Biometric Data Fusion Based on Subjective Logic [1]

Audun Jøsang
University of Oslo
Norway
josang@mn.uio.no

Thorvald H. Munch-Møller
University of Oslo
Norway
thorvahm@ifi.uio.no

***Abstract*—Biometric identification and authentication is affected by varying levels of reliability depending on the inherent strength of a specific biometric modality, the specific techniques used to analyse the modality, as well as the quality of biometric samples. In many situations there are multiple biometric samples of the same modality available, or there are samples of different modalities available. In such situations the analysis includes some form of data fusion in order to optimize the accuracy of the biometric analysis. Biometric data fusion can be performed at different levels of the analysis and decision making process. This paper describes how subjective logic can be applied for biometric data fusion for multiple samples, multiple modalities and at different fusion levels.**

I. Introduction

Biometric analysis can be used in *authentication mode* where a $1:1$ match is assessed, or in *identification mode* where a $1:N$ match is assessed for a population of N individuals. A biometric modality or characteristic is a type or class of biometric system, for example: face recognition, fingerprint recognition, iris recognition, and so forth. Each biometric modality has different properties with regard to their accuracy, ease of use and practical implementation. The following four criteria are critical in order for a biometric modality to be used in a practical implementation.

1) **Universality:** Each person should be able to express the biometric characteristic.
2) **Distinctiveness:** Any two persons should be sufficiently different in terms of the characteristic.
3) **Permanence:** The characteristic should be sufficiently invariant (with respect to the matching criterion) over a period of time.
4) **Collectability:** The characteristic should be measurable quantitatively.

In addition there are practical considerations that must also be taken into account when implementing a specific biometric modality in a biometric system, such as

- **Accuracy:** The achievable accuracy of the biometric analysis, commonly expressed in terms of ERR (Equal Error Rate), where a low ERR value is desirable.
- **Performance:** The achievable speed of analysis, and the economy of resources required to achieve the desired speed of analysis.
- **Acceptability:** The extent to which people are willing to accept the use of a particular biometric characteristic.
- **Circumvention resistance**: The difficulty of fooling the biometric system. This criterion is related to the accuracy criterion above.
- **Safety:** Whether the biometric system is safe to use.

The most important practical consideration is the accuracy of a biometric modality. To improve the accuracy through biometric data fusion is the topic of this paper.

In the operation phase it is assumed that a biometric template has already been extracted for each person registered in the biometric system. The template consists of digitally represented features of a biometric characteristic. During operation, features from the new sample captured by the biometric sensor are compared against features from the stored template sample. Based on this comparison a score s is derived, where a better match leads to higher score value.

The threshold T determines the system decision. In particular, a *match* or *assumed user* is when the comparison of samples generates a score $s \geq T$, which leads to an accept decision. Alternatively, a *non-match* or *assumed imposter* is when the comparison of samples generates a score $s < T$, which leads to a reject decision.

Biometric samples from genuine users can of course generate relatively low scores, e.g. in case a fingerprint sample is collected from a dirty or injured finger, in which case a genuine user typically is falsely rejected. Similarly, it is possible that samples from an imposter can generate a relatively high score, e.g. in case of similar biometric characteristics such as when user and imposter have similarly looking faces, in which case an imposter can be falsely accepted as a specific user.

The recognition capability of a biometric system is characterised by the score distributions resulting from exposing the system to a number of known users, as well as to a number of known imposters. The FMR (False Match Rate) and FNMR (False Non-Match Rate) are then determined based on the score distributions together with the chosen threshold T.

Assuming that a biometric system is used in authentication mode, then FMR and FNMR are determined according to:

$$\begin{cases} \text{FMR} = \dfrac{\#\text{ of matching imposter samples}}{\text{total }\#\text{ of imposter samples}} \\ \\ \text{FNMR} = \dfrac{\#\text{of non-matching user samples}}{\text{total }\#\text{ of user samples}} \end{cases} \tag{1}$$

[1] In the proceedings of the 17th International Conference on Information Fusion (FUSION 2014), Salamanca, July, 2014.

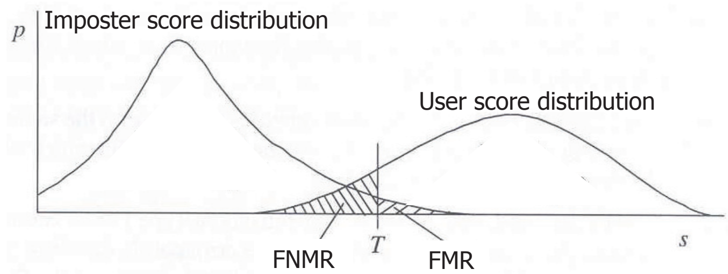

Figure 1. Determining FMR and FNMR from score distributions and threshold T.

The relationship between the score distributions, the threshold and the rates FMR and FNMR is illustrated in Fig.1.

The accuracy of a biometric system is typically expressed in terms of EER (Equal Error Rate) which is is determined by tuning the threshold T so that FMR = FNMR (= ERR). The smaller the ERR the higher the accuracy, with perfect accuracy when ERR $= 0\%$. Unfortunately no practical biometric system can have perfect accuracy [1]. Efforts are therefore being invested to improve the accuracy and performance of existing systems, and to create new biometric systems with superiour performance, better accuracy or a combination of both.

The combination of multiple different samples or the combination of multiple biometric modalities is a natural approach for improving the quality of biometric systems, which is general is called biometric fusion. This approach depends on specific methods for data fusion during the analytical process. This paper proposes a framework for biometric data fusion based on the application of subjective logic, called BDF-SL.

II. Biometric Data Fusion

Data fusion can be performed at different levels of the biometric analysis process, and can fuse data obtained from the same modality, or from different modalities. Relevant data fusion levels are 1) sensor level, 2) feature level, 3) opinion level and 4) decision level. Each of these fusion levels are briefly described below.

1) **Sensor Level Data Fusion**
   At this level the same biometric characteristics are recorded with two or more physical sensors. This is necessarily unimodal data fusion because it is assumed that multiple sensors record data relate to the same modality.
   Combination of sensor data can provide noise cancellation, blind source separation, and so forth. [2].
   An extension of this principle is to fuse unimodal data in order to create richer data models, such as in face recognition when multiple cameras can be used to record frontal and profile images for deriving three-dimensional face models, which in turn can be used for feature extraction.
2) **Feature Level Data Fusion**
   Feature level data fusion can be applied to both unimodal and multimodal biometric systems.
   In case of a unimodal system it is possible to extract multiple features from a single biometric recording. Through combination of these features it is possible to derive a set of fused features that are assumed to be of higher quality than the individual features in isolation. An example of a unimodal feature fusion is the combination of instantaneous and transitional features in voice recognition.
   In case of multi-modal feature fusion, extracted features from different modalities can be fused to create a combined set of features. An example of multi-modal feature fusion is the combination of face recognition features and iris features, to create a richer set of features related to face images.
3) **Opinion Level Data Fusion**
   Opinion level data fusion can be applied to both unimodal and multimodal biometric systems. Most commonly, scores from multiple samples of the same or of different modalities are combined, in which case it is called score-level data fusion. However, it is also possible to combine other metrics such as ranks, in which case it is called rank-level data fusion.
   In case of a unimodal system multiple scores derived from multiple samples of the same modality. For example, multiple photos that are know to represent the same person can be compared with the stored face template of a given person. Each photo will give a different score, so some form of combination of each score can provide higher certainty about the likeness than each photo separately.
   In case of multimodal systems the scores are often derived from different scales, so scaling and normalisation might be required.
   The crucial aspect of opinion level data fusion is the weight that is to be given to the various scores, where specific methods can be weighted sum or average, weighted product, and decision trees. Intuitively, combining scores from the same modality is very different from combining scores from different modalities. This is discussed in more detail below.
4) **Decision Level Data Fusion**
   Decision Level Data Fusion can be applied to unimodal or multimodal biometric systems. In both cases each classifier provides a sub-decision, where the final de-

cision is determined by some form of combination of each sub decision.

Methods for determining the final decision can be based on a voting scheme, boolean operators such as AND and OR, or other decision algorithms. Intuitively, combining multiple decisions based on the same modality is quite different from combining multiple decisions based on different modalities. This is discussed in more detail below.

It can be argued that low-level biometric data fusion (e.g. signal or feature-level fusion) has the potential to offer higher accuracy than high-level biometric data fusion [3]. However, it is also important to consider the simplicity of implementation and performance issues, so that decision level fusion also can have many advantages, and score-level fusion might provide a good trade-off between potential accuracy, performance and ease of implementation [4].

A general observation of biometric analysis is that when the quality of the measure is questionable, then the derived score values and the decisions become uncertain. We therefore propose to perform biometric data fusion based on subjective logic which is a framework which explicitly can handle degrees of uncertainty in respect to biometric samples.

We first describe the opinion representation in subjective logic as well as the relevant fusion models in Section III. Section VI proposes how these fusion models can be applied to situations of biometric fusion. Section VII rounds off the paper with a discussion and indication of future work.

III. Subjective Logic

In this section, we will first introduce the notation and formation of subjective opinions used for biometric data fusion.

A. Opinion Representations

A subjective opinion expresses belief about states of *frame of discernment* or *frame* for short, which is equivalent to a traditional state space. Let $X = \{x_1, x_2, \ldots, x_k\}$ be a frame of cardinality k, where x_i $(1 \leq i \leq k)$ represents a specific state. An opinion distributes belief mass over the reduced powerset of the frame denoted as $\mathcal{R}(X)$ defined as:

$$\mathcal{R}(X) = \mathcal{P}(X) \setminus \{X, \emptyset\} \ , \tag{2}$$

where $\mathcal{P}(X)$ denotes the powerset of X and $|\mathcal{P}(X)| = 2^k$. All proper subsets of X are states of $\mathcal{R}(X)$, but the frame X and the empty set $\emptyset$ are not states of $\mathcal{R}(X)$, in line with the hyper-Dirichlet model [5]. $\mathcal{R}(X)$ has cardinality $\kappa = 2^k - 2$.

An opinion is a composite function that consists of a belief vector b, an uncertainty parameter u and base rate vector a that take values in the interval $[0,1]$ and that satisfy the following additivity requirements.

$$\text{Belief additivity: } u_X + \sum_{x_i \in \mathcal{R}(X)} b_X(x_i) = 1. \tag{3}$$

$$\text{Base rate additivity: } \sum_{i=1}^{k} a_X(x_i) = 1, \ \text{ where } \ x_i \in X. \tag{4}$$

User A's opinion over frame X is denoted $\omega_X^A = (b_X, u_X, a_X)$, where b_X is a belief vector over the states of $\mathcal{R}(X)$, u_X is the complement uncertainty mass, and a_X is a base rate vector over X, all seen from the viewpoint of source A. Base rates of elements $x_i \in \mathcal{R}(X)$ can be computed as:

$$a_X(x_i) = \sum_{(x_j \cap x_i) \neq \emptyset, \, x_j \in X} a_X(x_j), \ \forall \, x_i \in \mathcal{R}(X) \ . \tag{5}$$

The belief vector b_X has $(2^k - 2)$ parameters, whereas the base rate vector a_X only has k parameters. The uncertainty parameter u_X is a simple scalar. Thus, a general opinion contains $(2^k + k - 1)$ parameters. However, given that Eq.(3) and Eq.(4) remove one degree of freedom each, opinions over a frame of cardinality k only have $(2^k + k - 3)$ degrees of freedom. The probability projection of hyper opinions is the vector E_X expressed as:

$$\mathrm{E}_X(x_i) = \sum_{x_j \in \mathcal{R}(X)} a_X(x_i/x_j) \, b_X(x_j) + a_X(x_i) \, u_X, \quad \forall x_i \in \mathcal{R}(X) \tag{6}$$

where $a_X(x_i/x_j)$ denotes relative base rate, i.e. the base rate of subset x_i relative to the base rate of (partially) overlapping subset x_j, defined as follows:

$$a_X(x_i/x_j) = \frac{a_X(x_i \cap x_j)}{a_X(x_j)}, \ \forall \, x_i, x_j \in \mathcal{R}(X). \tag{7}$$

Equivalent probabilistic representations of opinions, e.g. as Beta PDF (probability density function) or a Dirichlet PDF, offer an alternative interpretation of subjective opinions in terms of traditional statistics [6].

The term *hyper opinion* is used for a general opinion [6]. A *multinomial opinion* is when the belief vector b_X only applies to elements $x_i \in X$, not in $\mathcal{R}(X)$. *Binomial opinions* apply to binary frames and have a special notation as described below.

Let $X = \{x, \overline{x}\}$ be a binary frame, then a binomial opinion about the truth of state x is the ordered quadruple $\omega_x = (b, d, u, a)$ where:

b,	*belief*:	belief mass in support of x being true;
d,	*disbelief*:	belief mass in support of $\overline{x}$ (NOT x);
u,	*uncertainty*:	uncertainty about probability of x;
a,	*base rate*:	non-informative prior probability of x.

The special case of Eq.(3) in case of binomial opinions is expressed by Eq.(8).

$$b + d + u = 1. \tag{8}$$

Similarly, the special case of the probability expectation value of Eq.(6) in case of binomial opinions is expressed by Eq.(9).

$$\mathrm{E}_x = b + au. \tag{9}$$

Binomial and multinomial opinions can be visualised as a point inside a simplex. Binomial opinions can thus be visualised as a point inside an equal sided triangle, and a trinomial opinion as a point inside a tetrahedron. The case of binomial opinions is illustrated in Fig.2

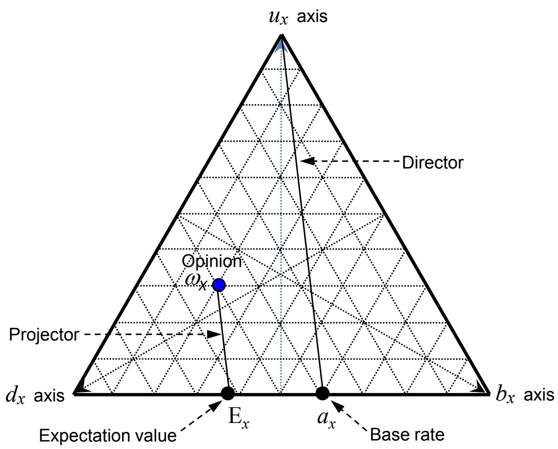

Figure 2. Opinion triangle with example binomial opinion

The opinion $\omega_x = (0.2,\, 0.5,\, 0.3,\, 0.6)$ with expectation value $E_x = 0.38$ is shown as an example. The base rate[1], is shown as a point on the base line, and the probability expectation, E_x, is formed by projecting the opinion point onto the base, parallel to the base rate director line.

Characteristics of various binomial opinion classes are listed below. For example, a binomial opinion:

where $b = 1$	is equivalent to binary logic TRUE,
where $d = 1$	is equivalent to binary logic FALSE,
where $b + d = 1$	is equivalent to a probability,
where $b + d < 1$	expresses degrees of uncertainty,
where $b + d = 0$	expresses total uncertainty.

General opinions are also called *hyper opinions*. A *multinomial opinion* is when belief mass only applies to singleton statements in the frame. A *binomial opinion* is when the frame is binary. A *dogmatic opinion* is an opinion without uncertainty, i.e. where $u = 0$. A *vacuous opinion* is an opinion that only contains uncertainty, i.e. where $u = 1$.

Equivalent probabilistic representations of opinions, e.g. as a Beta PDF (probability density function) in case of binomial opinions, as a Dirichlet PDF in case of multinomial opinions, or as a hyper Dirichlet PDF in case of hyper opinions offer an alternative interpretation of subjective opinions in terms of traditional statistics [6].

IV. Determining Opinions from Biometric Sensors

In our approach, binomial opinions are used to represent biometric opinion scores. Each specific feature extracted from a biometric sample can be matched against the corresponding features stored in the template for a given person. Assume N features denoted by f_i where $i = 1 \ldots N$. We assume that the degree of match of each feature f_i is given a rating $r(f_i)$ where $r(f_i) \in [0,1]$. This might require normalisation in case the underlying model for feature comparison provides ratings on a different scale. The more features analysed, the more ratings collected, and the smaller the uncertainty of the biometric score opinion becomes. The score opinion ω_X can be determined from the ratings $r(f_i)$ according to Eq.(10):

$$\omega_X \begin{cases} b = \frac{\sum r(f_i)}{W+N} \\ d = \frac{\sum(1-r(f_i))}{W+N} \\ u = \frac{W}{W+N} \end{cases} \qquad (10)$$

where $W = 2$ is the non-informative prior weight with default value dictated by the requirement of having a uniform PDF over binary frames in case of absence of evidence. The value would e.g. be $W = 3$ in case it were required to have a uniform PDF over a ternary frame. However, higher values for W make the probability distribution less sensitive to new evidence, so the value $W = 2$ is adopted [7].

Fig.3 illustrates the principle for deriving biometric opinion scores and using those scores for making decisions.

An opinion derived according to Eq.(10) can thus represent a biometric score which can always be mapped to a probability value whenever required. The advantage of the opinion representation is that it directly takes into account the amount of evidence supporting the score.

V. Subjective Opinion Fusion Models

Belief fusion involve belief arguments from multiple sources that must be fused in some way to produce a single belief argument. The purpose of opinion fusion is to produce a new opinion that is more correct or representative.

Different opinions can be fused in various ways, each having an impact on how the specific type of biometric fusion is modeled. It can be challenging to determine the correct or the most appropriate fusion operator for a specific situation. This section characterizes and describes various fusion models defined for subjective logic.

A. Constraining Fusion

Constraining opinion fusion is when it is assumed that (a) each opinion can dictate the correctness of the outcome, and (b) conflicting belief between the two sources is assigned to common states considered correct by both sources. In this fusion class, if two belief arguments express totally conflicting beliefs, i.e. no common state is considered correct by both sources, then they effectively veto each other's beliefs - which means that no state is correct. An example is when two persons try to agree on seeing a movie at the cinema. If their preferences include some common movies they can decide to see one of them. Yet, if their preferences do not have any movies in common then there is no solution, so the rational consequence is that they will not watch any movie together.

The belief constraint operator is an extension of Dempster's rule which in Dempster-Shafer belief theory is often presented as a method for fusing evidence from different sources [8] in order to identify the most likely hypothesis from the frame. Many authors have however demonstrated that Dempster's rule is not an appropriate operator for evidence fusion [9], and that

[1]Also called *relative atomicity*

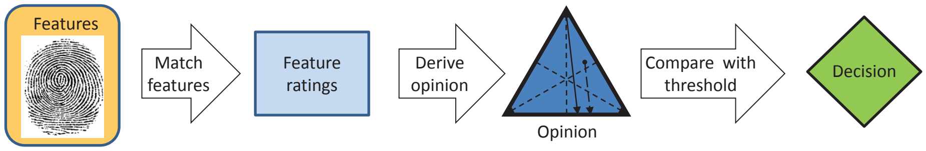

Figure 3. Deriving biometric opinion scores

it is better suited as a method for combining constraints [10], [11], which is also our view.

Assume two opinions ω_X^A and ω_X^B over the frame X of cardinality k with reduced powerset $\mathcal{R}(X)$ of cardinality κ. The superscripts A and B are attributes that identify the respective belief sources or belief owners. These two opinions can be mathematically merged using the belief constraint operator denoted as '$\odot$', which can be expressed as:

$$\text{Constraining Belief Fusion:} \quad \omega_X^{A\&B} = \omega_X^A \odot \omega_X^B \ . \tag{11}$$

Belief source combination denoted with the symbol '&' thus corresponds to opinion fusion with '$\odot$'. The algebraic expression of the belief constraint operator for subjective opinions is described next.

$$\omega_X^{A\&B} = \tag{12}$$

$$\begin{cases} \vec{b}^{A\&B}(x_i) & = \frac{\text{Har}(x_i)}{(1-\text{Con})}, \quad \forall\, x_i \in \mathcal{R}(X),\ x_i \neq \emptyset \\ u_X^{A\&B} & = \frac{u_X^A u_X^B}{(1-\text{Con})} \\ \vec{a}^{A\&B}(x_i) & = \frac{\vec{a}^A(x_i)(1-u_X^A)+\vec{a}^B(x_i)(1-u_X^B)}{2-u_X^A-u_X^B}, \quad \forall\, x_i \in X,\ x_i \neq \emptyset \end{cases}$$

The term $Har(x_i)$ represents the degree of *Harmony* (overlapping belief mass) on x_i. The term *Con* represents the degree of *Conflict* (non-overlapping belief mass) between ω_X^A and ω_X^B. These are defined below:

$$Har(x_i) = \vec{b}^A(x_i)u_X^B + \vec{b}^B(x_i)u_X^A + \sum_{y \cap z = x_i} \vec{b}^A(y)\vec{b}^B(z) \tag{13}$$

$$Con = \sum_{y \cap z = \emptyset} \vec{b}^A(y)\vec{b}^B(z) \tag{14}$$

The divisor $(1 - Con)$ in Eq.(12) normalizes the derived belief mass; it ensures belief mass and uncertainty mass additivity. The use of the belief constraint operator is mathematically possible only if ω^A and ω^B are not totally conflicting, i.e., if $Con \neq 1$.

The belief constraint operator is commutative and non-idempotent. Associativity is preserved when the base rate is equal for all agents. Associativity in case of different base rates requires that all preference opinions be combined in a single operation which would require a generalization of Eq.(12) for multiple agents, i.e. for multiple input arguments, which is relatively trivial.

B. Cumulative Fusion

Cumulative opinion fusion is when it is assumed that it is possible to collect an increasing amount of independent evidence by including more and more arguments, and that certainty about the most correct state increases with the amount of evidence accumulated. A typical case depicting this type of fusion is when one makes statistical observations about possible outcomes, i.e. the more observations the stronger the analyst's belief about the likelihood of each outcome. For example, a mobile network operator could observe the location of a subscriber over time, which will produce increasing certainty about the most frequent locations of that subscriber. However, the result would not necessarily be suitable for indicating the exact location of the subscriber at a specific time.

The cumulative fusion rule is equivalent to *a posteriori* updating of Dirichlet distributions. Its derivation is based on the bijective mapping between the belief and evidence notations described in [6].

The symbol '$\diamond$' denotes the cumulative fusion of two observers A and B into a single imaginary observer $A \diamond B$.

Let ω^A and ω^B be opinions respectively held by agents A and B over the same frame X of cardinality k with reduced powerset $\mathcal{R}(X)$ of cardinality κ. Let $\omega^{A\diamond B}$ be the opinion where:

Case I: For $u^A \neq 0 \ \vee \ u^B \neq 0$:

$$\begin{cases} b^{A\diamond B}(x_i) & = \frac{b^A(x_i)u^B + b^B(x_i)u^A}{u^A+u^B-u^A u^B} \\ u^{A\diamond B} & = \frac{u^A u^B}{u^A+u^B-u^A u^B} \end{cases} \tag{15}$$

Case II: For $u^A = 0 \ \wedge \ u^B = 0$:

$$\begin{cases} b^{A\diamond B}(x_i) & = \gamma^A b^A(x_i) + \gamma^B b^B(x_i) \\ u^{A\diamond B} & = 0 \end{cases} \tag{16}$$

$$\text{where} \quad \begin{cases} \gamma^A = \lim\limits_{\substack{u^A \to 0 \\ u^B \to 0}} \frac{u^B}{u^A+u^B} \\ \gamma^B = \lim\limits_{\substack{u^A \to 0 \\ u^B \to 0}} \frac{u^A}{u^A+u^B} \end{cases}$$

Then $\omega^{A\diamond B}$ is the cumulatively fused opinion of ω^A and ω^B, representing the combination of independent opinions of A and B. By using the symbol '$\oplus$' to designate this belief operator, cumulative fusion is expressed as:

$$\text{Cumulative Belief Fusion:}\quad \omega_X^{A\diamond B} = \omega_X^A \oplus \omega_X^B\ . \tag{17}$$

The cumulative fusion operator is commutative, associative and non-idempotent. In Eq.(16) the associativity depends on the preservation of relative weights of intermediate results through the weight variable γ, in which case the cumulative rule is equivalent to the weighted average of probabilities.

C. Averaging Fusion

Averaging opinion fusion is when dependence between arguments is assumed. In other words, including more arguments does not mean that more evidence is supporting the conclusion. An example of this type of situation is when a jury tries to reach a verdict after having observed the court proceedings. Because the evidence is limited to what was presented to the court, the certainty about the verdict does not increase by having more jury members expressing their beliefs, since they were all exposed to the same evidence.

Averaging belief fusion is derived from averaging arguments represented as evidence (not belief) through the bijective mapping between evidence and belief in subjective logic [6].

The symbol '$\underline{\diamond}$' denotes the averaging fusion of two observers A and B into a single imaginary observer $A\diamond B$.

Let ω^A and ω^B be the respective opinions of agents A and B over the same frame X of cardinality k with reduced powerset $\mathcal{R}(X)$ of cardinality κ. Let $\omega^{A\underline{\diamond}B}$ be the opinion such that:

Case I: For $u^A \neq 0 \ \vee\ u^B \neq 0$:

$$\begin{cases} b^{A\underline{\diamond}B}(x_i) &= \frac{b^A(x_i)u^B + b^B(x_i)u^A}{u^A+u^B} \\ u^{A\underline{\diamond}B} &= \frac{2u^Au^B}{u^A+u^B} \end{cases} \tag{18}$$

Case II: For $u^A = 0 \ \wedge\ u^B = 0$:

$$\begin{cases} b^{A\underline{\diamond}B}(x_i) &= \gamma^A b^A(x_i) + \gamma^B b^B(x_i) \\ u^{A\underline{\diamond}B} &= 0 \end{cases} \tag{19}$$

$$\text{where} \begin{cases} \gamma^A = \lim\limits_{\substack{u^A\to 0\\ u^B\to 0}} \frac{u^B}{u^A+u^B} \\ \gamma^B = \lim\limits_{\substack{u^A\to 0\\ u^B\to 0}} \frac{u^A}{u^A+u^B} \end{cases}$$

Then $\omega^{A\underline{\diamond}B}$ is the averaging opinion of ω^A and ω^B, representing the combination of the possibly dependent opinions of A and B. By using the symbol '$\underline{\oplus}$' to designate this belief operator, averaging fusion is expressed as:

$$\text{Averaging Belief Fusion:}\quad \omega_X^{A\underline{\diamond}B} = \omega_X^A \underline{\oplus} \omega_X^B\ . \tag{20}$$

It can be verified that the averaging fusion rule is commutative and idempotent; but it is *not* associative.

D. Consensus & Compromise Fusion

Consensus & Compromise fusion is when no single opinion argument alone can dictate that specific states of the frame are the most correct. In this fusion class the analyst naturally wants to preserve shared beliefs from each argument, and in addition transform conflicting beliefs into new shared beliefs on union subsets. In this way consensus belief is preserved when it exists and compromise belief is formed when necessary. In case of totally conflicting opinions on a binary frame, then the resulting fused belief is totally uncertain. An example is when two sensors produce identical scores, then consensus & compromise fusion would say the same, because there is a consensus. However, when assuming that two sensors produce totally conflicting scores, then consensus & compromise fusion would return the result that the fused score is totally uncertain, because uncertainty is the best compromise in case of totally conflicting opinions.

CC-fusion (Consensus & Compromise) satisfies the requirements of being idempotent, having a neutral element, and where conflicting beliefs result in compromise beliefs. This shows that it is possible to design fusion models to fit particular requirements.

Assume two opinions ω_X^A and ω_X^B over the frame X of cardinality k with reduced powerset $\mathcal{R}(X)$ of cardinality κ. The superscripts A and B are attributes that identify the respective belief sources or belief owners. These two opinions can be mathematically merged using the CC-fusion operator denoted as '$\textcircled{cc}$' which can be expressed as:

$$\text{Consensus \& Compromise Fusion:}\ \omega_X^{A\heartsuit B} = \omega_X^A \textcircled{cc}\, \omega_X^B. \tag{21}$$

Belief source combination denoted with '$\heartsuit$' thus corresponds to opinion fusion with '$\textcircled{cc}$'. The CC-operator is formally described next. It is a two-step operator where the consensus step comes first, and then the compromise step.

1) Consensus Step: The consensus step simply consists of determining shared belief mass between the two arguments, which is stored as the belief vector $\vec{b}_X^{\text{cons}}$ expressed by Eq.(22).

$$\vec{b}_X^{\text{cons}}(x_i) = \min\left(\vec{b}_X^A(x_i),\ \vec{b}_X^B(x_i)\right)\ . \tag{22}$$

The sum of consensus belief denoted b_X^{cons} is expressed as:

$$b_X^{\text{cons}} = \sum_{x_i\in\mathcal{R}(X)} \vec{b}_X^{\text{cons}}(x_i) \tag{23}$$

The residue belief masses of the arguments are:

$$\begin{cases} \vec{b}_X^{\text{res}A}(x_i) = \vec{b}_X^A(x_i) - \vec{b}_X^{\text{cons}}(x_i) \\ \vec{b}_X^{\text{res}B}(x_i) = \vec{b}_X^B(x_i) - \vec{b}_X^{\text{cons}}(x_i) \end{cases} \tag{24}$$

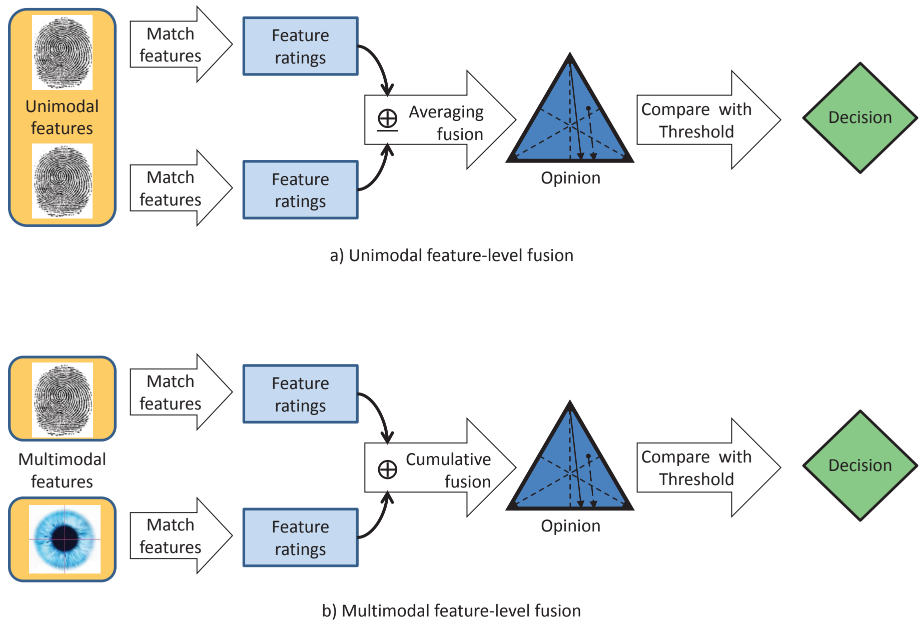

Figure 4. Biometric feature-level fusion models based on subjective logic

2) Compromise Step: The compromise step redistributes conflicting residue belief mass to produce compromise belief mass, stored in $\vec{b}_X^{\text{comp}}$ expressed by Eq.(25).

$$\begin{aligned}\vec{b}_X^{\text{comp}}(x_i) = \; & \vec{b}^{\text{res}A}(x_i)u_X^B + \vec{b}^{\text{res}B}(x_i)u_X^A \\ + & \sum_{\{y \cap z\} = x_i} \vec{a}_X(y/z)\,\vec{a}_X(z/y)\,\vec{b}^{\text{res}A}(y)\,\vec{b}^{\text{res}B}(z) \\ + & \sum_{\substack{\{y \cup z\} = x_i \\ \{y \cap z\} \neq \emptyset}} (1 - \vec{a}_X(y/z)\,\vec{a}_X(z/y))\,\vec{b}^{\text{res}A}(y)\,\vec{b}^{\text{res}B}(z) \\ + & \sum_{\substack{\{y \cup z\} = x_i \\ \{y \cap z\} = \emptyset}} \vec{b}^{\text{res}A}(y)\,\vec{b}^{\text{res}B}(z)\;, \quad \text{where } x_i \in P(X)\;.\end{aligned} \tag{25}$$

The preliminary uncertainty u_X^{pre} is computed as:

$$u_X^{\text{pre}} = u_X^A\, u_X^B\;. \tag{26}$$

The sum of compromise belief denoted b_X^{comp} is:

$$b_X^{\text{comp}} = \sum_{x_i \in P(X)} \vec{b}_X^{\text{comp}}(x_i)\;. \tag{27}$$

In general $b_X^{\text{cons}} + b_X^{\text{comp}} + u_X^{\text{pre}} < 1$, so normalisation of $\vec{b}_X^{\text{comp}}$ is required. The normalisation factor denoted f_{norm} is:

$$f_{\text{norm}} = \frac{1 - (b_X^{\text{cons}} + u_X^{\text{pre}})}{b_X^{\text{comp}}}\;. \tag{28}$$

Because belief on X is uncertainty, the fused uncertainty is:

$$u_X^{A \heartsuit B} = u_X^{\text{pre}} \;+\; f_{\text{norm}}\, \vec{b}_X^{\text{comp}}(X)\;. \tag{29}$$

After computing the fused uncertainty the compromise belief mass on X must be set to zero, i.e.

$$\vec{b}_X^{\text{comp}}(X) = 0\;. \tag{30}$$

After normalisation the resulting CC-fused belief is:

$$\vec{b}_X^{A \heartsuit B}(x_i) = \vec{b}_X^{\text{cons}}(x_i) \;+\; f_{\text{norm}}\, \vec{b}_X^{\text{comp}}(x_i)\;, \quad \forall x_i \in R(X)\;. \tag{31}$$

The CC-operator is commutative, idempotent and semi-associative, with the vacuous opinion as neutral element. Semi-associativity means that 3 or more arguments must first be combined together in the Consensus Step, and then together again in the Compromise Step before normalisation.

VI. Biometric Fusion with Subjective Logic

In this section we propose 4 fusion models for biometric data fusion, whereof 2 feature-level fusion models and 2 opinion level fusion models. The appropriate fusion model depends on the level and on whether the fusion is unimodal or multimodal.

A. Feature-Level Fusion Models

The feature-level fusion models are described below and illustrated in Fig.4.

1) Unimodal case: It is assumed that multiple measurements of the same biometric feature are recorded. Because the same feature is measured it does not increase the set of features and thereby the information coverage, rather it results in more accurate measures of the same set of features. Averaging fusion is therefore the natural fusion model for this situation.

2) Multimodal case: It is assumed that measurements of multiple sets of features are recorded. The collection and measurement of different features leads to an increase in the information coverage, so that it is natural to use the cumulative fusion model.

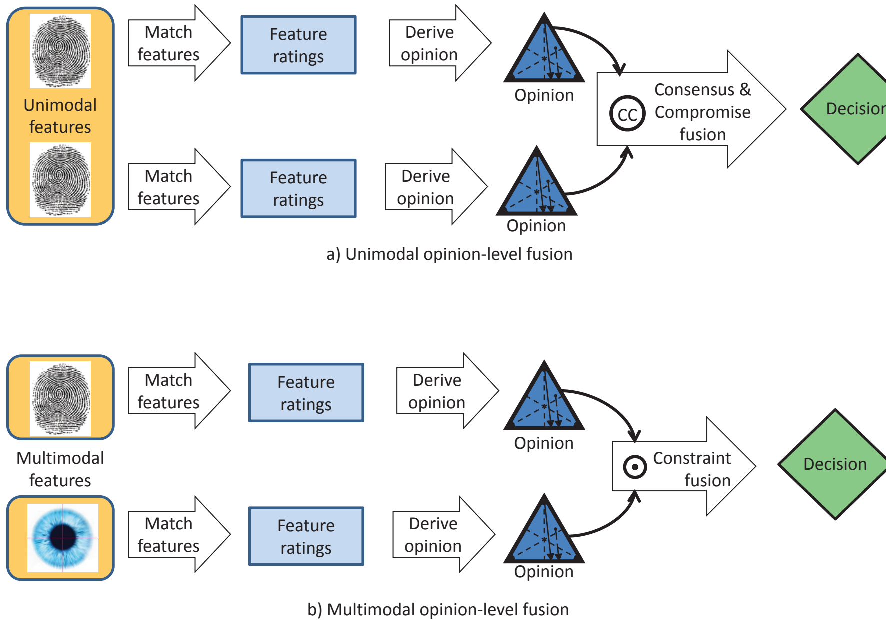

Figure 5. Biometric opinion-level fusion models based on subjective logic

B. Opinion-Level Fusion Models

The opinion-level fusion models are described below and illustrated in Fig.5.

1) Unimodal ase: It is assumed that multiple score opinions of the same modality are combined. It is here crucial to consider how to handle situations of consistent and inconsistent score opinions, as well as the situation where one score opinion is highly uncertain. it is natural to preserve the common belief from more or less consistent score opinions, to let inconsistent belief produce average belief, and to let highly uncertain score opinions carry little weight in the fusion process. The CC-model (Consensus & Compromise is therefore the natural fusion model for this situation.

2) Multimodal case: It is assumed that multiple score opinions from different modalities are combined. It is normally assumed that all modalities must produce a high score in order to produce a fused biometric score. It is therefore natural to use the constrain fusion operator in this situation.

VII. Conclusions

The concept of biometric fusion is vague in the sense that it can mean different things, thus making it very challenging to define the best data fusion method. An aspect of biometric measurements that is often ignored is that the scores are derived based on varying degrees of firm evidence which naturally leads to uncertainty about the score values. The advantage of subjective logic is precisely that levels of uncertainty can be taken explicitly into account. It is therefore natural to apply subjective logic when deriving biometric scores and also for biometric data fusion.

Different operators for biometric data fusion should be used when applied to different levels of abstraction as well as when applied in different situations of unimodal or multimodal system. Subjective logic offers multiple operators for opinion fusion which represent a good fit to various fusion situations.

This work is limited in the sense that it only proposes new approaches to biometric fusion based on subjective logic. As part of an ongoing project we plan to extend this work by applying the proposed methods to real situations of analysing and fusing various biometric data samples. While the current work is intuitive, it is necessary to also provide empirical evidence for the suitability of the proposed models.

References

[1] S. A. Magnet, *When Biometrics Fail: Gender, Race, and the Technology of Identity*. Durham, NC: Duke University Press, 2011.

[2] A. Hyvärinen, I. Karhunen, and E. Oja, *Independent Component Analysis*. John Wiley & Sons, 2001.

[3] A. Ross, K. Nandakumar, and A. Jain, *Handbook of Multibiometrics*. Springer Science and Business Media, 2006.

[4] N. Morizet and J. Gilles, "A New Adaptive Combination Approach to Score Level Fusion for Face and Iris Biometrics Combining Wavelets and Statistical Moments," in *Proceedings of the 4th International Symposium on Visual Computing (Advances in Visual Computing), LNCS, vol.5359*, G. Bebis, Ed., 2008, pp. 661–671.

[5] R. K. Hankin, "A generalization of the dirichlet distribution," *Journal of Statistical Software*, vol. 33, no. 11, pp. 1–18, February 2010.

[6] A. Jøsang and R. Hankin, "Interpretation and Fusion of Hyper Opinions in Subjective Logic." in *Proceedings of the 15th International Conference on Information Fusion (FUSION 2012)*, Singapore, July 2012.

[7] P. Walley, "Inferences from Multinomial Data: Learning about a Bag of Marbles," *Journal of the Royal Statistical Society*, vol. 58, no. 1, pp. 3–57, 1996.

[8] G. Shafer, *A Mathematical Theory of Evidence*. Princeton University Press, 1976.

[9] L. Zadeh, "Review of Shafer's A Mathematical Theory of Evidence," *AI Magazine*, vol. 5, pp. 81–83, 1984.

[10] A. Jøsang, "Multi-Agent Preference Combination using Subjective Logic," in *International Workshop on Preferences and Soft Constraints (Soft'11)*, Perugia, Italy, 2011.

[11] A. Jøsang and S. Pope, "Dempster's Rule as Seen by Little Colored Balls," *Computational Intelligence*, vol. 28, no. 4, November 2012.