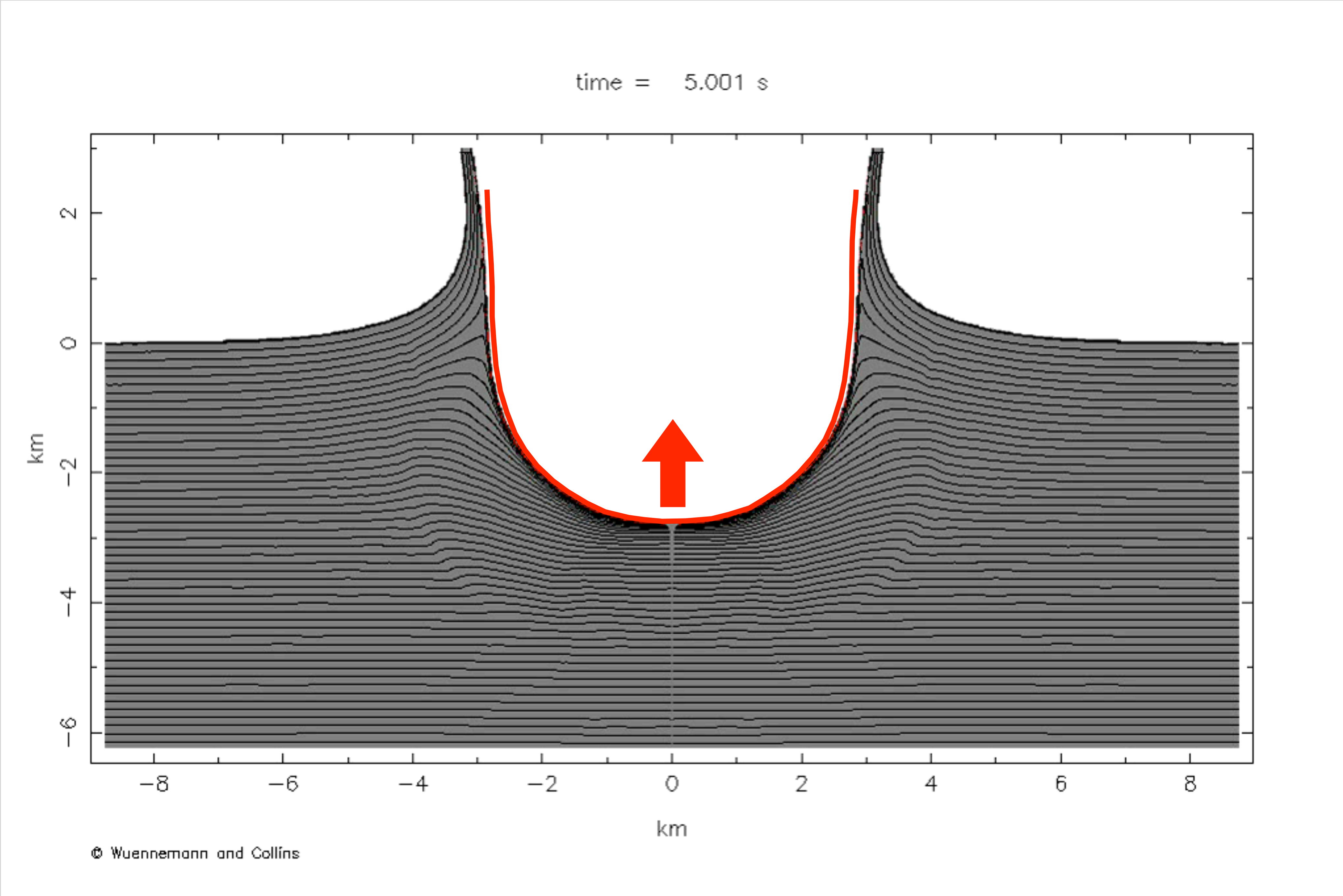

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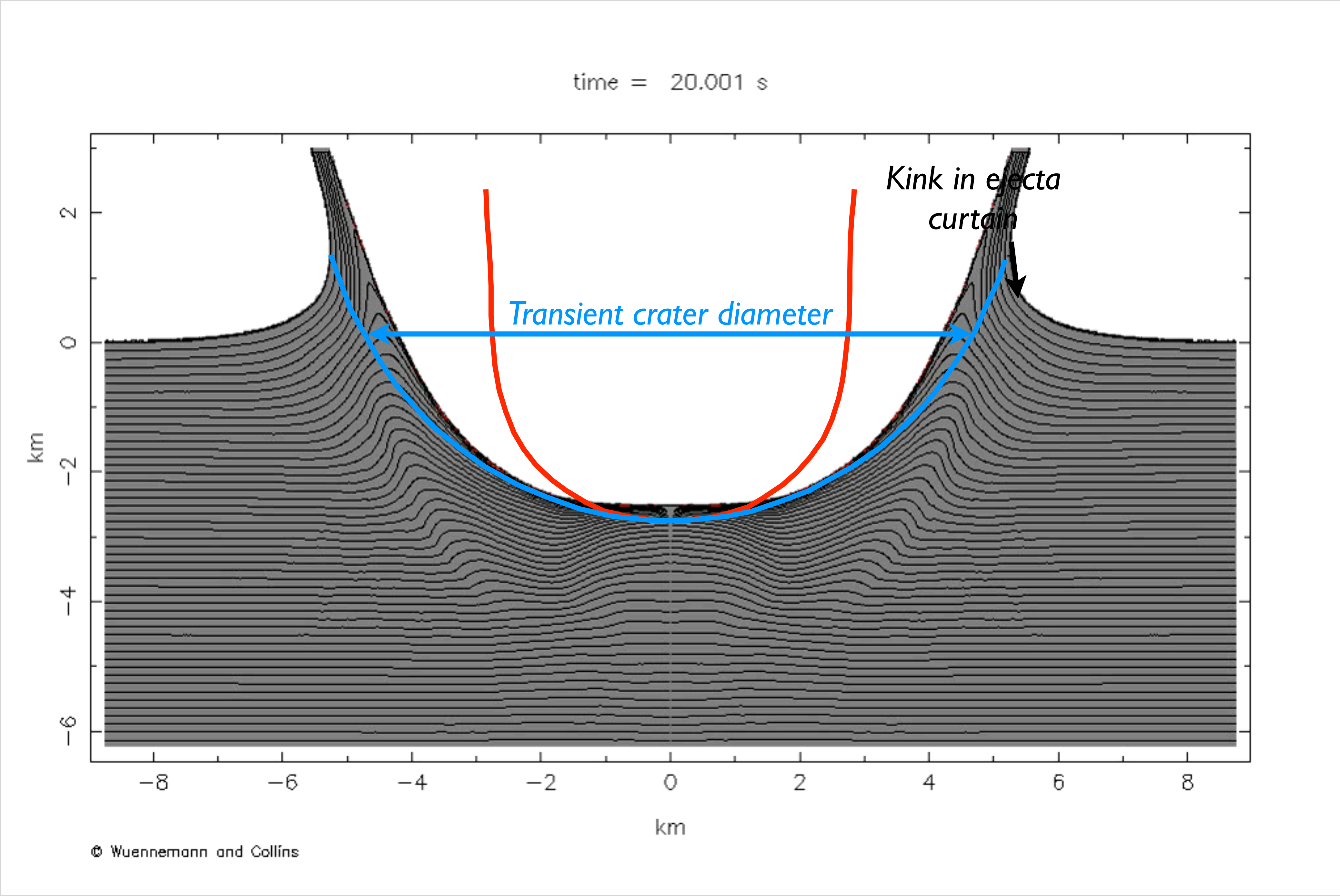

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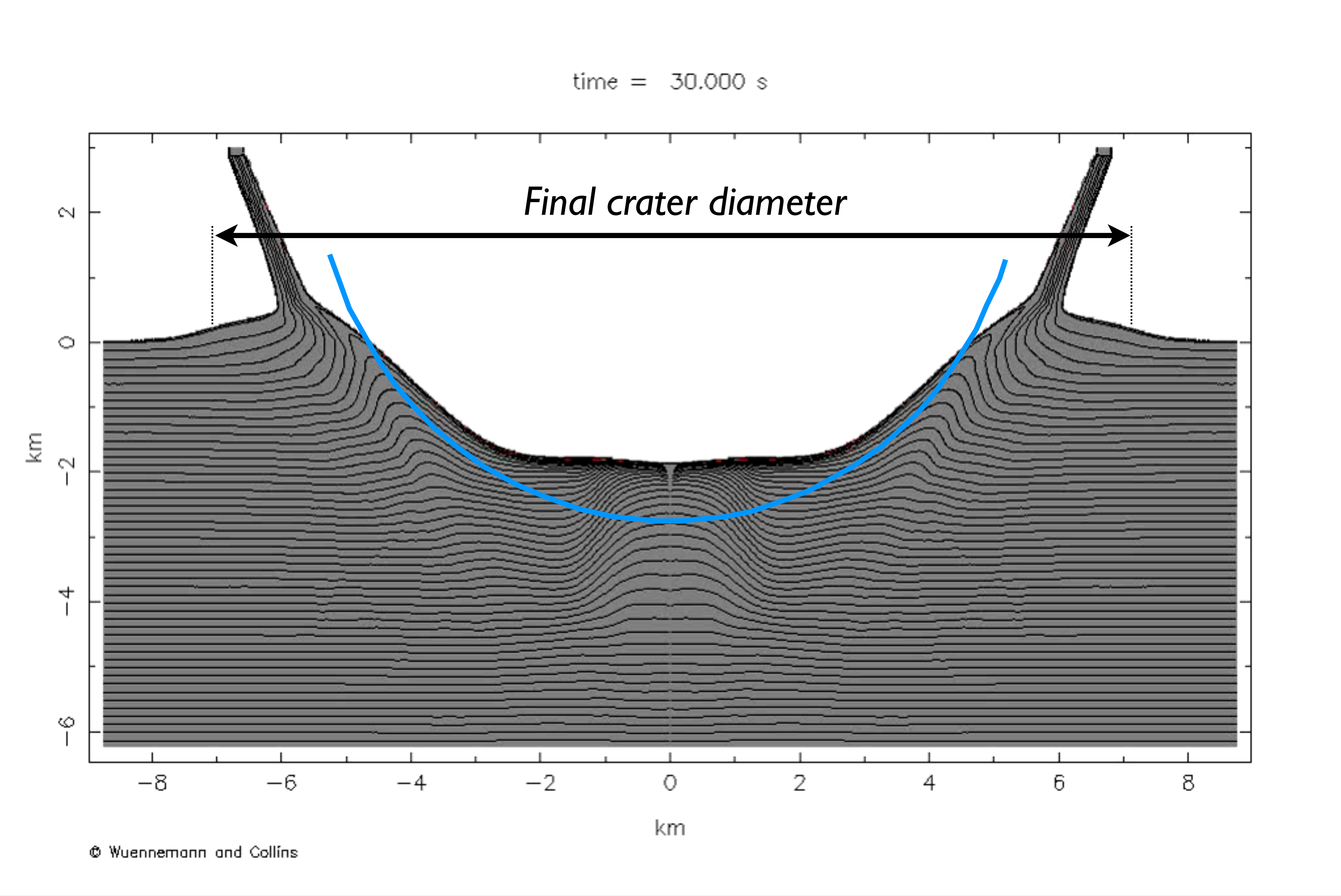
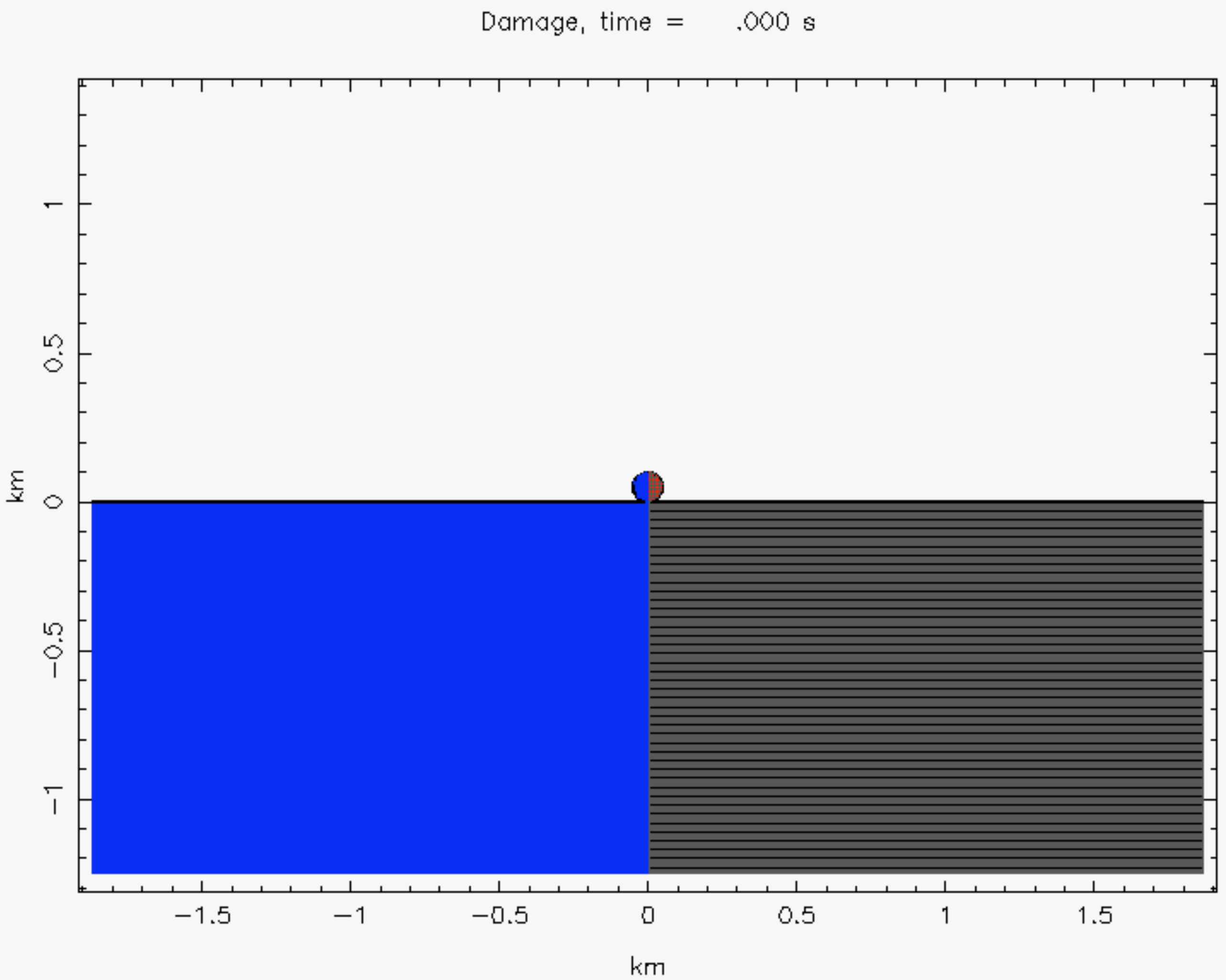
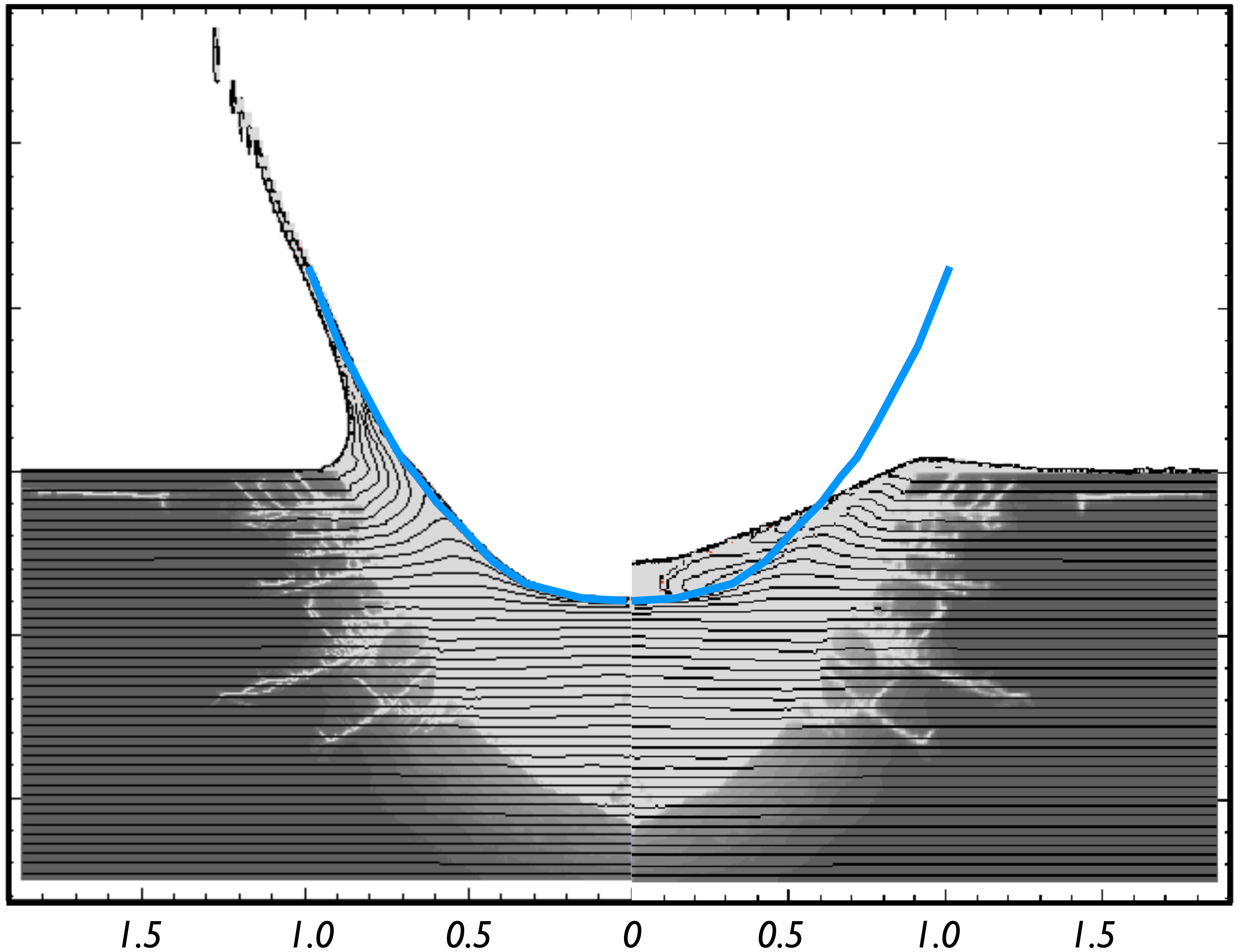




Simple crater - gravity dominated

(craters > 10th of meters on earth and all craters in granular material - no cohesion)

slumping of oversteepened crater rim causes the formation of a breccia lens inside the crater and slightly enlarges crater diameter and reduces crater depth



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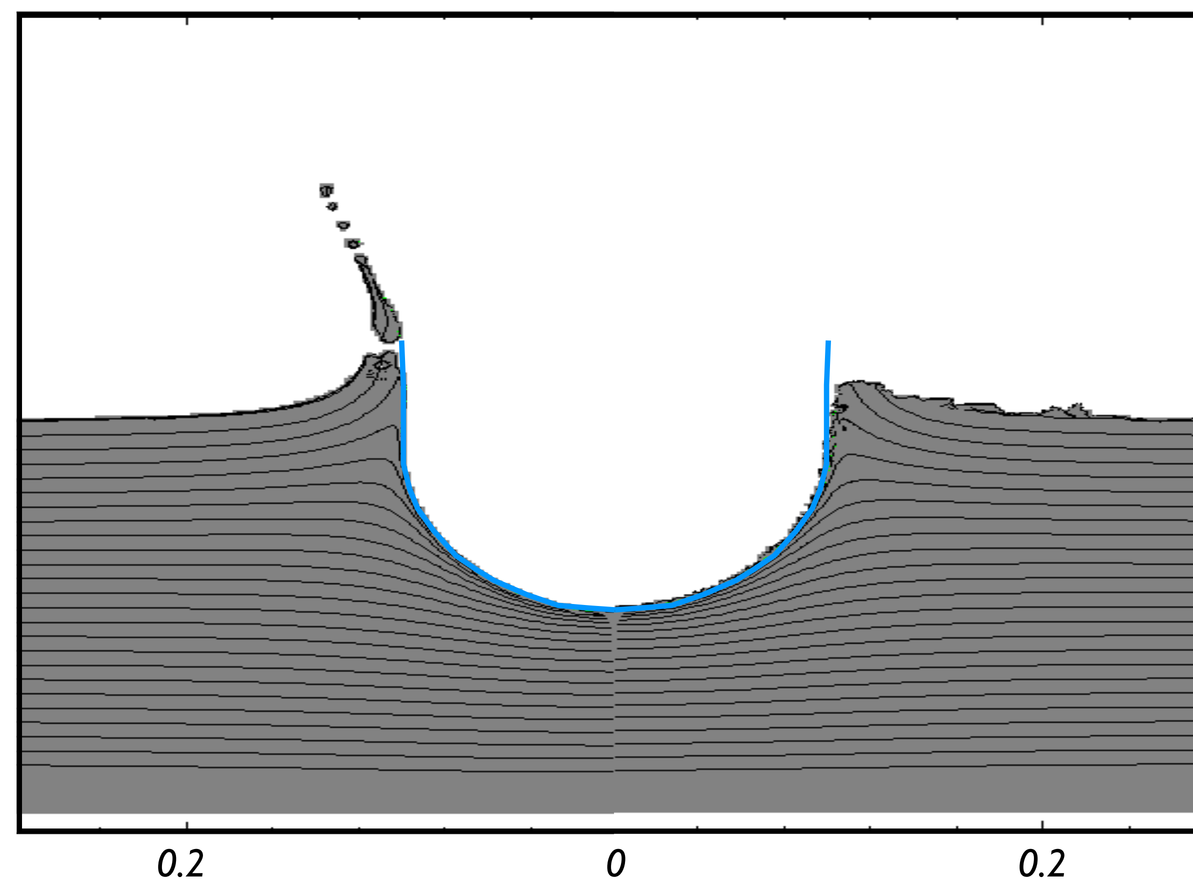
(craters > 10th of meters on earth and all craters in granular material - no cohesion)

slumping of oversteepened crater rim causes the formation of a breccia lens inside the crater and slightly enlarges crater diameter and reduces crater depth

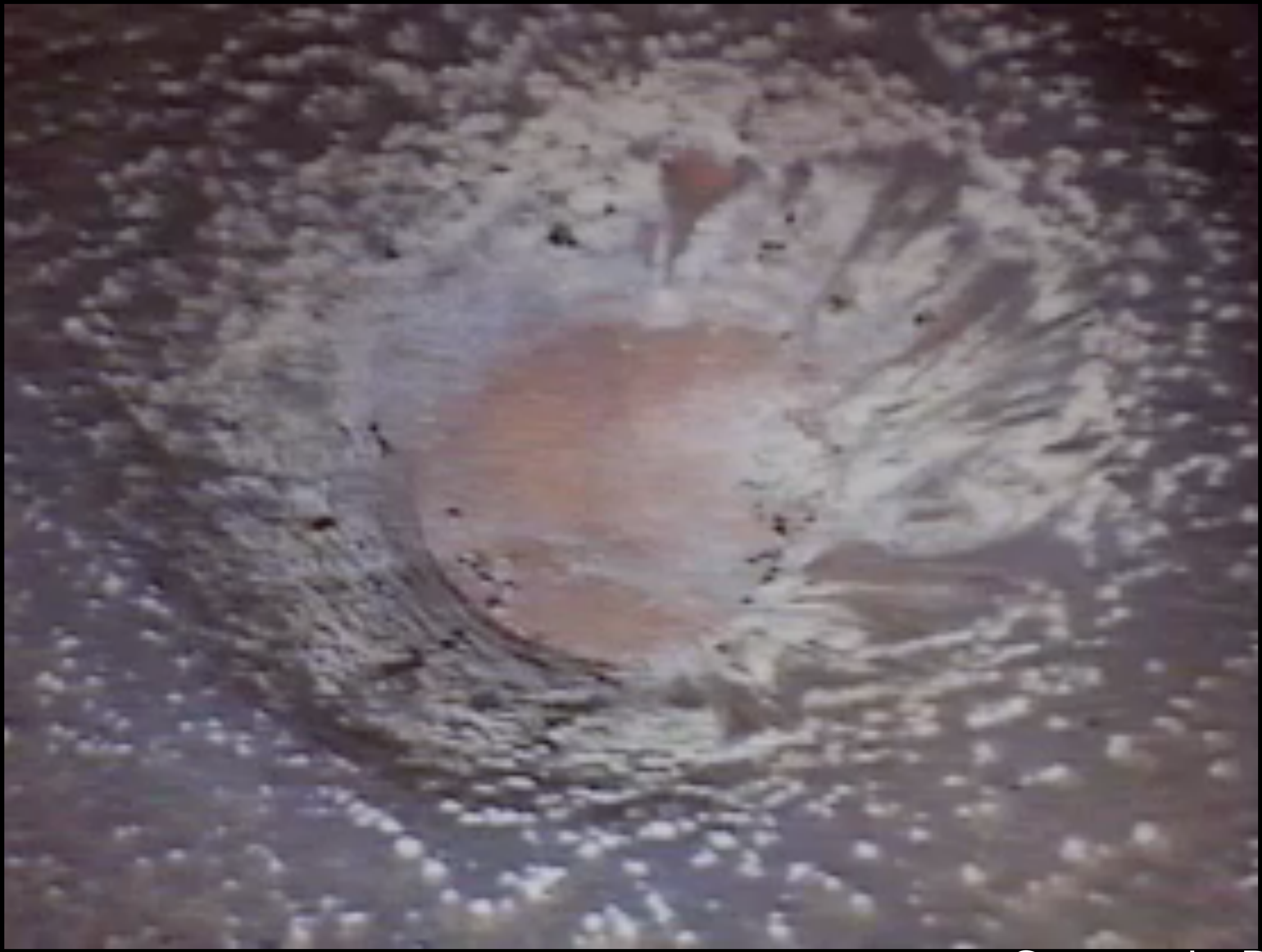
Simple crater - strength dominated

(laboratory-size craters in competent rock)

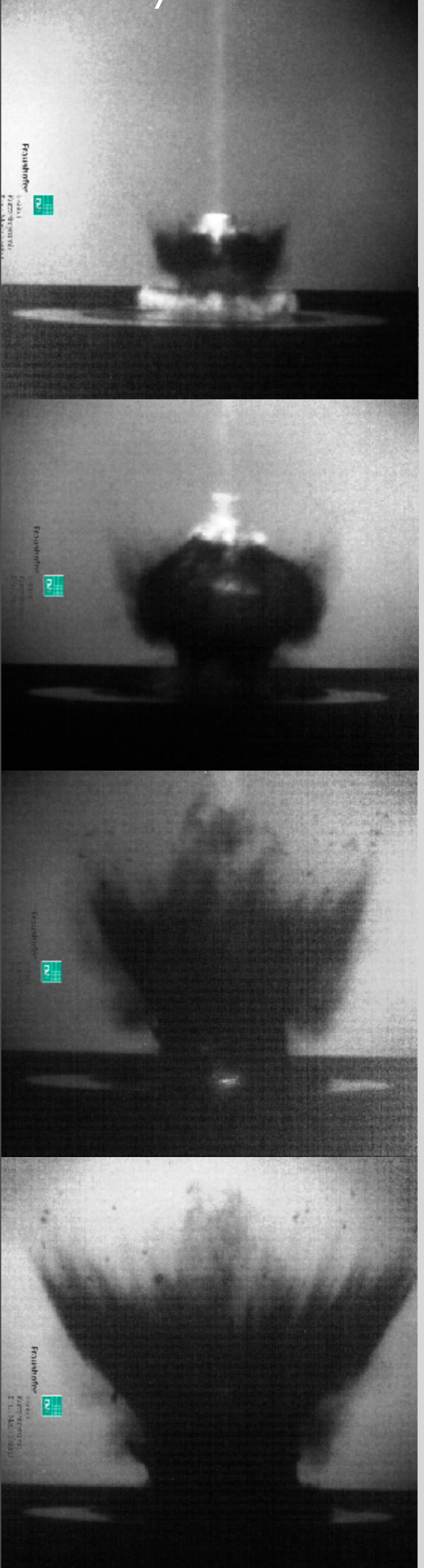
material strength stops crater growth, final crater corresponds almost exactly to the transient crater



Impact experiments in sand (1978-1981), NASA Ames Vertical Gun Facility

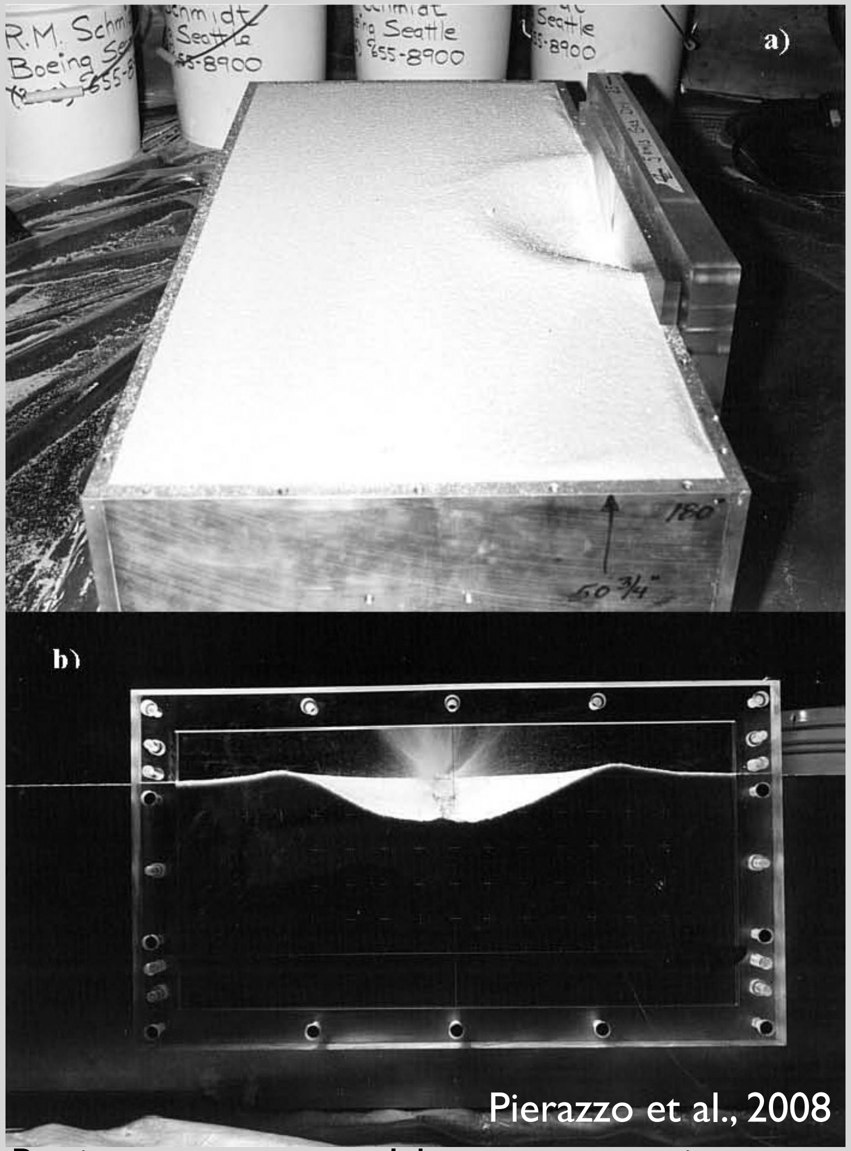


Courtesy by Dieter Stöffler



MEMIN-experiments courtesy EMI

Laboratory experiments - *high technical demands*



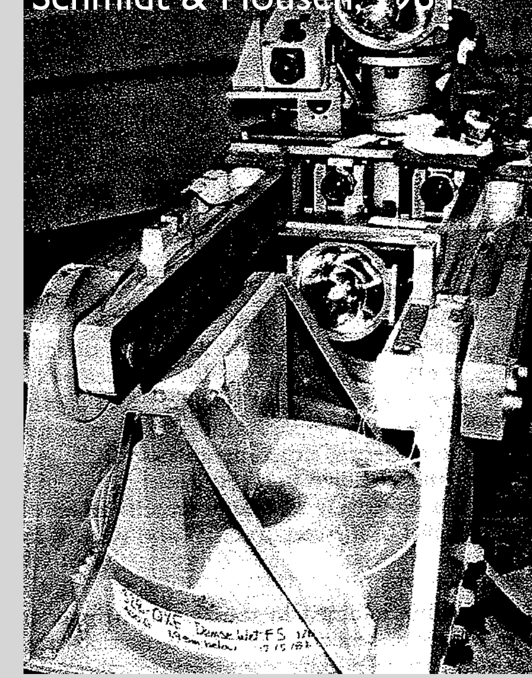
Pierazzo et al., 2008

Boeing quarter space laboratory experiment

Range of material properties is limited and cannot be varied independently

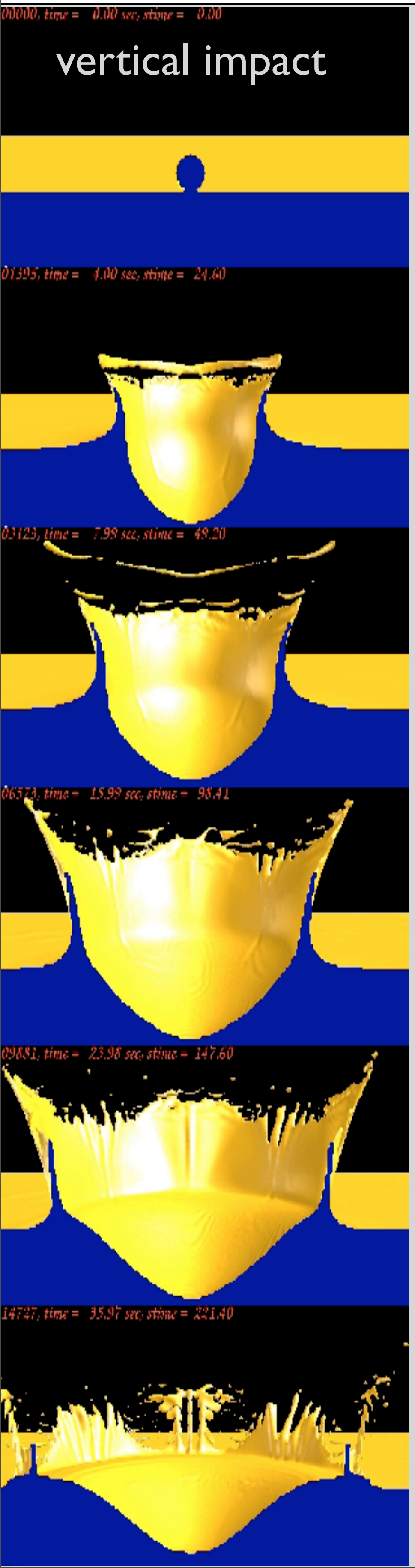
The effect of gravity on crater growth can be only investigated in granular target materials

To vary gravity large centrifuges are required
→ only vertical impacts can be simulated



Schmidt & Housen, 1984

Impact velocity is limited to ~10 km/s at most; however, most experiments are carried out at 2-5 km/s



Numerical modeling - development of appropriate codes

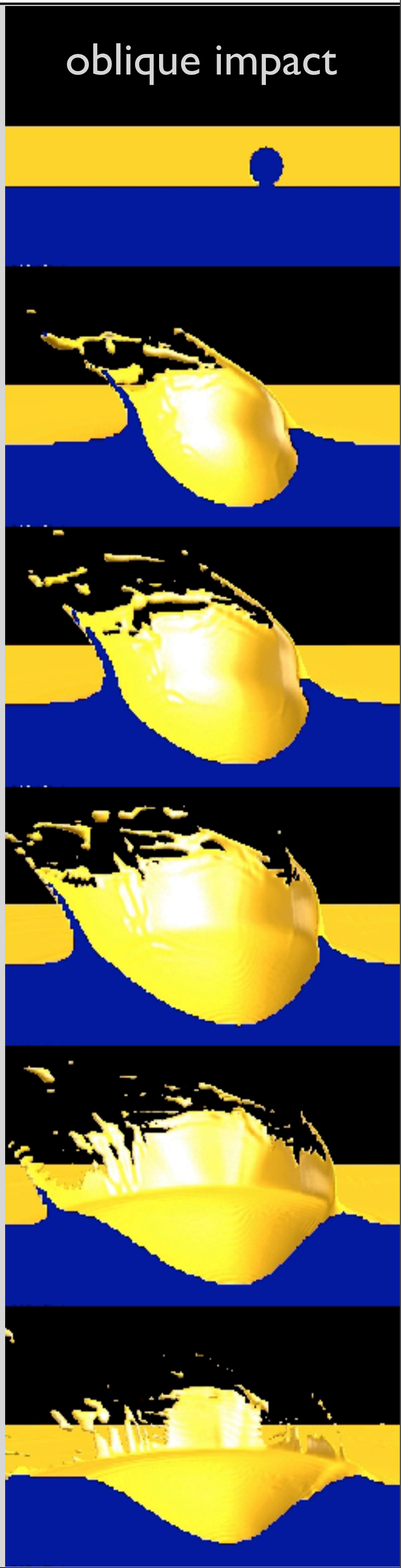
Crater formation can be studied in the course of time

Simulation of crater formation at realistic size-scale, gravity conditions,and arbitrary impact angle and velocity are possible

All parameters can be varied independently, thus the effect of a single material property can be studied separate from other effects

Numerical models are „cheap";
however, simulation of laboratory-size experiments can be computationally very expensive

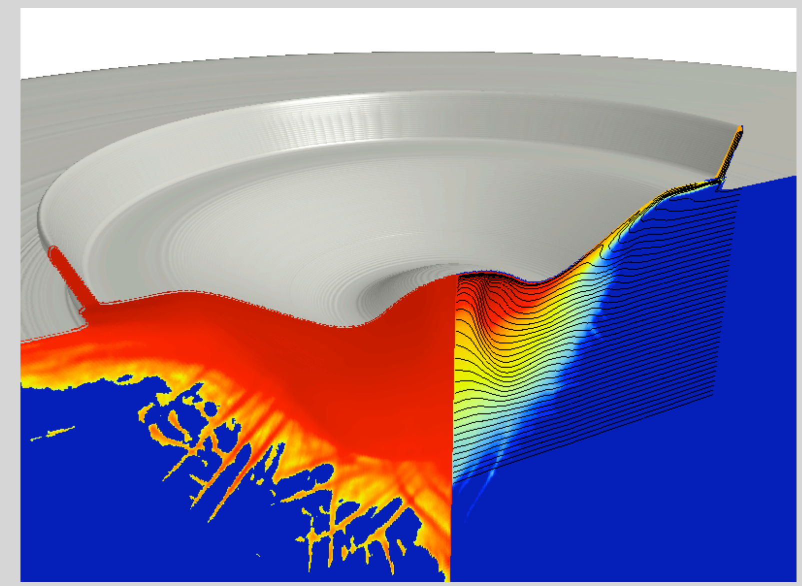
Codes (hydrocodes) must be validated against experiments to assure accuracy



To compare small-scale laboratory experiments, large-scale natural craters and numerical modeling dimensionless measures of crater dimensions are required

Boeing quarter space laboratory experiment, Pierazzo et al., 2008

Lunar complex crater „Euler“, diameter 28 km, depth 2.5 km, © NASA/JPL



iSALE 2D hydrocode simulation of a complex impact crater

Most successful approach in dimesnional analysis uses the so-called „Pi-theorem“[1] which has bee used the develop so-called Pi-group scaling[2] for impact cratering

[1] Buckingham (1914), Bridgman (1949);

[2] e.g. Schmidt (1980), Schmidt & Housen (1987), Holsapple(1993)

Dimensionless ratios for crater morphometry

(Holsapple & Schmidt 1987)

volume V, diameter D, depth d

$$\left.\begin{matrix} V \\ D \\ d \end{matrix}\right\} = F(\underbrace{U, \rho, \delta, Y, g, m}_{\text{6 variables}})$$

dimensionless ratios

$$\left.\begin{matrix} \pi_V \\ \pi_D \\ \pi_d \end{matrix}\right\} = F'(\underbrace{\pi_2, \pi_3, \pi_4}_{\text{3 variables}})$$

dimensionless geometric parameters

$$\pi_V = \frac{\rho V}{m}$$

crater efficiency

$$\pi_D = D\left(\frac{\rho}{m}\right)^{1/3}$$

$$\pi_d = d\left(\frac{\rho}{m}\right)^{1/3}$$

gravity-scaled size

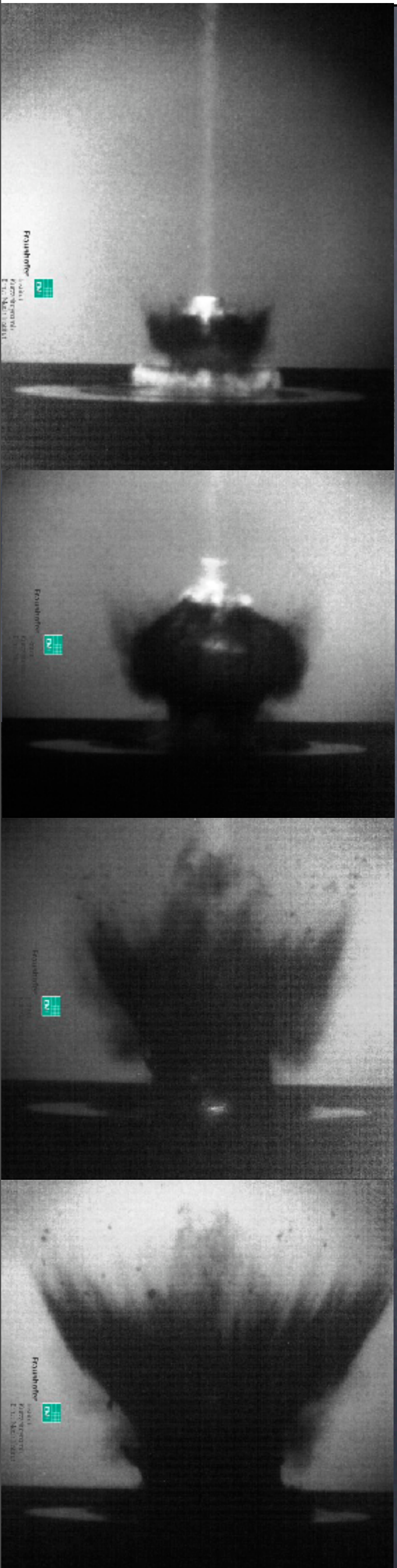
$$\pi_2 = 1.61\frac{gL}{U^2}$$

strength-scaled size

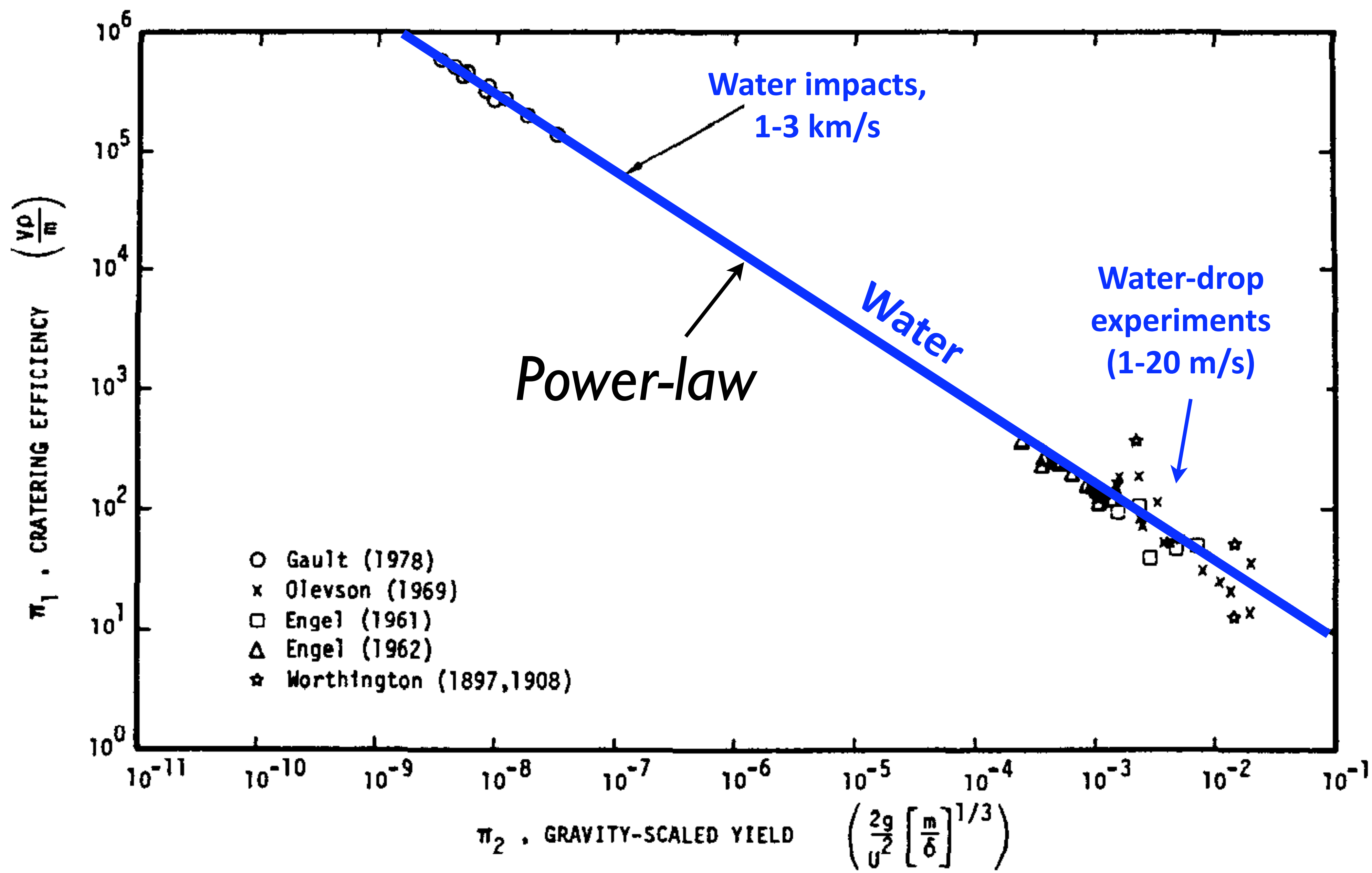
$$\pi_3 = \frac{Y}{\delta U^2}$$

other scaling parameters

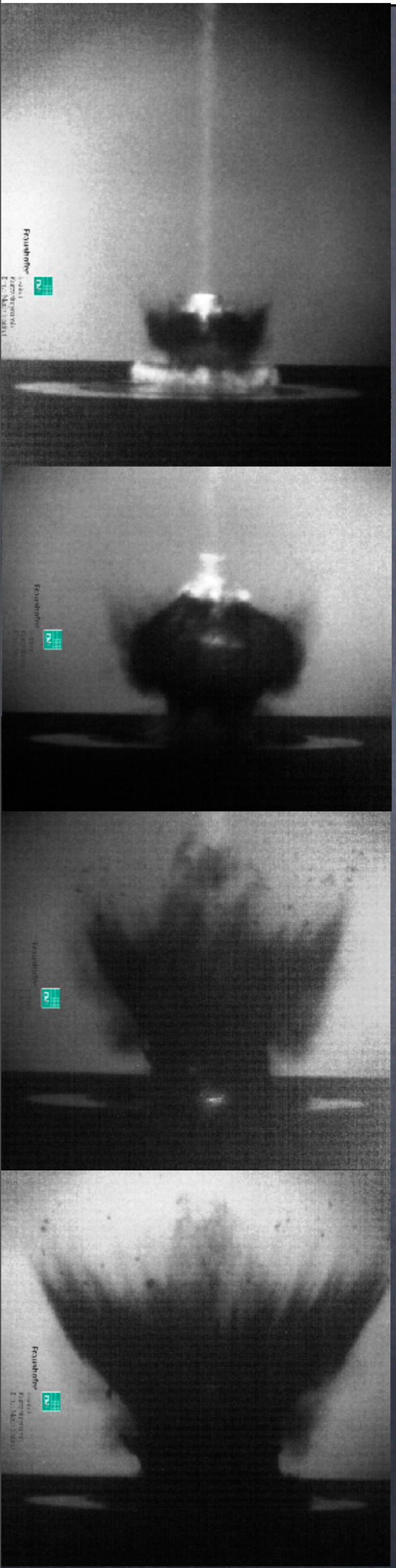
$$\pi_4 = \frac{\rho}{\delta}$$



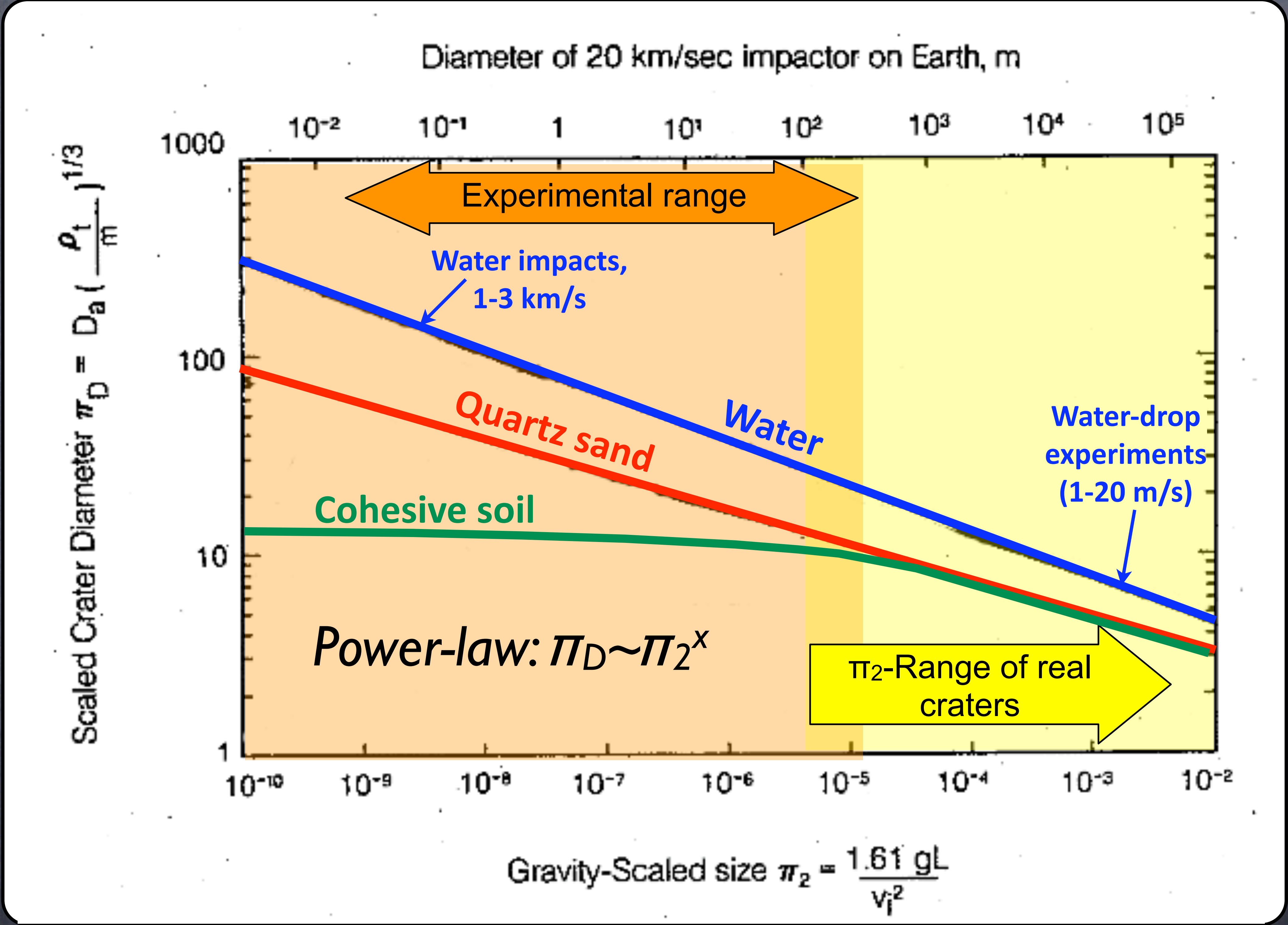
Analog impact experiments in water



after Holsapple (1993)

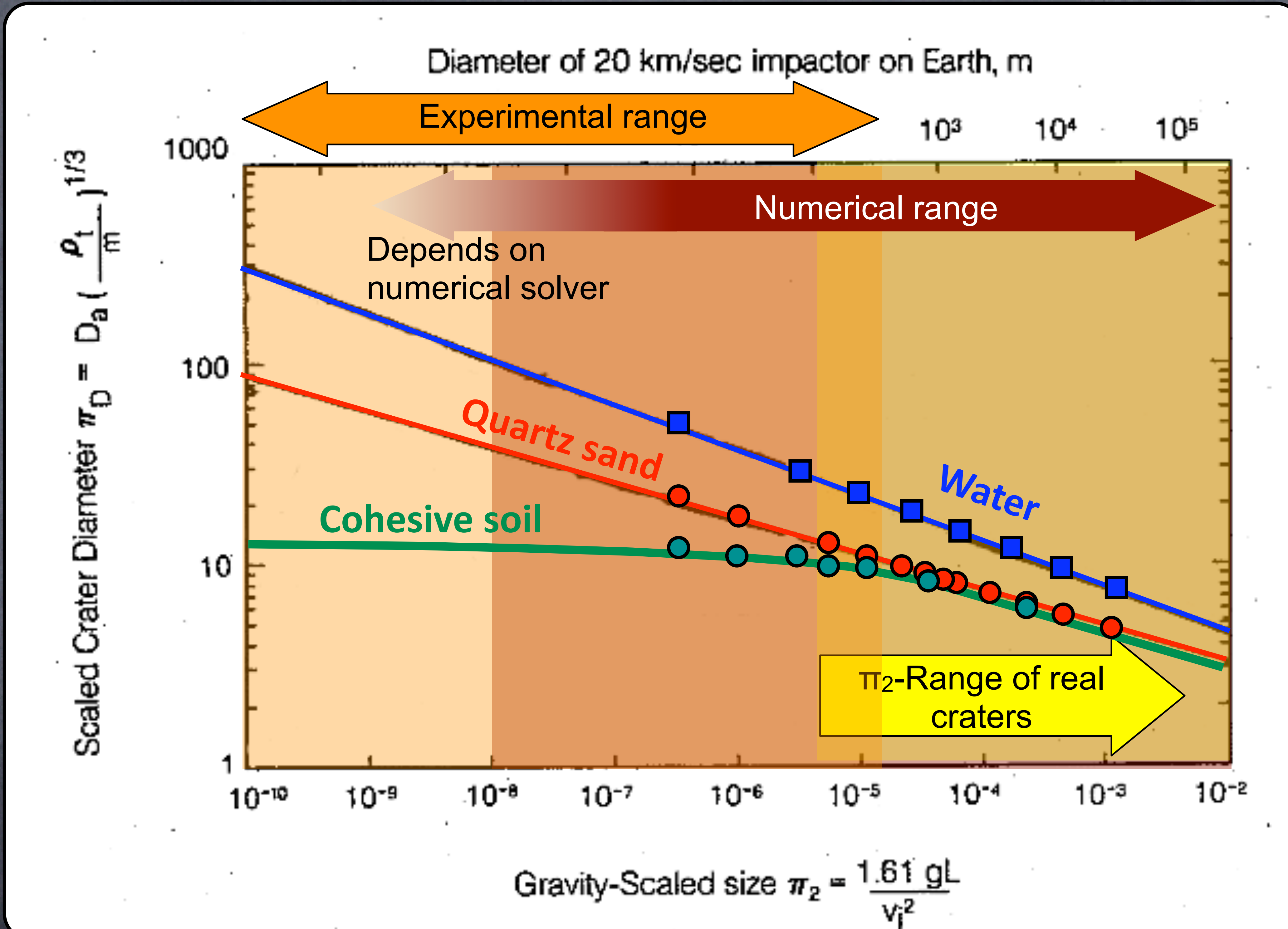


Centrifuge experiments in different materials



Schmidt (1980), Schmidt & Housen (1987), Holsapple (1993), Gault & Wedekind (1977), Gault & Sonett (1982)

Numerical experiments in different materials

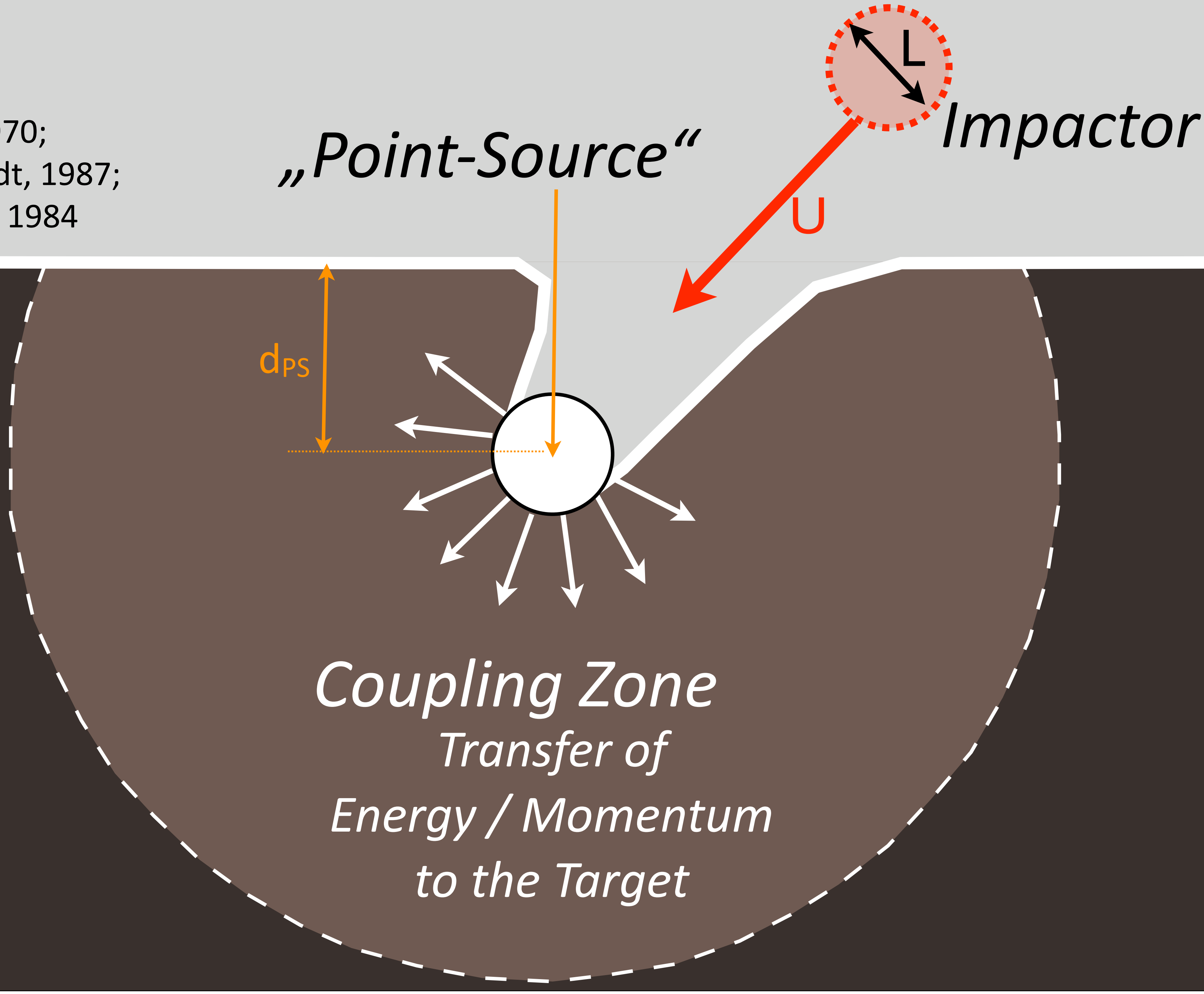


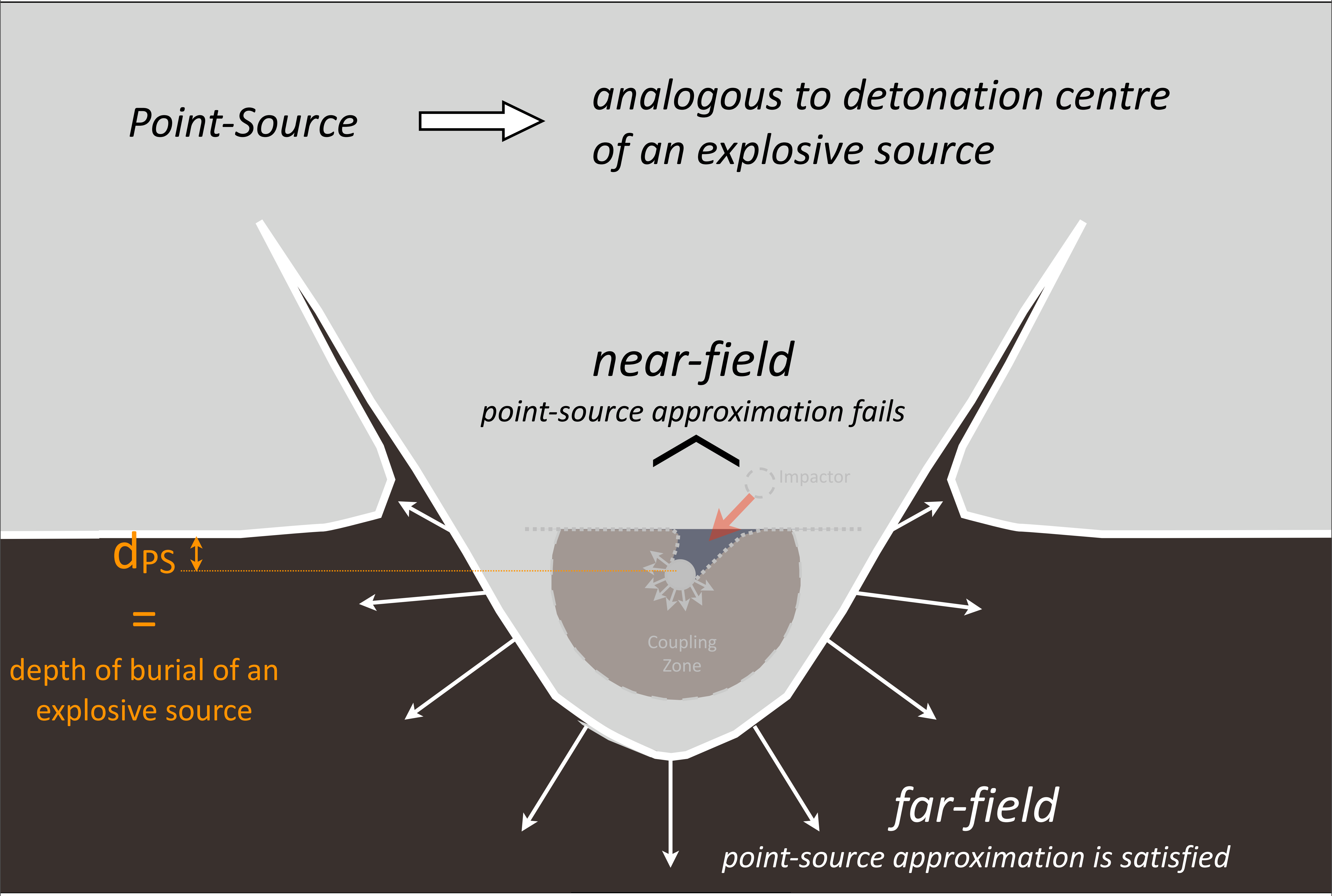
Wünnemann et al., 2007, 2008

Theoretical solution:

Impacts can be approximated by a „point source“.

Dienes & Walsh, 1970;
Holsapple & Schmidt, 1987;
Schmidt & Housen, 1984




Point-Source
analogous to detonation centre
of an explosive source
near-field
point-source approximation fails
Impactor
Coupling
Zone
d_{PS}
=
depth of burial of an
explosive source
far-field
point-source approximation is satisfied

The coupling parameter

*At **sufficient distance** point-source approximation holds true and energy/momentum transfer by the impactor can be described by a single parameter, the **coupling parameter***

$$C = L \cdot U^{\mu} \cdot \delta^{\nu}$$

(Holsapple, 1993)

If the size of an event scales with energy

$$C \sim E^{1/3} = \left[\frac{1}{2} m U^2\right]^{1/3} = \frac{1}{12}\pi \cdot \underbrace{L^3 \cdot U^{2/3} \cdot \delta^{1/3}}_{=C}$$

If the size of an event scales with momentum

$$C \sim M^{1/3} = [mU]^{1/3} = \frac{1}{6}\pi \cdot \underbrace{L^3 \cdot U^{1/3} \cdot \delta^{1/3}}_{=C}$$

***The coupling parameter** is the sole measure of the coupling of energy and momentum of the impactor into the target*

(Holsapple, 1993)

$$C = L \cdot U^{\mu} \cdot \delta^{\nu}$$

energy-momentum
2/3 - 1/3

Implications and physical meaning:

- *Impact events with same C produce same-sized craters*
- *All far-field processes (e.g. crater growth) are related by power-laws*
- *The velocity exponent μ depends on dissipative target properties, e.g. porosity, friction, ...?*
- *The angle of impact is not considered in the coupling parameter*

point-source approximation enables derivation of power-laws to scale crater size

(and other processes, e.g. ejection, crater growth, ...)

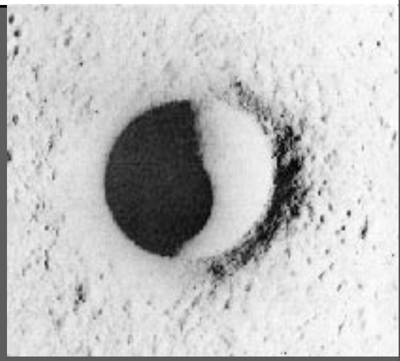
point-source approximation applies to both regimes, gravity and strength dominated crater formation

Note, the coupling zone has to be „small" in comparison to the crater size so that point-source approximation holds true

→projectile small in comparsion to the size of transient crater (at least 2-3 times smaller in diameter; Holsapple, 1993)

This is satisfied for most hypervelocity impacts with some restrictions to very large impacts ...

Scaling-laws for gravity regime



applies to almost all crater structures on planetary surfaces; critical size depends on material strength and gravity

crater efficiency

$$\pi_V = K_V \; (\pi_2)^{-\frac{3\mu}{2+\mu}}$$

crater diameter

$$\pi_D = K_D \; (\pi_2)^{-\frac{\mu}{2+\mu}} \quad = -\beta$$

$$\pi_V = \frac{\rho V}{m} \qquad \pi_D = D \left(\frac{\rho}{m}\right)^{1/3} \qquad \pi_2 = 1.61 \frac{gL}{U^2} \qquad \pi_4 = \frac{\rho}{\delta}$$

(for ρ=δ)

μ, K_D, K_V need to be determined in laboratory or numerical experiments and depend on material properties (friction, porosity ...).

Note, μ and ν are the same parameters for strength and gravity regime!

(e.g. Housen & Holsapple, 2003)

Scaling-laws for strength regime

applies to laboratory-sized craters in cohesive material and very small natural craters in competent rock

crater efficiency

$$\pi_V = Q_V \; (\pi_3)^{-\frac{3\mu}{2}} \; (\pi_4)^{1-3\nu+\frac{3\mu}{2}}$$

crater diameter

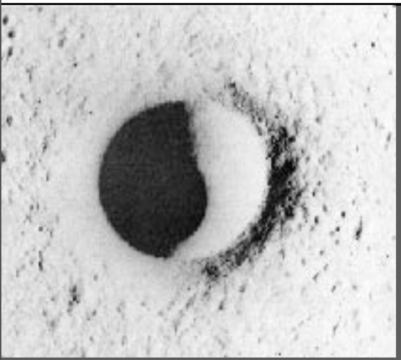
$$\pi_D = Q_D \; (\pi_3)^{-\frac{\mu}{2}} \; (\pi_4)^{\frac{1}{3}-\nu+\frac{\mu}{2}}$$

$$\pi_V = \frac{\rho V}{m} \quad \pi_D = D\left(\frac{\rho}{m}\right)^{1/3} \quad \pi_3 = \frac{Y}{\delta U^2} \quad \pi_4 = \frac{\rho}{\delta}$$

(for ρ=δ)

μ, Q_D, Q_V need to be determined in laboratory or numerical experiments and depend on material properties (friction, porosity ...).

(e.g. Housen & Holsapple, 2003)



It is possible to formulate one equation that satisfies both regimes, strength and gravity

(Holsapple & Housen, 2007)

crater diameter

$$\frac{D}{L} = J_D \left[\left(1.61 \frac{gL}{U^2}\right) \left(\frac{\rho}{\delta}\right)^{\frac{2\nu}{\mu}} + \left(\frac{Y}{\rho U^2}\right)^{\frac{2+\mu}{2}} \left(\frac{\rho}{\delta}\right)^{\frac{\nu(2+\mu)}{\mu}} \right]^{-\frac{\mu}{2+\mu}}$$

$$\Rightarrow \quad \frac{D}{L} = J_D \left[\pi_2 \pi_4^{\frac{2\nu}{\mu}} + \underbrace{\pi_3^{\frac{2+\mu}{2}} \pi_4^{\frac{\nu(2+\mu)}{\mu}}} \right]^{-\frac{\mu}{2+\mu}}$$

=0, If strength negligible (Y=0)

$$\Rightarrow \quad \frac{D}{L} = J_D \left[\pi_2^{-\frac{\mu}{2+\mu}} \pi_4^{-\frac{2\nu}{2+\mu}} \right]$$

multiply both sides of the equation with $\left(\frac{\rho}{\frac{1}{6}\Pi\delta}\right)$

$$\Rightarrow \quad \frac{D}{L} \left(\frac{\rho}{\frac{1}{6}\Pi\delta}\right) = J_D \left(\frac{6}{\Pi}\right)^{\frac{1}{3}} \left[\pi_2^{-\frac{\mu}{2+\mu}} \pi_4^{-\frac{2+\mu-6\nu}{3(2+\mu)}} \right]$$

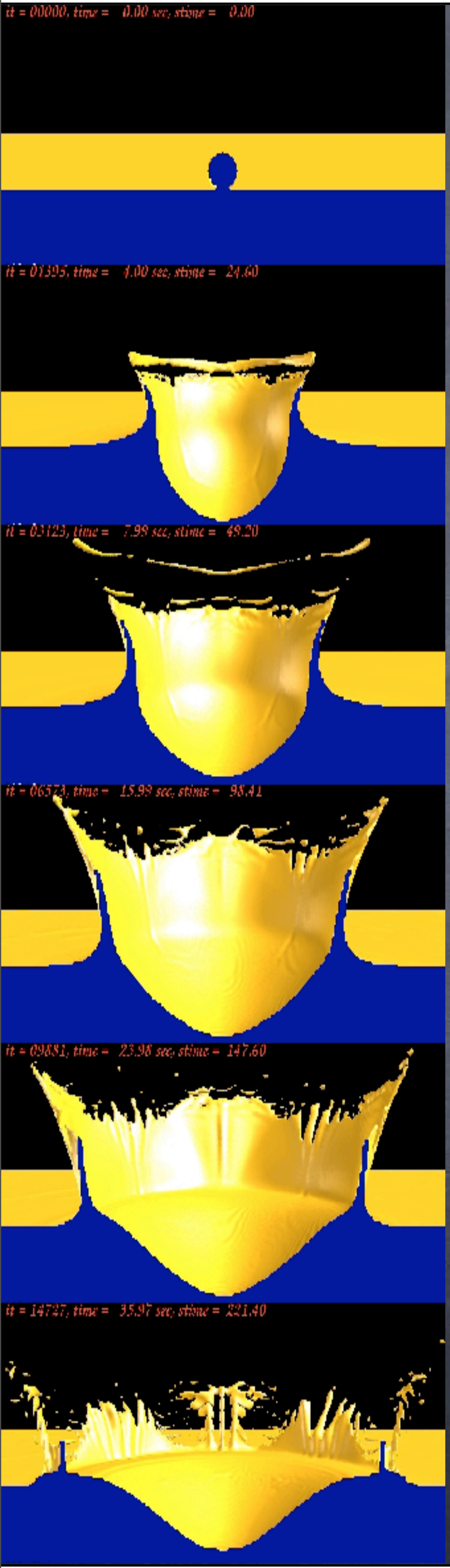
↑ *assuming spherical projectile!*

$$\Rightarrow \quad \pi_D = K_D(\pi_2)^{-\frac{\mu}{2+\mu}} \underbrace{(\pi_4)^{-\frac{2+\mu-6\nu}{3(2+\mu)}}}$$

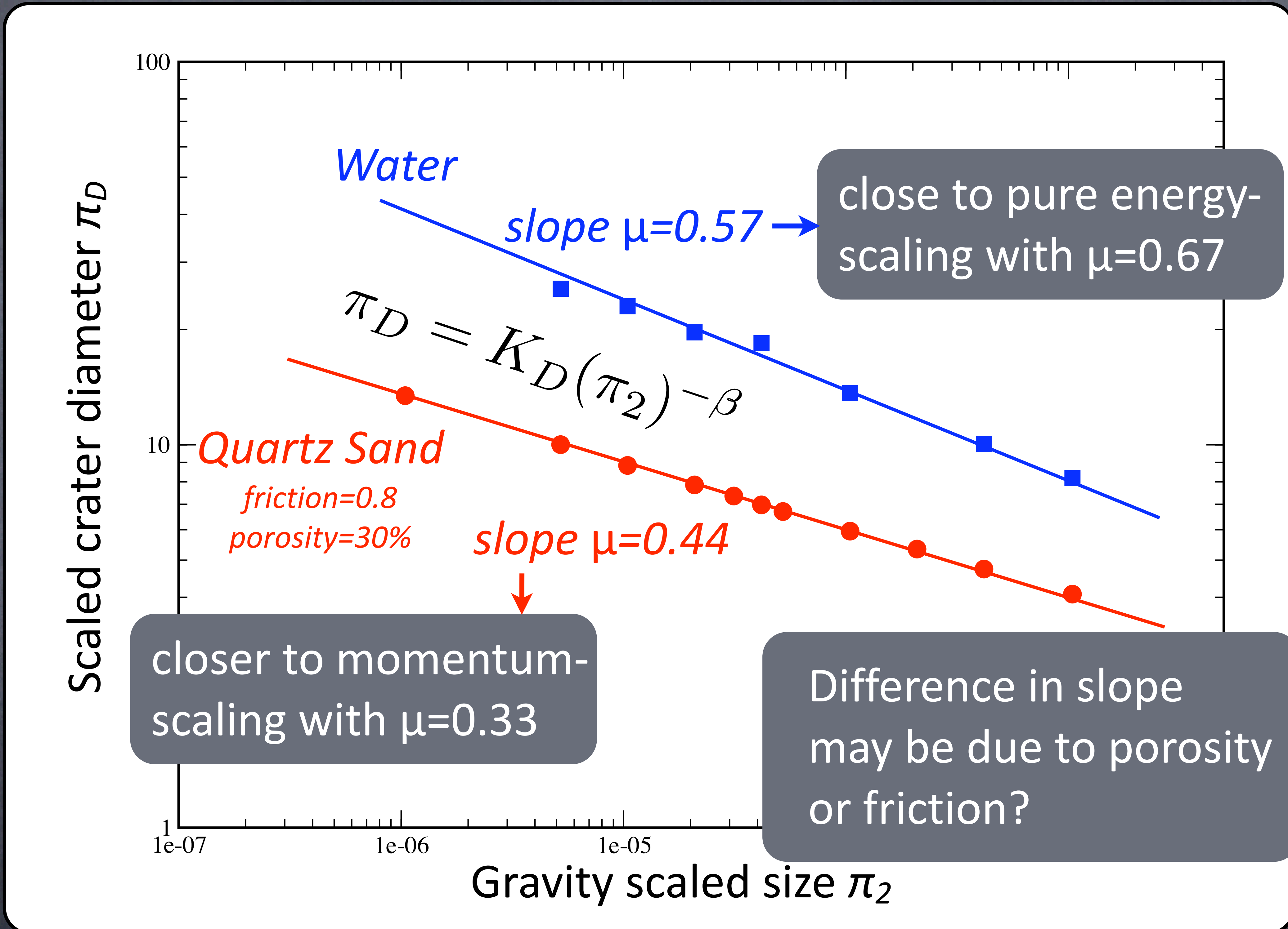
=1, if ρ=δ (π_4=1)

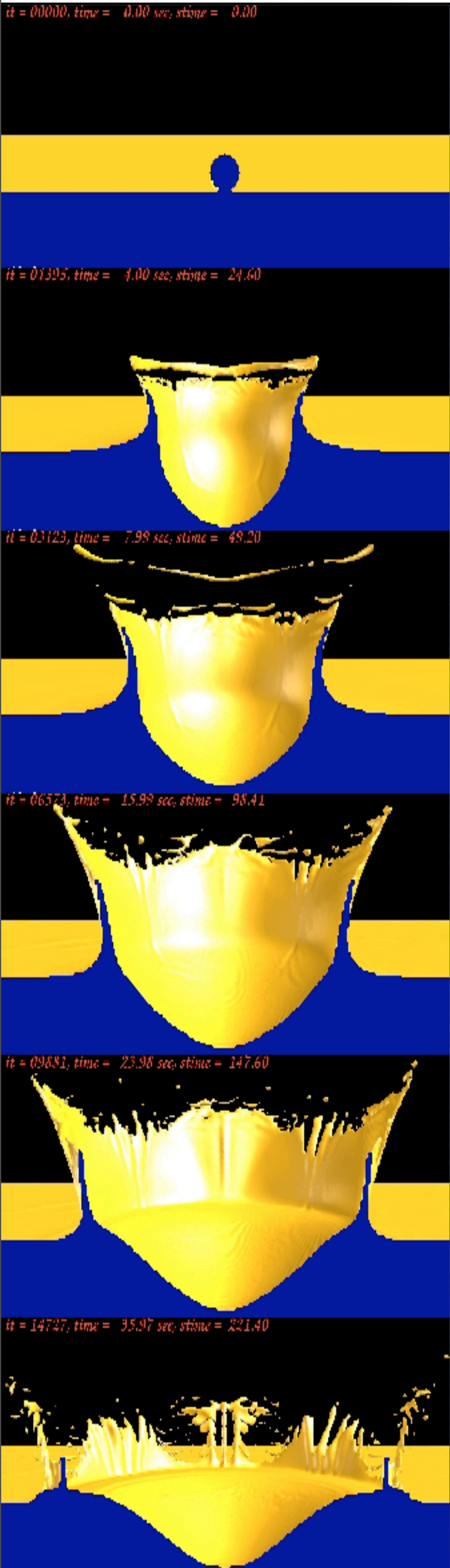
$$\Rightarrow \quad \pi_D = K_D(\pi_2)^{-\beta}$$

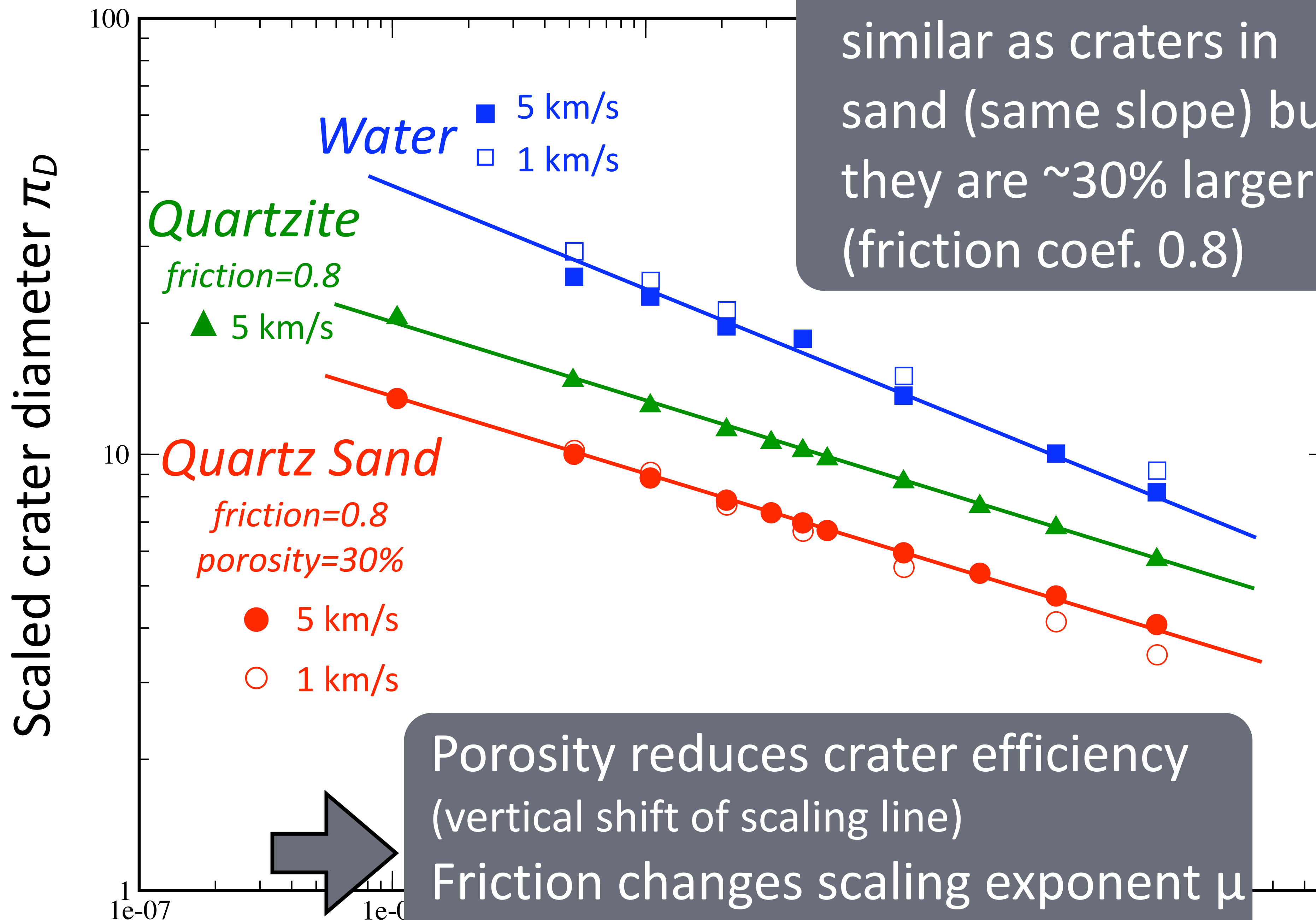
β=0.25 for energy-scaling
β=0.14 for momentum-scaling



Numerical experiments in different materials







craters in compact rock (nonporous) scale similar as craters in sand (same slope) but they are ~30% larger (friction coef. 0.8)

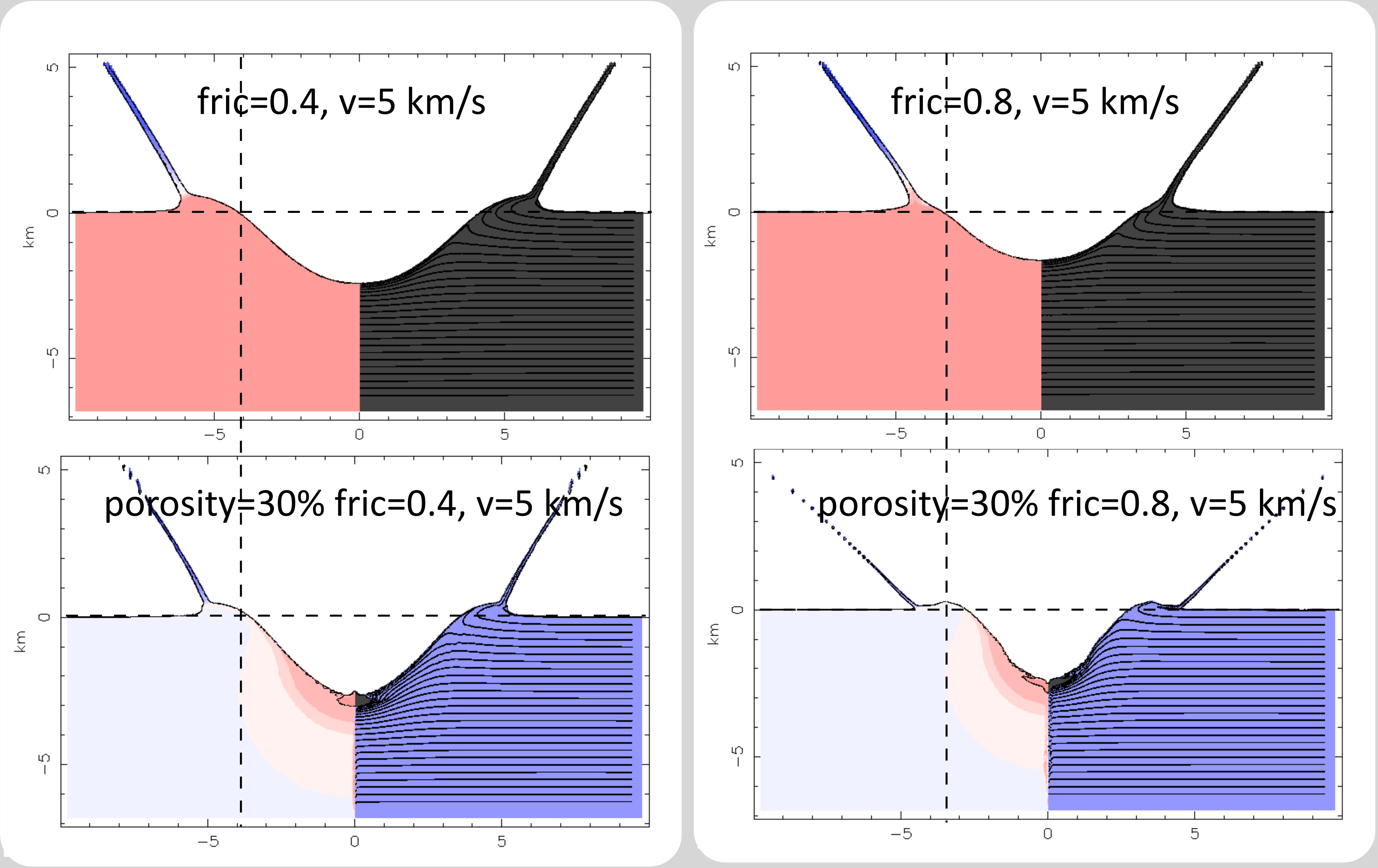
Porosity reduces crater efficiency
(vertical shift of scaling line)

Friction changes scaling exponent μ
(change in slope of scaling line)

The fact that craters in porous targets can be smaller than in competent rock may seem a bit unintuitive at first glance

Competent rock

Porous rock



Note, crater excavation is a result of release from **shock compression***. The crushing of pore space consumes a lot of energy, thus shock pressure and therefore crater size is smaller!*

Summary: scaling of transient crater size (vertical impacts)

Dimensionless form

$$\pi_D = K_D(\pi_2)^{-\frac{\mu}{2+\mu}}$$

Absolute parameters

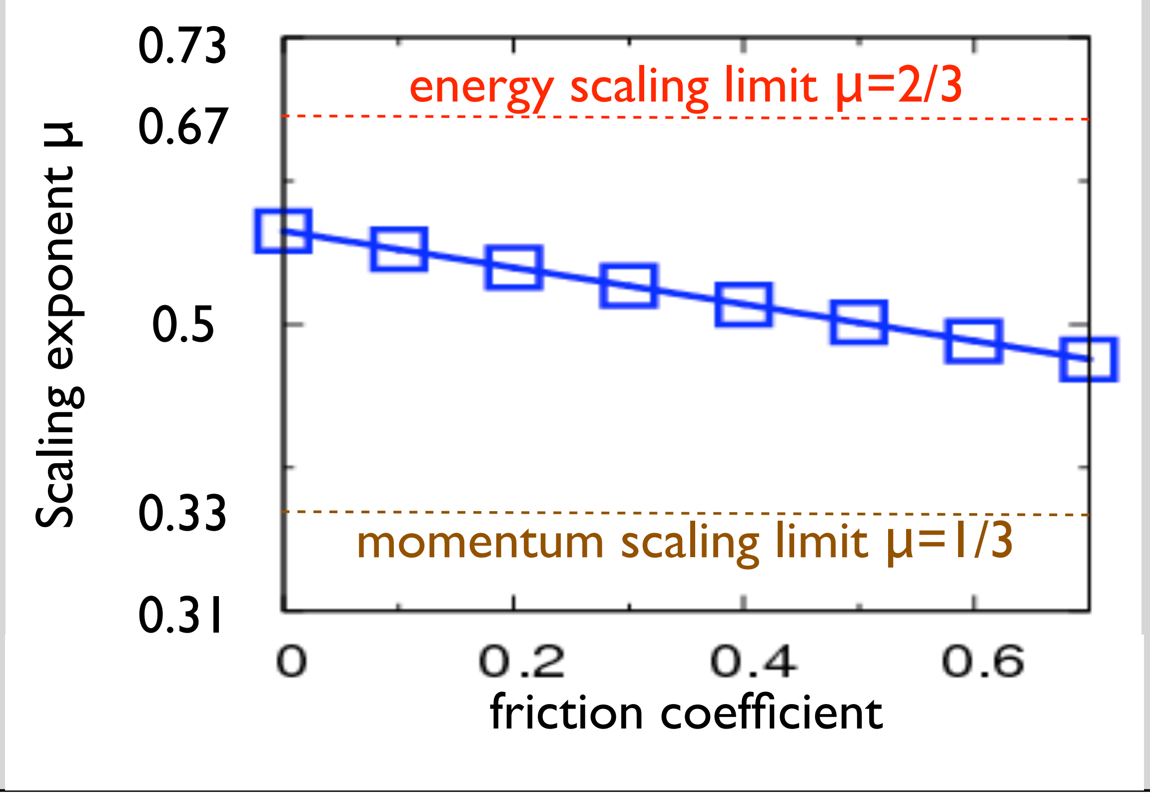
$$\frac{D}{L} = J_D\left(1.61\frac{gL}{U^2}\right)^{-\frac{\mu}{2+\mu}}$$

If strength is negligible and density of projectile and target are equal

(Note difference between K_D, J_D, K_I, ...) → *where* $K_D = J_D\left(\frac{6}{\Pi}\right)^{\frac{1}{3}}$ *and* $\beta = \frac{\mu}{2+\mu}$

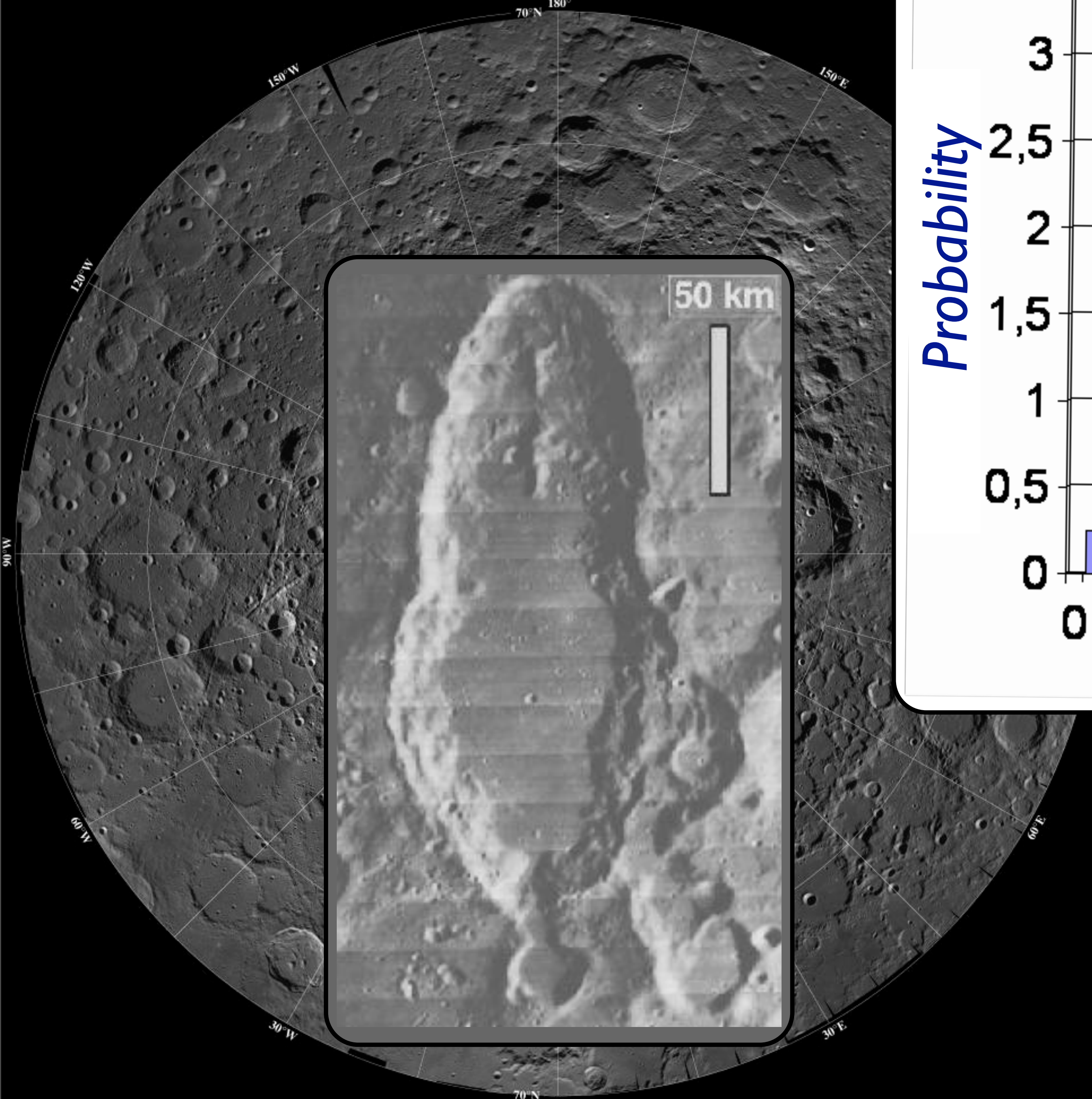
Numerical modeling enables determination of K_D, μ, ν, ... as a function of friction and porosity.

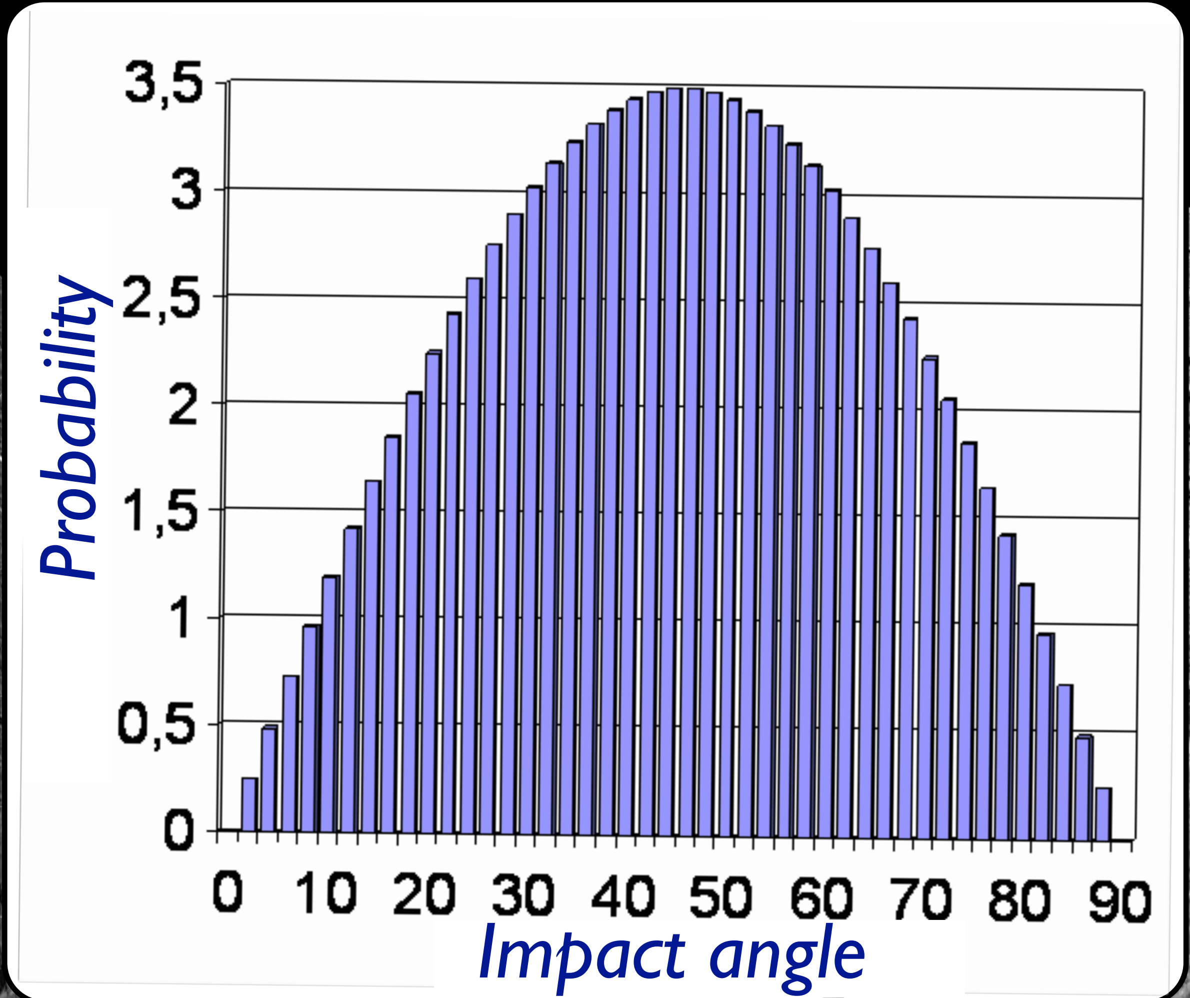
Scaling exponent μ as a function of friction



All scaling laws discussed so far apply only to vertical impact!

What about the effect of the impact angle on crater efficiency?

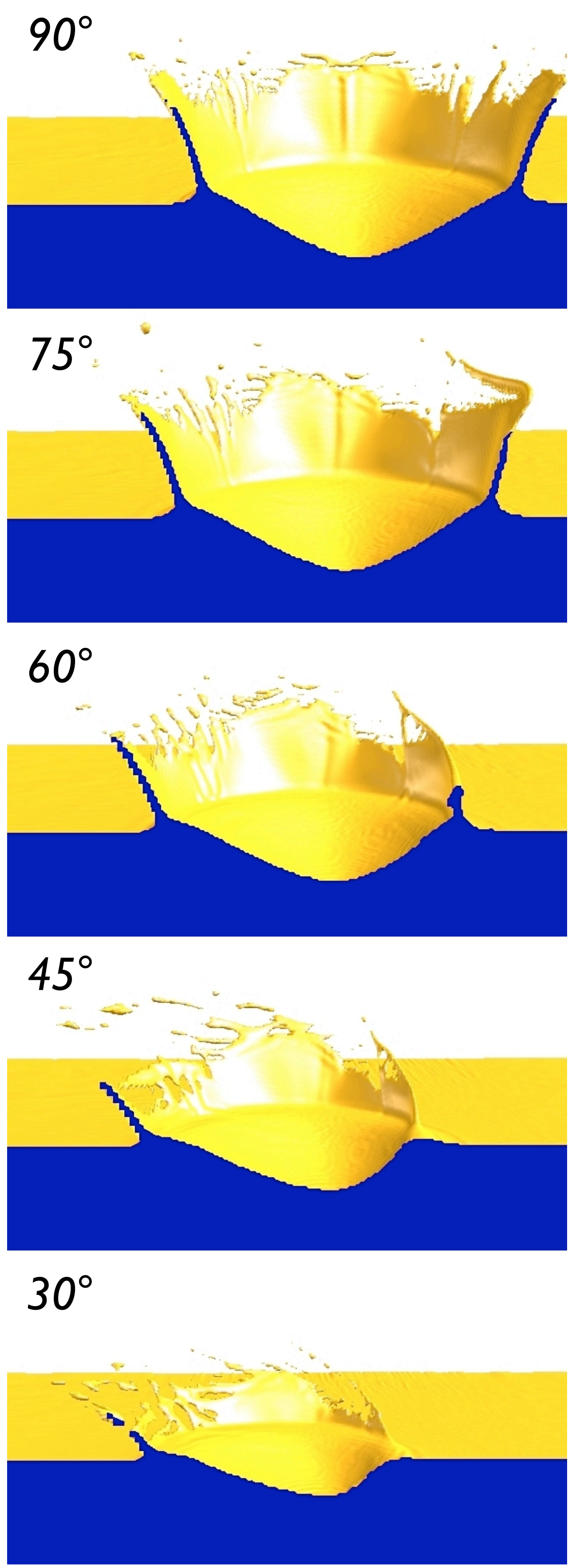


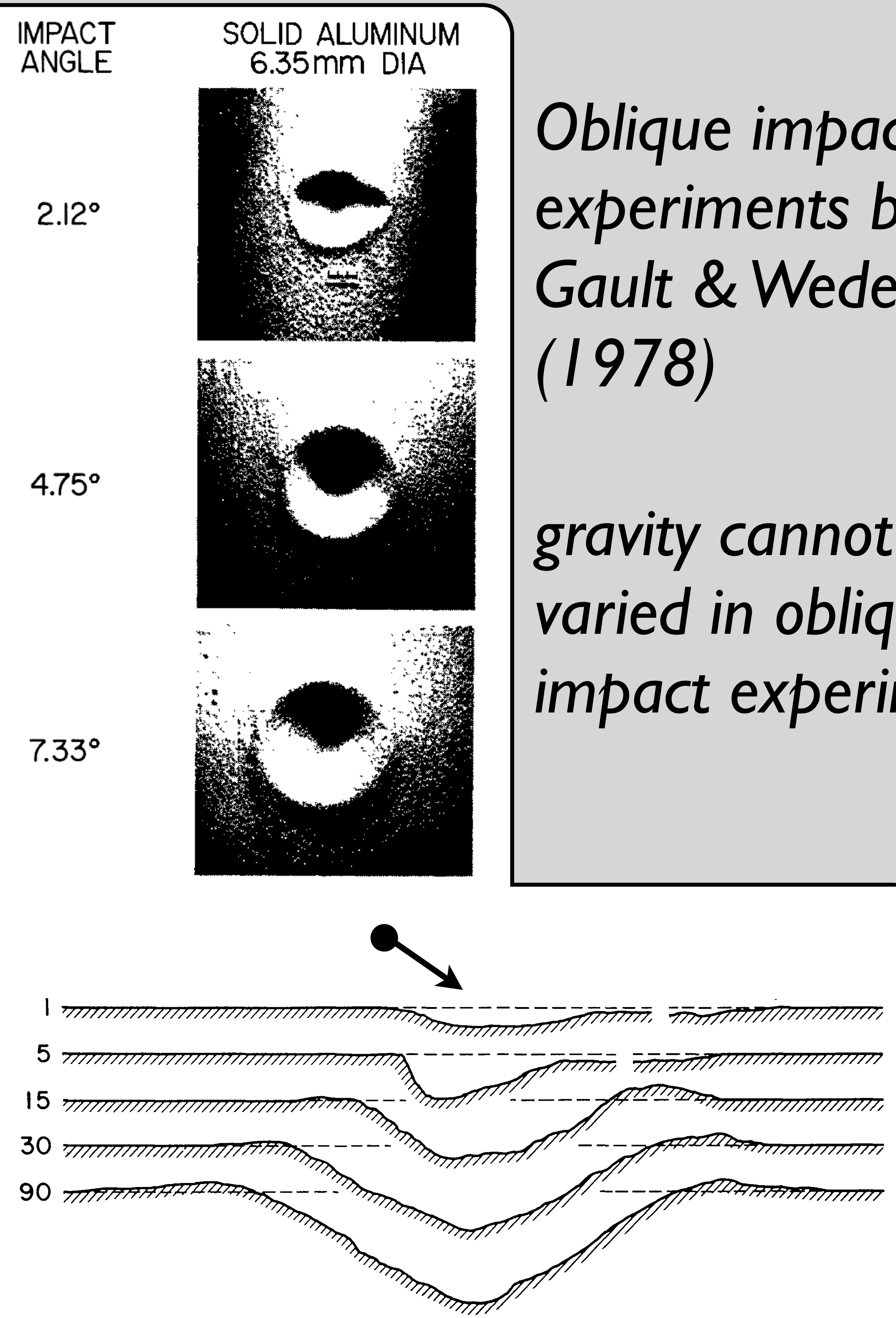


Impact angle		Probability
30°-60°	:	50%
20-70°	:	76.6%
<15°	:	6.7 %
>5°	:	0.75%

To simulate oblique impacts 3D hydrocodes are required

Computation time much higher; parameter studies time consuming

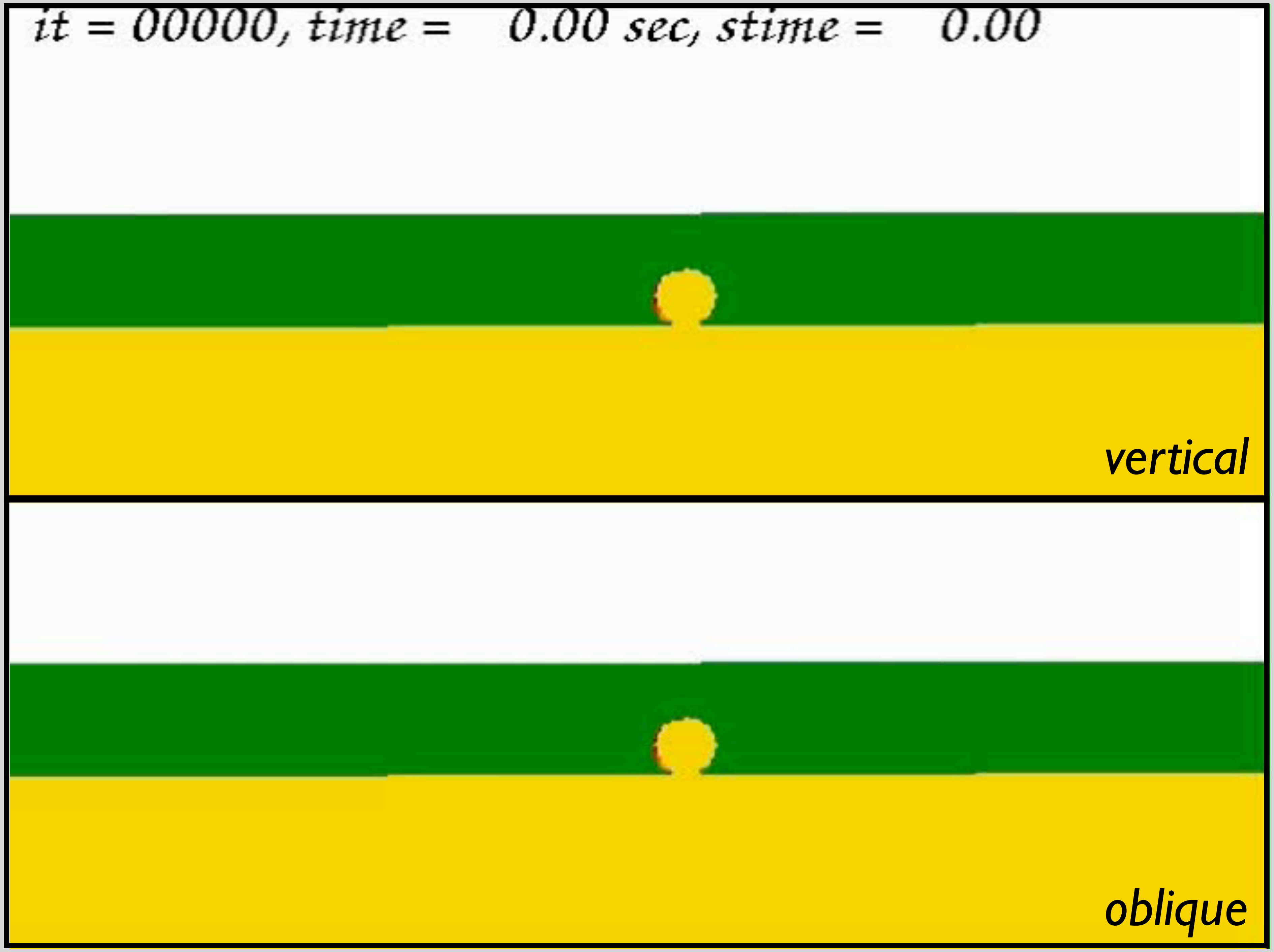




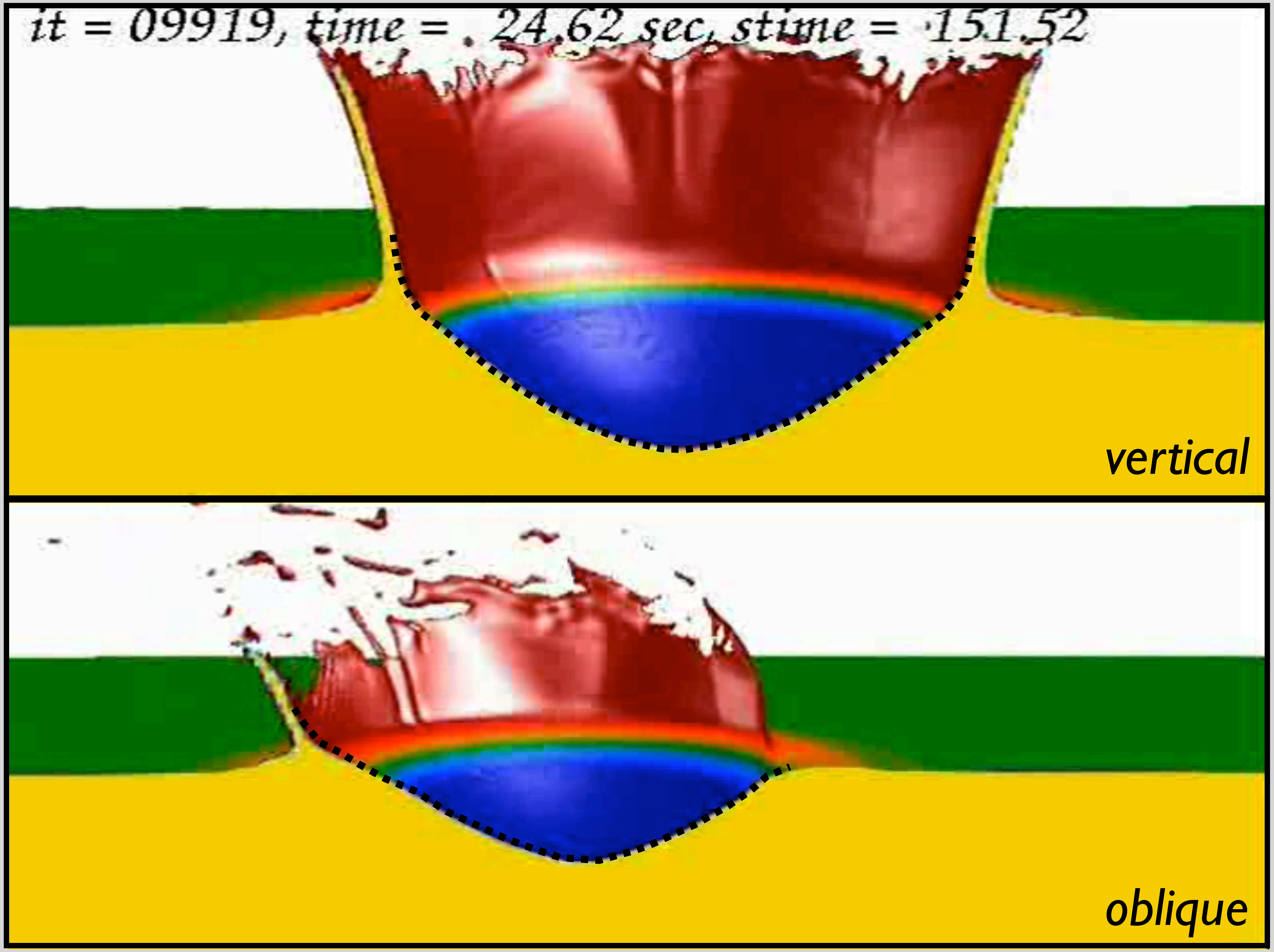
Oblique impact experiments by Gault & Wedekind (1978)

gravity cannot be varied in oblique impact experiments

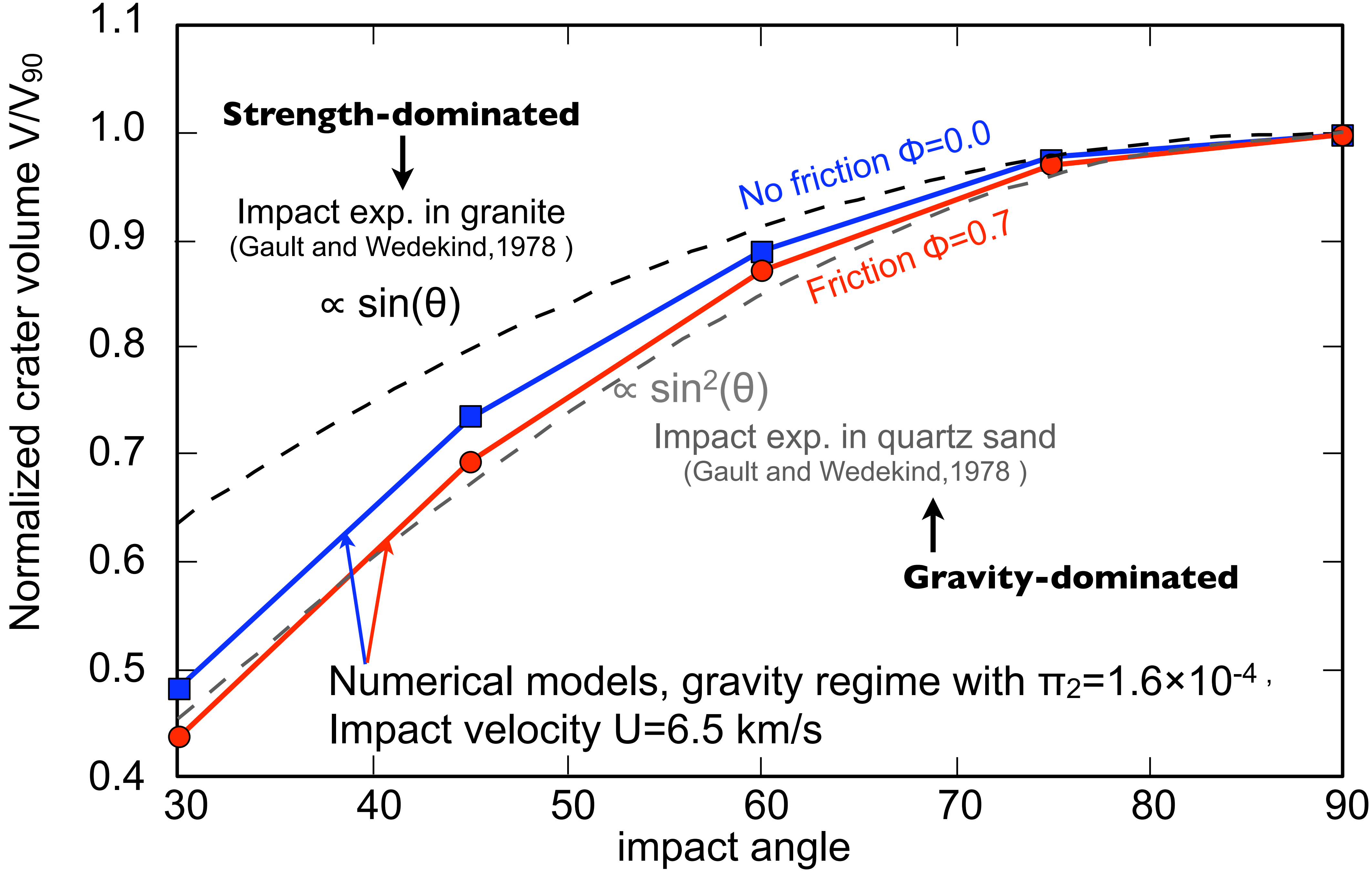
Comparison between numerical model of vertical and oblique impact



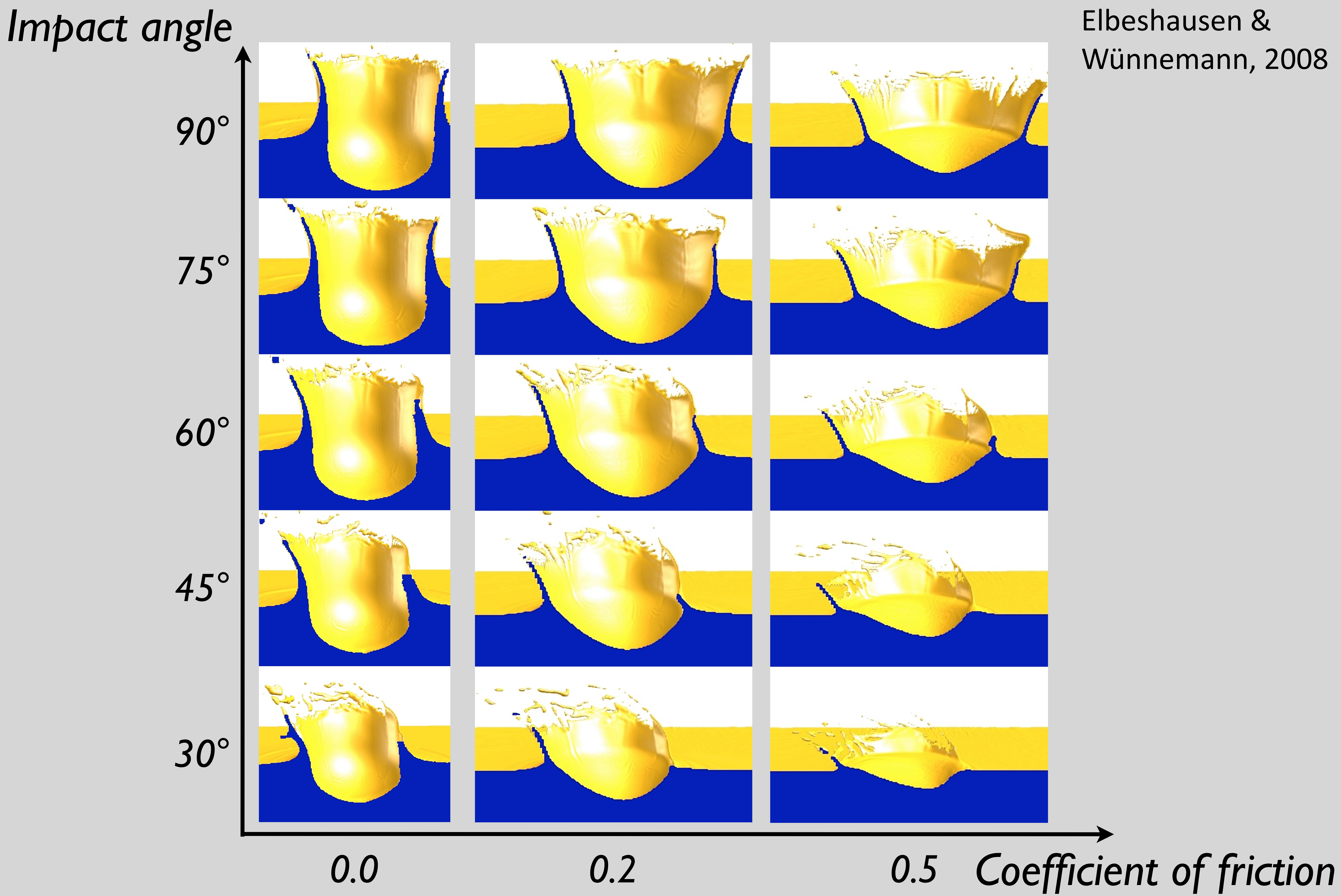
Comparison if the size of the transient crater between vertical and oblique impact



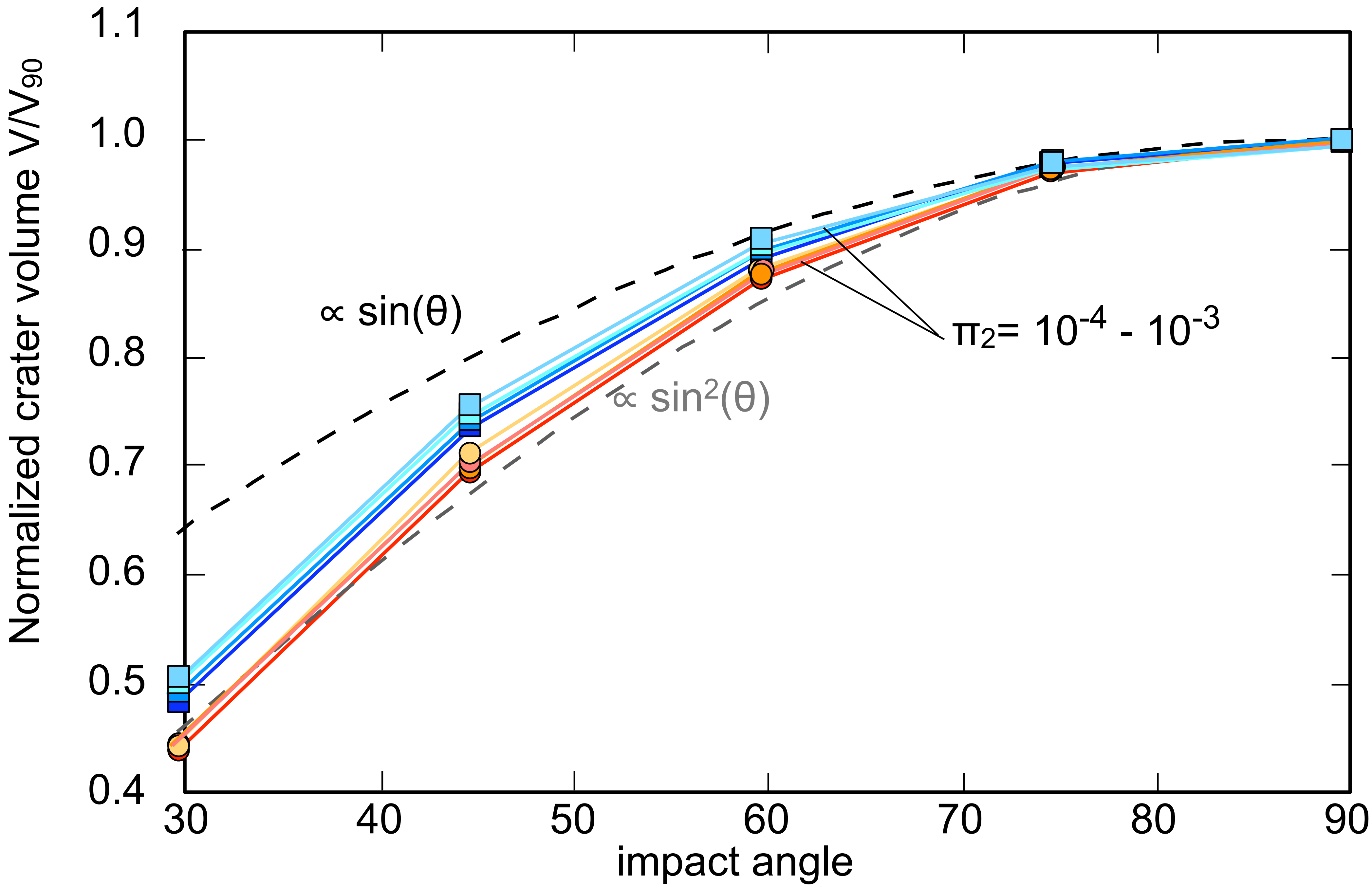
Crater volume decreases with decreasing impact angle



Elbeshausen & Wünnemann, 2007, 2008


Impact angle
Elbeshausen & Wünnemann, 2008
90°
75°
60°
45°
30°
0.0
0.2
0.5
Coefficient of friction

The effect of impact angle on crater efficiency does not depend on π_2



How can we consider the impact angle in the scaling equations?

Scaling for crater efficiency for vertical impacts

$$\pi_V = K_V \ (\pi_2)^{-\frac{3\mu}{2+\mu}}$$

$$\pi_V = K_V \ \left(1.61 \frac{gL}{U^2}\right)^{-\frac{3\mu}{2+\mu}}$$

Chapman and McKinnon (1986) suggested to replace U by the vertical component U sin(θ)

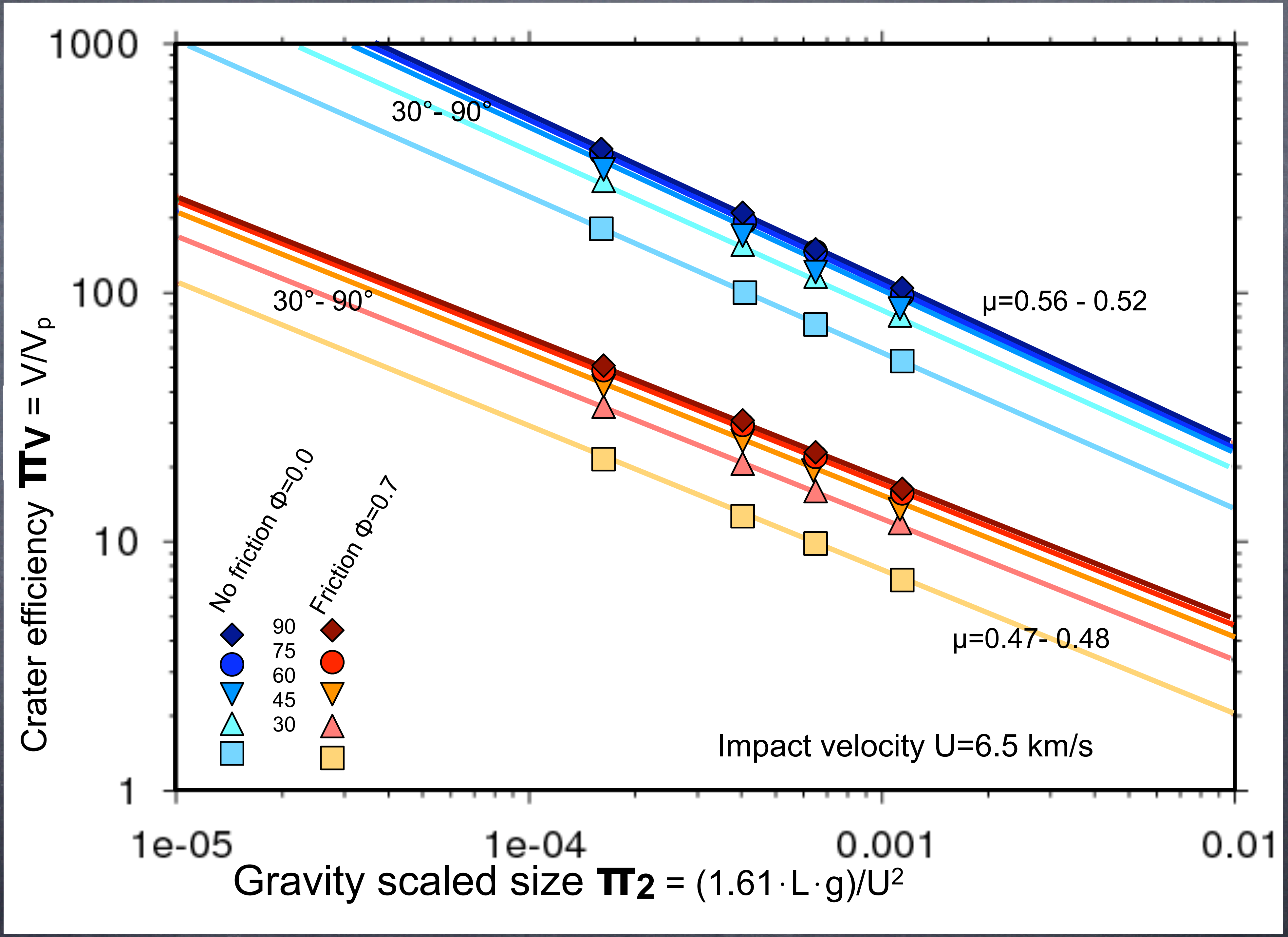
$$\pi_V = K_V \ \left(1.61 \frac{gL}{U^2 \sin(\theta)^2}\right)^{-\frac{3\mu}{2+\mu}}$$

$$\kappa = 1 \text{ or } 2$$

Depending on material (?), friction (?), ...

$$\pi_V = K_V \ \left(1.61 \frac{gL}{U^2}\right)^{-\frac{3\mu}{2+\mu}} \cdot \sin(\theta)^{\kappa \frac{3\mu}{2+\mu}}$$

The effect of impact angle on crater scaling



Summary: scaling the diameter of the transient crater size

$$\pi_D = K_D \ (\pi_2)^{-\frac{\mu}{2+\mu}}$$

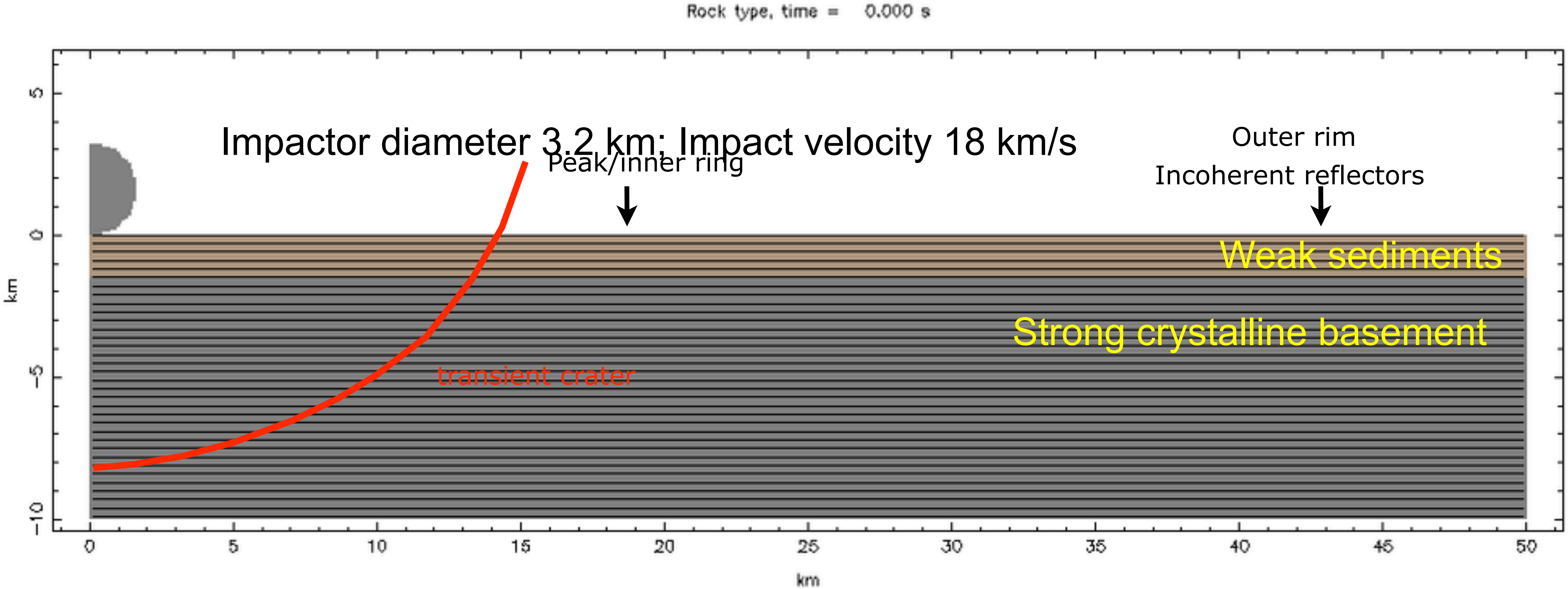
$$D\left(\frac{\rho}{m}\right)^{1/3} = K_D \ \left(\frac{gL}{U^2}\right)^{-\frac{\mu}{2+\mu}}$$

↑ Considering impact angle

↑ Considering density ratio projectile/target

There is a big difference between the size of the transient crater and the diameter of the final crater in particular for complex crater morphology

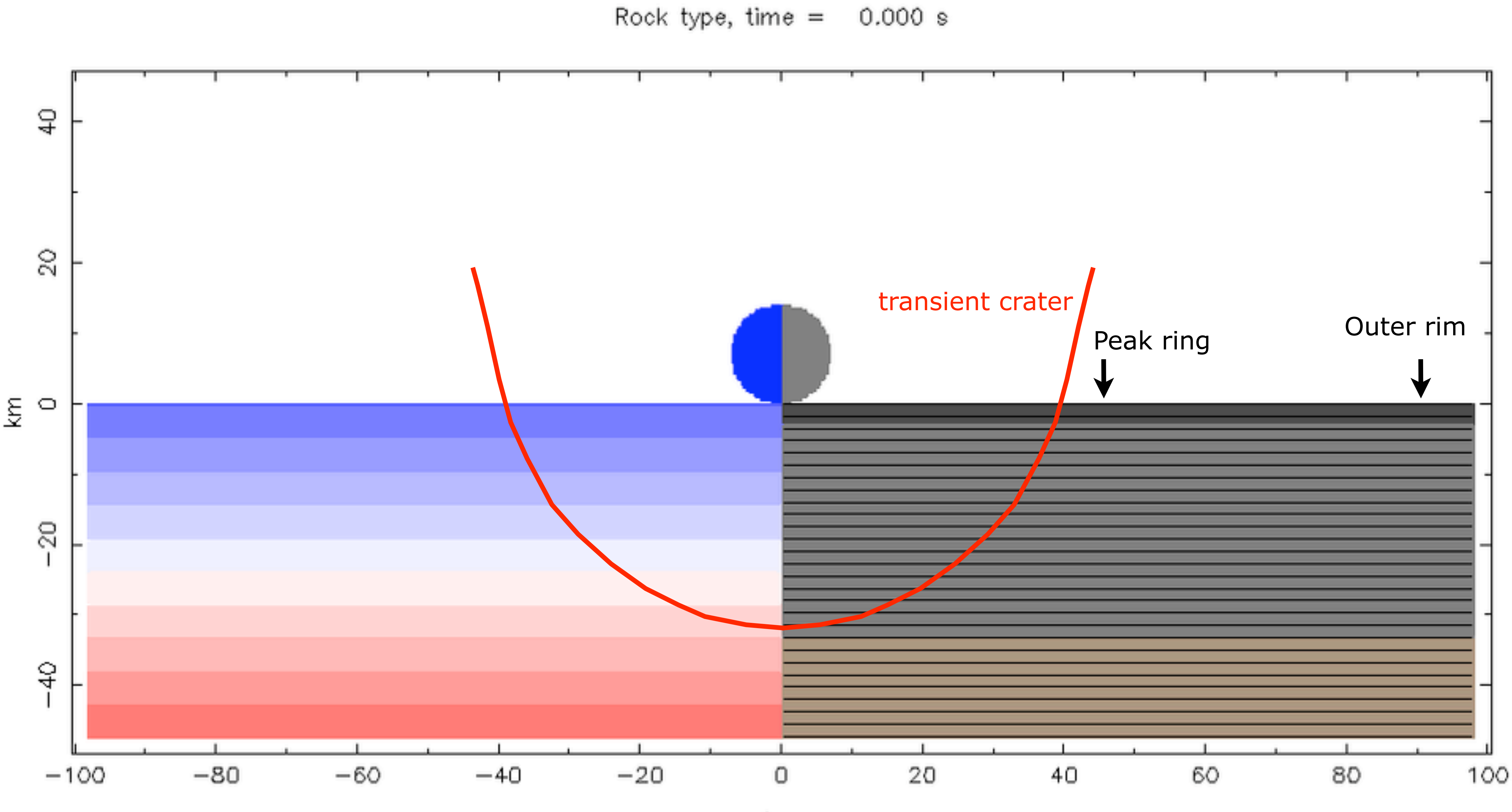
Chesapeake Bay crater formation model



Collins and Wünnemann, 2005

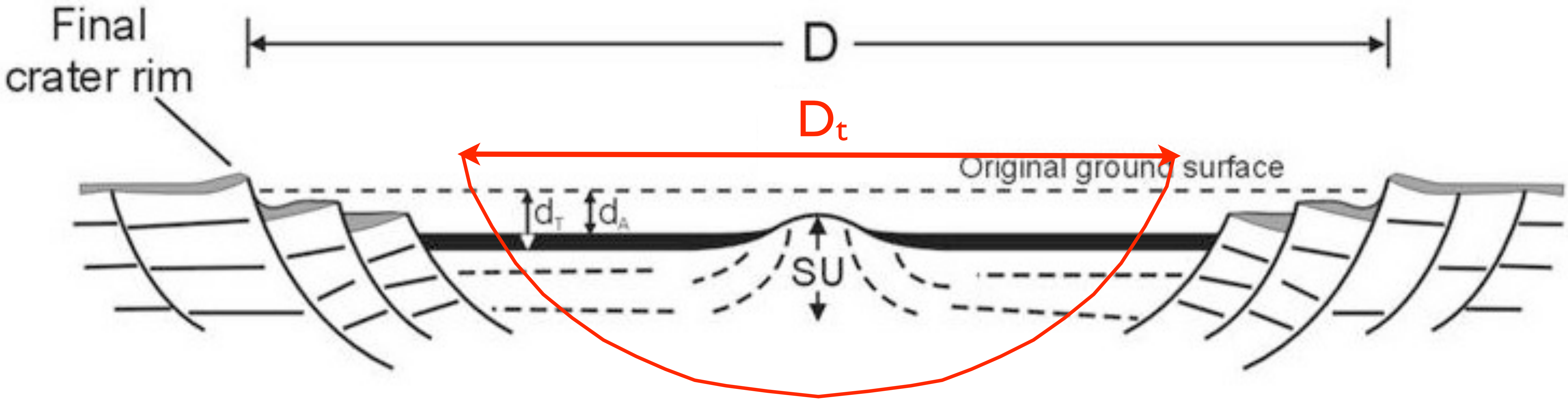
- *Final crater (~85 km) is enlarged by inwards collapsing weak, water-saturated sediments*
- *more diagnostic crater size ~36 km in diameter*
- *transient crater size ~28 km in diameter*

Chicxulub crater formation model



Wünnemann, Collins and Ivanov

Scaling from the transient crater size to the final crater diameter



Scaling based on complex lunar craters

↓

$$D = 1.17 \frac{D_t^{1.13}}{D_c^{0.13}}$$

McKinnon and Schenk, 1985

with $D_c = 3.2\ km$ *for the earth*

Transient crater	
28.0 km	*Chesapeake Bay*
40.0 km	*Chicxulub*

↳ *Scaling laws provide only a very rough estimate*

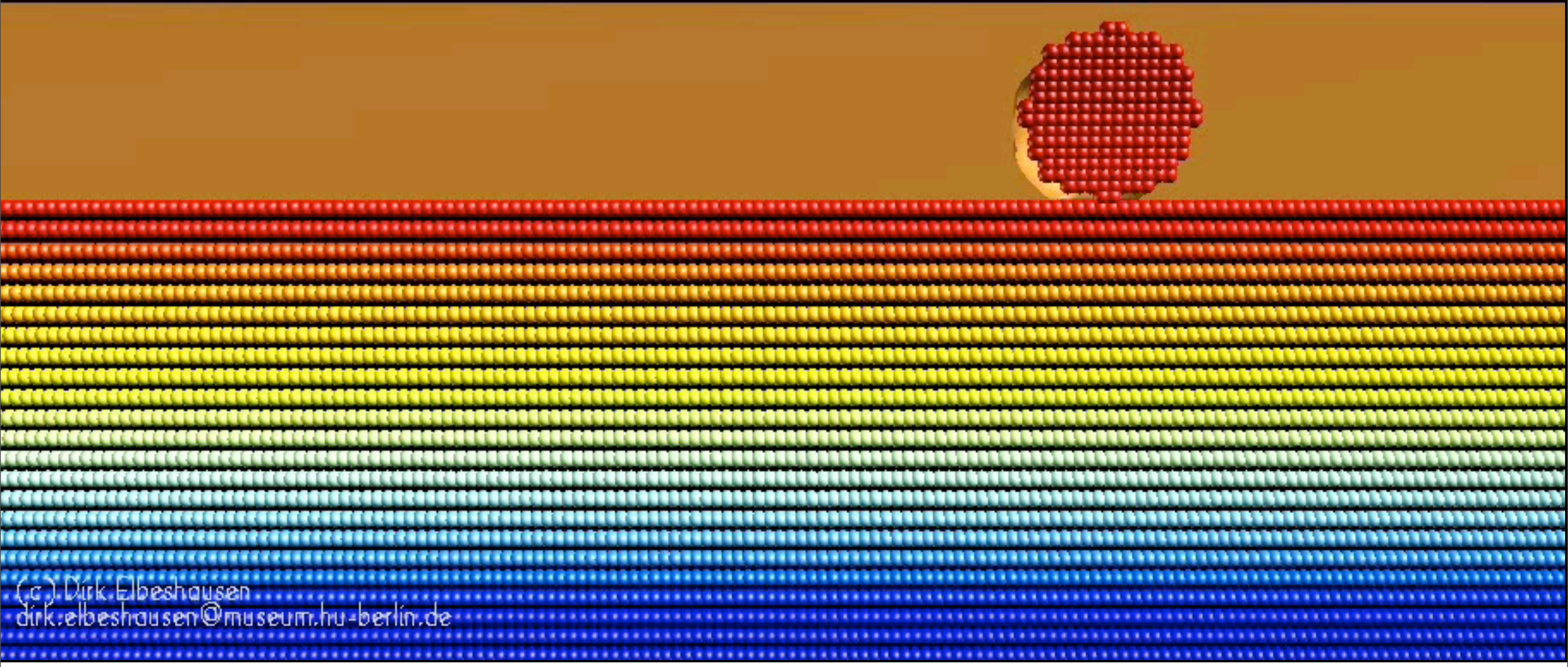
Take-Home-Messages

- *Almost all impact craters on planetary surfaces are „gravity dominated“ - crater size is controlled by gravity not strength!*
- *Scaling laws relate the kinetic energy of an impact to the size of the transient crater (crater diameter, depth, efficiency)*
- *Scaling laws apply only to processes, such as crater growth, that take place in the „far-field“ - sufficiently far enough from the „coupling zone“*
- *Scaling parameters (velocity exponent μ, intercept K) can be determined by laboratory or numerical experiments and depend on material properties such as friction and porosity*
- *Only numerical experiments can be used to study the effect of the impact angle on crater scaling (diameter, depth, ...) → using the vertical component of the impact velocity holds true only for frictional targets comparable to sand*
- *Final crater size can be roughly estimated from the transient crater by empirical estimates from the lunar crater record; however, more precise estimates can only be obtained from numerical modeling*

Thanks

to

Henning, Elin, and NiR



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