Combining Recommender and Reputation Systems to Produce Better Online Advice*

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Abstract. Although recommender systems and reputation systems have quite different theoretical and technical bases, both types of systems have the purpose of providing advice for decision making in e-commerce and online service environments. The similarity in purpose makes it natural to integrate both types of systems in order to produce better online advice, but their difference in theory and implementation makes the integration challenging. In this paper, we propose to use mappings to subjective opinions from values produced by recommender systems as well as from scores produced by reputation systems, and to combine the resulting opinions within the framework of subjective logic.

1 Introduction

Recommender systems [1] and reputation systems [7, 14] are similar in the sense that both collect data of members in a community in order to provide advice to those members. However, there are also fundamental differences. Recommender systems assume that different people inherently have different tastes, and hence value things subjectively. In contrast, reputation systems assumption that all members in a community value things under the same criteria, i.e. objectively. Said differently, when a recommender system indicates that a user probably does not like a given resource, it does not mean that there is anything wrong with it. However, when a reputation system produces a low value for a resource, one can assume that its quality is poor. We use the term *"resource"* to denote the thing (or item) being rated. The purpose of recommender systems is mainly to generate suggestions about resources that a user *a priori* is not aware of but would probably be interest in. The purpose of reputation systems is to provide advice about resources that the user already is aware of and interested in. On this background there is a strong potential for combining the two types of systems.

However, it is quite challenging to make an effective integration of the output results produced by recommender and reputation systems, given the following

* 10th International Conference on Modeling Decisions for Artificial Intelligence (MDAI 2013), Barcelona, November 2013.

three-fold. First, in general the advices generated from different systems are distinct and heterogeneous. This is because different systems may use different forms of feedback and evaluate the performance based on different criteria. Second, the result from reputation systems reflects the collective opinions of a whole community whereas the result from recommender systems only represents the collective opinions of a local community, i.e. the users with similar preference. Third, the uncertainty of the generated advice should be taken into consideration. The uncertainty is typically due to the small number of received ratings, and will hinder the usefulness of ratings in decision making. To address these issues, we propose to use mappings to subjective opinions from the respective output results of recommender and reputation systems, so that the outputs are homogeneous and hence can be easily integrated and fused. Subjective logic [11] is a probabilistic framework capable of coping with the uncertainty in evidences.

We denote *recommendation values* and *reputation scores* as the outputs derived from recommender systems and reputation systems, respectively. Reputation systems produce reputation scores, e.g. in the range 0 – 5 stars. We assume a Bayesian/Dirichlet reputation system where the collected feedback ratings can be converted to subjective opinions, see Section 4.1 for details. Besides, a recommender system derives predictive recommendation values in the range $[0, 1]$[6], which will be converted to subjective opinions, see Section 4.2 for details. To integrate both reputation scores and recommendation values, we introduce the CasMin operator in Section 5. Finally, in Section 5.3 we show via an example that the advice produced by our approach is better than that produced by either a recommender system or a reputation system alone. To the authors' best knowledge, our work is the first effort in the literature to fuse outputs from recommender systems and reputation systems in order to produce better advice.

2 Related Work

Both recommender systems and reputation systems have been extensively and separately studied for decades. Recommender systems, as an essential component of e-commerce and online service applications, provide users with personalized high-quality recommendations to mitigate the well-known *information overload* problem. Collaborative filtering (CF) is a widely adopted technique to generate recommendations using the ratings of like-minded users [1]. The basic principle is that users with similar tastes in the past will also favour the same resources in the future. CF techniques can be classified into memory-based and model-based approaches. However, CF inherently suffers from two severe issues: *data sparsity* and *cold start* [1], due to the fact that users – especially new users – typically have rated only a few resources. The uncertainty of predictions arises from such conditions where none or only few ratings are available for recommendations.

Many approaches have been proposed to reduce the uncertainty and improve the accuracy of recommendations. One direction of work is to develop new similarity measures in order to identify more reliable similar users [2]. However, the

[6] The ratings given by users are normalized in the range [0,1] if necessary.

uncertainty due to few ratings of similar users cannot be handled. Model-based approaches [12, 15] generally handle these issues better than memory-based approaches in terms of efficiency and accuracy. This is because global rating data is used to train a prediction model whereas memory-based approaches work on local rating data. The main drawback is that the trained static model is difficult to adapt to real-time increasing ratings. Another direction is to incorporate social relationships, such as trust-aware recommender systems [13]. The underlying principle assumption is that trust and taste are strongly and positively correlated [7]. Our work follows this general direction, i.e. to integrate taste and trust. The difference is that our approach takes the global perspective of resources (reputation scores) rather than the local perspective of users (social ties). In addition, the integration that we study is based on directly fusing taste and trust, rather than on moderating taste recommendations with trust.

Attacks against recommender systems are usually summarized as *shilling attacks* [3, 4] where bogus rating profiles are injected to promote or degrade some resources. Although effective methods have been designed for memory-based CF, the research on robust model-based CF are not well studied [4]. Reputation systems are often built upon the assumption that user feedback may be fake and unreliable, and that various kinds of attacks could be conducted to influence the formation of reputation scores [10].

Reputation systems also suffer from the cold start problem. Remember that reputation systems generate scores based on feedback (or ratings) from members in a community [14, 7]. When only little feedback is available, it is like a cold start situation where the derived reputation scores will be less reliable. Uncertainty can also increase when feedback greatly conflicts [17]. Users also tend to give mostly positive feedback which results in the derived reputation scores being less distinguishable and hence less useful.

In a nutshell, combining scored from both recommender systems and reputation systems can provide users with more accurate and robust online advice than either of the scores can in isolation. However, to date the integration of the two types of systems has not been studied in the literature.

3 Subjective Opinions

In this section, we will first introduce the notation and formation of subjective opinions used for fusing taste and trust. We also depict the mappings to binomial opinions from the multinomial ratings which is the common form of feedback in reputation systems and recommender systems.

3.1 Opinions Formation and Representation

A subjective opinion expresses belief about states of a state space called a *"frame of discernment"* or *"frame"* for short. In practice, a state in a frame can be regarded as a statement or proposition, so that a frame contains a set of statements. Let $X = \{x_1, x_2, \ldots, x_k\}$ be a frame of cardinality k, where x_i $(1 \leq i \leq k)$

represents a specific state. An opinion distributes belief mass over the reduced powerset of the frame denoted as $\mathcal{R}(X)$ defined as:

$$\mathcal{R}(X) = \mathcal{P}(X) \setminus \{X, \emptyset\}\ , \tag{1}$$

where $\mathcal{P}(X)$ denotes the powerset of X and $|\mathcal{P}(X)| = 2^k$. All proper subsets of X are states of $\mathcal{R}(X)$, but the frame X and the empty set $\emptyset$ are not states of $\mathcal{R}(X)$, in line with the hyper-Dirichlet model [5]. $\mathcal{R}(X)$ has cardinality $\kappa = 2^k - 2$.

An opinion is a composite function that consists of a belief vector $\boldsymbol{b}$, an uncertainty parameter u and base rate vector $\boldsymbol{a}$ that take values in the interval $[0, 1]$ and that satisfy the following additivity requirements.

$$\text{Belief additivity: } u_X + \sum_{x_i \in \mathcal{R}(X)} \boldsymbol{b}_X(x_i) = 1. \tag{2}$$

$$\text{Base rate additivity: } \sum_{i=1}^{k} \boldsymbol{a}_X(x_i) = 1, \quad \text{where} \quad x_i \in X. \tag{3}$$

The opinion of user A over the frame X is denoted as $\omega_X^A = (\boldsymbol{b}_X, u_X, \boldsymbol{a}_X)$, where $\boldsymbol{b}_X$ is a belief vector over the states of $\mathcal{R}(X)$, u_X is the complementary uncertainty mass, and $\boldsymbol{a}_X$ is a base rate vector over X, all seen from the viewpoint of belief owner A.

The belief vector $\boldsymbol{b}_X$ has $(2^k - 2)$ parameters, whereas the base rate vector $\boldsymbol{a}_X$ only has k parameters. The uncertainty parameter u_X is a simple scalar. Thus, a general opinion contains $(2^k + k - 1)$ parameters. However, given that Eq.(2) and Eq.(3) remove one degree of freedom each, opinions over a frame of cardinality k only have $(2^k + k - 3)$ degrees of freedom. The probability projection of hyper opinions is the vector $\mathbf{E}_X$ expressed as:

$$\mathbf{E}_X(x_i) = \sum_{x_j \in \mathcal{R}(X)} \boldsymbol{a}_X(x_i/x_j)\, \boldsymbol{b}_X(x_j) + \boldsymbol{a}_X(x_i)\, u_X, \quad \forall x_i \in \mathcal{R}(X) \tag{4}$$

where $\boldsymbol{a}_X(x_i/x_j)$ denotes relative base rate, i.e. the base rate of subset x_i relative to the base rate of (partially) overlapping subset x_j, defined as follows:

$$\boldsymbol{a}_X(x_i/x_j) = \frac{\boldsymbol{a}_X(x_i \cap x_j)}{\boldsymbol{a}_X(x_j)},\ \forall\ x_i, x_j \subset X. \tag{5}$$

Equivalent probabilistic representations of opinions, e.g. as Beta pdf (probability density function) or a Dirichlet pdf, offer an alternative interpretation of subjective opinions in terms of traditional statistics [11].

The term *hyper opinion* is used for a general opinion [11]. A *multinomial opinion* is when the belief vector $\boldsymbol{b}_X$ only applies to elements $x_i \in X$, not in $\mathcal{R}(X)$. *Binomial opinions* apply to binary frames and have a special notation as described below. Let $X = \{x, \overline{x}\}$ be a binary frame, then a binomial opinion about the truth of state x is the ordered quadruple $\omega_x = (b, d, u, a)$ where:

b, *belief*: belief mass in support of x being true;
d, *disbelief*: belief mass in support of $\overline{x}$ (NOT x);
u, *uncertainty*: uncertainty about probability of x;
a, *base rate*: non-informative prior probability of x.

The special case of Eq.(2) in case of binomial opinions is expressed by Eq.(6).

$$b + d + u = 1. \tag{6}$$

Similarly, the special case of the probability expectation value of Eq.(4) in case of binomial opinions is expressed by Eq.(7).

$$\mathrm{E}_x = b + au. \tag{7}$$

Binomial and multinomial opinions can be visualised as a point inside a simplex. Binomial opinions can thus be visualised as a point inside an equal sided triangle, and a trinomial opinion as a point inside a tetrahedron.

3.2 Mapping to Binomial Opinions

Multinomial opinions represent a generalisation of binomial opinions, and hyper opinions represent a generalisation of multinomial opinions. It can be necessary to project hyper opinions onto multinomial opinions, or to project multinomial opinions onto binomial opinions. For example, a reputation system where ratings are given in the form of 1-5 stars can represent reputation scores as multinomial opinions over a frame of five states, each of which represents a specific number of stars. In this case, a reputation score represented as a multinomial opinion can be projected to a binomial opinion over a binary frame, as explained below.

Let $X = \{x_1, \ldots, x_k\}$ be a frame where the k states represent linearly increasing rating levels, e.g. so that x_i represents an i-star rating. Let $Y = \{y, \overline{y}\}$ be a binary frame where y and $\overline{y}$ indicate *high quality* and *low quality* of a resource, respectively. Assume that a reputation score or recommendation value is represented as the multinomial opinion $\omega_X = (\boldsymbol{b}_X, u_X, \boldsymbol{a}_X)$ over the frame X, and that a binomial opinion $\omega_y = (b_y, d_y, u_y, a_y)$ over Y is required. The linear projection from the multinomial opinion ω_X on X onto the binomial opinion ω_y on Y is defined by:

$$\begin{cases} u_y = u_X \\ b_y = \sum_{i=1}^{k} b_{x_i} \frac{(i-1)}{(k-1)} \\ d_y = 1 - b_y - u_y \\ a_y = \sum_{i=1}^{k} a_{x_i} \frac{(i-1)}{(k-1)} \end{cases} \tag{8}$$

where $\frac{(i-1)}{(k-1)}$ indicates the relative weight, and hence the belief in a higher level x_i will have more weight in the formation of binary belief and base rate. As the default base rates on X is defined by $a_{x_i} = 1/k$, the default base rate of y is computed as follows:

$$a_y = \sum_{i=1}^{k} \frac{1}{k} \frac{(i-1)}{(k-1)} = \frac{1}{k(k-1)} \sum_{i=1}^{k} (i-1) = \frac{1}{k(k-1)} \frac{k(k-1)}{2} = 1/2 \; . \tag{9}$$

The advantage of the projection of Eq.(8) is to provide the flexibility of analysing reputation scores and recommendation values independently of the frame cardinality.

4 Determining Opinions

This section details the procedures to derive subjective opinions, i.e. reputation scores and recommendation values from reputation systems and recommender systems, respectively.

4.1 Opinions Derived from Reputation Systems

A reputation system generally applies to services or goods that can be rated on one or multiple aspects, such as the set (*expected quality*, *seller communication*, *shipment timeliness*, *shipment charges*) in case of `eBay.com`. In case only a single aspect can be rated, it is typically the overall quality of a specific service or target. Each aspect can be rated with a specific level out of l levels such as 1 to 5 stars. It is also common that an aspect is rated with only two possible levels such as *Thumbs Up* and *Thumbs Down*.

Opinions for each aspect can be derived from such ratings. The frame for each aspect is the set of discrete rating levels, so that in case ratings can be given as 1 to 5 stars the frame has five states. Let X denote the frame of cardinality k, $r(x_i)$ be the number of ratings of type x_i, and $\omega_X = (\boldsymbol{b}_X, u, \boldsymbol{a}_X)$ be a multinomial opinion on X. The more ratings collected, the smaller the uncertainty becomes. The opinion ω_X can be determined from the ratings $r(x_i)$ according to Eq.(10):

$$\forall x_i \in X \quad \begin{cases} \boldsymbol{b}(x_i) = \frac{\boldsymbol{r}(x_i)}{W + \sum_{i=1}^{\kappa} \boldsymbol{r}(x_i)} \\ \\ u \quad = \frac{W}{W + \sum_{i=1}^{\kappa} \boldsymbol{r}(x_i)} \end{cases} \tag{10}$$

where $W = 2$ is the non-informative prior weight with default value dictated by the requirement of having a uniform pdf (probability density functions) over binary frames when no evidence other than the domain base rate is available. The value would e.g. be $W = 3$ in case it were required to have a uniform pdf over a ternary frame. However, higher values for W make the probability distribution less sensitive to new evidence, so the value $W = 2$ is adopted [16].

An opinion derived according to Eq.(10) can thus represent a reputation score which can be mapped to a probability value, or to a simple user friendly representation e.g. in the form of 1 to 5 stars. A reputation score can also be adjusted as a function of time, reliability of the rater, etc. [6].

A rating is expressed as a specific level corresponding to a singleton state in the frame. In case there are more than two rating levels, the derived opinions are multinomial. In case only two types of ratings can be given, e.g. as *Thumbs Up* and *Thumbs Down*, the frame is binary so the opinions are binomial.

Reputation scores represented as multinomial opinions can be mapped to a binomial opinion according to Eq.(8) by assuming that each rating level corresponds to a value in the range $[0, 1]$, for more details see [8].

4.2 Opinions Derived from Recommender Systems

As an example, we describe a user-based CF method to generate recommendations [1]. The task of CF methods is to predict the preference (or rating) of a given resource (or item) for an active user, based on the rating histories of the active user as well as other participants in the community.

We keep the symbols s, v for users and i, j for items. Let $r_{v,i}$ denote a rating given by user v on item i, and let I_v denote the set of items that user v previously has rated. The mean rating of user v is computed by:

$$\overline{r}_v = \frac{1}{|I_v|} \sum_{i \in I_v} r_{v,i} \ . \tag{11}$$

Let $N_{s,j}$ denote the neighbourhood of an active user s constrained by having rated item j, i.e. the set of users who have rated (some of) the same items as user s and who have also rated the specific target item j. In general, only the top-K most similar users will be selected as the neighbourhood. The prediction $p_{s,j}$ for user s on target item j is computed by:

$$p_{s,j} = \overline{r}_s + \kappa \sum_{v \in N_{s,j}} w(s,v)(r_{v,j} - \overline{r}_v), \tag{12}$$

where κ is a normalisation factor and $w(s,v)$ represents the similarity between users s and v. There are several ways to compute user similarity, where the most commonly used method is the Pearson correlation coefficient [1]:

$$w(s,v) = \frac{\sum_{i \in I_{s,v}} (r_{s,i} - \overline{r}_s)(r_{v,i} - \overline{r}_v)}{\sqrt{\sum_{i \in I_{s,v}} (r_{s,i} - \overline{r}_s)^2 \sum_{i \in I_{s,v}} (r_{v,i} - \overline{r}_v)^2}}, \tag{13}$$

where $I_{s,v}$ represents the set of items that both users s and v has rated, and $w(s,v)$ is located in the range of $[-1,1]$. A problem for similarity computation is that in case of none or only few commonly rated items, i.e. the size of $I_{u,v}$ is small, the computed similarity is not reliable which results the predicted value uncertain. This problem is called *cold start*. However, when representing predictions in terms of subjective opinions, the degree of uncertainty can be explicitly expressed. Below is described a method by means of which subjective opinions can be derived from raw CF predictions.

The derivation is based on three intuitive assumptions. First, the uncertainty of the derived prediction opinion as expressed by Eq.(10) is a decreasing function of the number of ratings by similar users in $N_{s,j}$. Second, the probability expectation value of the derived prediction opinion as expressed by Eq.(7) is equal to the prediction of Eq.(12). Third, Eq.(6) also holds. Thus the set of equations below emerges.

$$\begin{cases} u_j^s \ = \frac{W}{W + \sum\limits_{v \in N_{s,j}} |I_{s,v}|} \\ p_{s,j} = b_j^s + a u_j^s \\ 1 \ = b_j^s + d_j^s + u_j^s \end{cases} \Rightarrow \begin{cases} u_j^s = \frac{W}{W + \sum\limits_{v \in N_{s,j}} |I_{s,v}|} \\ b_j^s = p_{s,j} - a u_j^s \\ d_j^s = 1 - b_j^s - u_j^s \end{cases} \tag{14}$$

where $W = 2$ is the non-informative prior weight. As before $N_{s,j}$ is the neighbourhood of user s constrained by having rated item j, and $I_{s,v}$ is the set of items that both users s and v have rated.

Although Eq.(14) is obtained from a user-based CF method, it can be easily adapted to item-based methods by:

$$\begin{cases} u_j^s \;= \frac{W}{W + \sum\limits_{i \in N_{s,j}} |U_{i,j}|} \\ p_{s,j} = b_j^s + a u_j^s \\ 1 \;\;= b_j^s + d_j^s + u_j^s \end{cases} \Rightarrow \begin{cases} u_j^s = \frac{W}{W + \sum\limits_{i \in N_{s,j}} |U_{i,j}|} \\ b_j^s = p_{s,j} - a u_j^s \\ d_j^s = 1 - b_j^s - u_j^s \end{cases} \tag{15}$$

where $N_{s,j}$ is the neighbourhood of item j, i.e. the set of items that have been rated by the users who also rated target item j as well as (some of) items rated by user s, and $U_{i,j}$ is the set of users who rated both items i and j. As before, generally only the top-K most similar items will be selected as the neighbourhood for rating prediction.

5 Combining Recommender and Reputation Values

After obtaining the subjective opinions from reputation systems and recommender systems respectively, the question is how they can be combined. We present the cascading minimum common belief fusion (CasMin) as a relatively conservative operator for fusing rating levels expressed as opinions. The detailed algorithm of CasMin fusion is also given below, and the usage is exemplified at the end of this section.

5.1 Cascading Minimum Common Belief Fusion

Various belief fusion models can be used to model specific situations. It is often challenging to determine the correct or the most appropriate fusion operator for a specific situation, see e.g. [9] for a discussion. We now present a new fusion model called *Cascading Minimum Common Belief Fusion* (CasMin) which is applicable when the states in the frame represent ordered levels.

When fusing belief masses on the highest order state in the frame, the greatest belief mass in one argument is reduced to match the belief mass in the other argument to produce the mutual minimum belief mass on that state. The amount of belief mass removed from the greatest belief mass is cascaded to the belief mass of the next inferior state in the frame and so forth until the lowest order state in the frame is reached. Belief mass from the least arguments can also be matched by uncertainty mass from the other argument, so that uncertainty typically is reduced, and belief mass in the lowest order states typically is increased.

An example situation is company investment where weighted ratings are given by analysts expressed as (1) *strong sell*, (2) *sell*, (3) *hold*, (4) *buy*, (5) *strong buy*. An investor might want to determine conservative company ratings based on the CasMin fusion model, so that in case a single analyst gives a low rating to a company on a specific level then the CasMin rating on that level is low even if all the other analysts give a high rating to the same company on that level. The conservative property of this fusion operator is useful in situations of possible bias in the arguments such as market analysis, where analysts tend to avoid negative opinions as they typically receive flack from the management teams and pressure that they may lose access to the companies that they cover.

The case that we are interested in is about giving advice that is confirmed by both recommendation values and reputation scores for resources. CasMin fusion provides a conservative fusion model for this situation because it takes the smallest of reputation score and recommendation value on each level, starting from the highest level, and on each level cascading the overshooting values down to the level below. A high CasMin fusion result, i.e. with large scores/values for high levels, can only be obtained when both reputation scores and recommendation values are high. In this way, the advice produced by CasMin fusion will be more conservative than that provided by reputation systems or recommender systems alone. We will describe the details of CasMin fusion in next sub section.

5.2 CasMin Fusion Operator

Let $X = \{x_1, \ldots x_k\}$ be an ordered frame where x_k is considered to be the highest order state predefined by a recommender or reputation system. The reduced powerset of X is denoted $\mathcal{R}(X)$ with cardinality κ. Assume that there are two opinions ω_X^A and ω_X^B over the frame X where the superscripts A and B represent the belief owners. The two opinions can be mathematically merged using the CasMin operator which in expressions is denoted as: $\omega_X^{(A \underline{\downarrow} B)} = \text{CasMin}(\omega_X^A, \omega_X^B)$.

The CasMin operator requires binomial or multinomial opinions, so in case of hyper opinion arguments, first project to binomial or multinomial opinions as described by Eq.(16), where the beliefs of the hyper opinion ω'_X are denoted as $\boldsymbol{b}'_X$, and the the beliefs of the multinomial opinion ω_X are denoted as $\boldsymbol{b}_X$.

$$\boldsymbol{b}_X(x_i) = \sum_{x_j \in \mathcal{R}(X)} \boldsymbol{a}_X(x_i/x_j)\, \boldsymbol{b}'_X(x_j)\ , \forall x_i \in X, \tag{16}$$

With multinomial opinions arguments the CasMin fusion operation proceeds according to the algorithm of Fig.1. Specifically, it first acts on the belief on the highest level state x_k and finally on the belief on lowest level state x_1. Line 2 ensures that the belief on the A-argument is always greater than that of the B-argument, by executing a swap operation if necessary. For each level x_i, there are two possible cases, i.e. whether the A-argument's belief is less than or equal to the sum of the B-argument's belief and uncertainty (lines 3-7) or not (lines 8-13). In either case, (a part of) the B-argument's uncertainty can compensate for it's belief value being less than that of the A-argument (lines 4, 9-10). The remaining minimum belief value will be assigned to both A and B's arguments (lines 5, 12), and then the differences between the new and previous belief values (lines 6, 11) will be cascaded to the next inferior state x_{i-1} (line 14). This procedure will be repeated until the frame is finished. Finally, user A's new opinion represents the fused result and will be returned (line 16).

FOR i = k to 2 DO {
    IF $\boldsymbol{b}_X^A(x_i) \leq \boldsymbol{b}_X^B(x_i)$ THEN {Swap(ω_X^A, ω_X^B);}
    IF $u_X^B > (\boldsymbol{b}_X^A(x_i) - \boldsymbol{b}_X^B(x_i))$ THEN {
        $u_X^B = u_X^B - (\boldsymbol{b}_X^A(x_i) - \boldsymbol{b}_X^B(x_i))$;
        $\boldsymbol{b}_X^B(x_i) = \boldsymbol{b}_X^A(x_i)$;
        $b_{\text{cascade}} = 0$;
    }
    ELSE {
        $\boldsymbol{b}_X^B(x_i) = \boldsymbol{b}_X^B(x_i) + u_X^B$;
        $u_X^B = 0$;
        $b_{\text{cascade}} = \boldsymbol{b}_X^A(x_i) - \boldsymbol{b}_X^B(x_i)$;
        $\boldsymbol{b}_X^A(x_i) = \boldsymbol{b}_X^B(x_i)$;
    }
    $\boldsymbol{b}_X^A(x_{i-1}) = \boldsymbol{b}_X^A(x_{i-1}) + b_{\text{cascade}}$;
}
$\omega_X^{(A \underline{\downarrow} B)} = \omega_X^A$;

Fig. 1. Algorithm for the CasMin belief fusion operator

The CasMin operator is commutative, associative and idempotent, and a totally uncertain opinion acts as the neutral element for the CasMin operator.

5.3 Example

We consider the case of providing advice about hotels through a web site such as e.g. `tripadvisor.com`. It is assumed that a recommender system tracks user preferences, and that a reputation system allows users to rate hotels.

With the method described in Eq.(10) the reputation system can produce scores expressed as multinomial opinions. With the method described in Eq.(8) the multinomial opinions can be transformed into binomial opinions.

The recommender system can also use a multi-aspect and multi-level representation of ratings. A user can rate general satisfaction high even if another aspect such as cleanliness is rated low, e.g. in case cleanliness is not an important preference for the user. The recommender system is thus able to identify hotels that match the users personal preference. The recommendation values for each hotel and each user are expressed as binomial opinions using Eq.(14) or Eq.(15).

The recommender system identifies a list of hotels based on the ratings given by the user and other travelers. The recommender system can predicted that the user will like the hotels because other users with similar tastes have rated the hotels with satisfaction. In contrast, the reputation system offers community-wide scores for each hotel. The CasMin operator produces conservative results in the sense that hotels must simultaneously have high recommendation values and high reputation scores. The numerical example of Table 1 illustrates the result of fusing two such opinions according to the CasMin algorithm of Fig.1.

Table 1. Fusion of reputation scores and recommendation values

Hotel	Rep. Ratings	Multinomial Rep. Score	Binomial Rep. Score	Rec. Value	CasMin Advice
Hotel I	$r(x_5) = 50$	$b_{x_5} = 0.65$	$b = 0.81$	$b = 0.1$	$b = 0.30$
	$r(x_4) = 10$	$b_{x_4} = 0.13$	$d = 0.16$	$d = 0.7$	$d = 0.70$
	$r(x_3) = 10$	$b_{x_3} = 0.13$	$u = 0.03$	$u = 0.2$	$u = 0.00$
	$r(x_2) = 0$	$b_{x_2} = 0.00$			
	$r(x_1) = 5$	$b_{x_1} = 0.06$			
		$u_X = 0.03$			
Hotel II	$r(x_5) = 5$	$b_{x_5} = 0.06$	$b = 0.16$	$b = 0.7$	$b = 0.19$
	$r(x_4) = 0$	$b_{x_4} = 0.00$	$d = 0.81$	$d = 0.1$	$d = 0.61$
	$r(x_3) = 10$	$b_{x_3} = 0.13$	$u = 0.03$	$u = 0.2$	$u = 0.20$
	$r(x_2) = 10$	$b_{x_2} = 0.13$			
	$r(x_1) = 50$	$b_{x_1} = 0.65$			
		$u_X = 0.03$			
Hotel III	$r(x_5) = 50$	$b_{x_5} = 0.65$	$b = 0.81$	$b = 0.7$	$b = 0.81$
	$r(x_4) = 10$	$b_{x_4} = 0.13$	$d = 0.16$	$d = 0.1$	$d = 0.19$
	$r(x_3) = 10$	$b_{x_3} = 0.13$	$u = 0.03$	$u = 0.2$	$u = 0.00$
	$r(x_2) = 0$	$b_{x_2} = 0.00$			
	$r(x_1) = 5$	$b_{x_1} = 0.06$			
		$u_X = 0.03$			

Table 1 shows the results of analysing three separate hotels called Hotel I, II and III, respectively. In case of Hotel I and Hotel II where the recommendation values and reputation scores are in conflict, the fused belief value is small. The only strong result is for Hotel III where both the recommendation value and reputation score are positive. In addition, as shown in cases of Hotel I and Hotel III, it is often the case for reputation systems that the scores have a strong positive bias, reducing the utility and discriminating power of the reputation system. The advantage of combining recommender systems and reputation systems is to amplify the discriminating power.

6 Conclusions

Since both recommender systems and reputation systems support decision making we believe that combining both types of systems may produce better advice than any individual systems can do alone. However, the significant differences in the underlying theory and implementation make such integration challenging. In this paper, we proposed a method to represent reputation scores and recommendation values within the framework of subjective logic. We also proposed the new CasMin fusion operator in order to fuse the results from recommender and reputation systems in a conservative fashion, i.e. so that high results can only be obtained when both reputation scores and recommendation values are high for a given resource. The proposed method was illustrated with a hypothetical example. In future research we intent to apply the method to real data in order to judge its usefulness.

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(♯) The work reported in this paper has been partially funded by UNIK.