# Simulation of Acoustic Fields from Medical Ultrasound Transducers of Arbitrary Shape

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### Introduction

In medical ultrasound a whole range of various transducers are common, including:

- 1. Pre-focused annular arrays divided into rings using the equal-area principle
- 2. Rectangular arrays divided into elements of dimension 0.5 2  $\lambda$  with pre-focusing in the short-axis dimension
- 3. Curved arrays divided into elements of dimension 1 2  $\lambda$  with pre-focusing in the short-axis dimension

In addition there is need to understand the properties of transducers of more complex shapes such as oval or elliptic ones, and to find the fields generated by 2-dimensional transducers. For this reason a general purpose simulator tool has been made.

## **Method and Examples**

In order to find the field it is common to assume that the Rayleigh integral applies:

$$\phi(r,t) = \iint_{S_s} \frac{u_n(r_o, t - r/c)}{2\pi r} dS \tag{1}$$

where the velocity potential is given by the surface velocity integrated over the active source. The source is assumed to be plane, i.e. the lateral dimensions and the radius of curvature are large compared to the wavelength [2], and thus curved transducers used in ultrasound are covered by this assumption.

In the impulse response method the Rayleigh integral is converted from a 2-dimensional to a 1-dimensional integral [1]. This assumes that the diffraction impulse response has been derived for the transducer shape used. In the described simulator, this method is not used. One of the reasons is that it is desirable to be quickly able to analyze new transducer shapes. This could also be done using the impulse response method by subdividing the radiating plane into smaller basic subtransducers with a known diffraction impulse response [4]. However it is also desirable to be able to analyze the field in an inhomoge-

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nous medium. One of the underlying assumptions of the impulse response method is that the path from the radiator to the summation point is independent on actual position. Thus this method has limitations when the field is to be found in an aberrating medium. In this case one has to give up the speed advantage and solve the Rayleigh integral directly taking the medium properties into account for each path from source to field point [6], [7].

The Rayleigh integral is solved by discretizing the radiating surface, assuming that the plane source vibrates in a single mode (thickness mode) [3] and thus that the surface velocity is separable:

$$u_n(r_o, t) = O(r_0)u(t)$$
 (2)

The observation plane is also discretized and the integration is done by finding the distance and quantized time delay [5] from each source point to each of the observation points. The time waveform is either continuous wave or a pulse that resembles the pressure pulse measured at the focal point on the acoustical axis. At this point one will get coherent summation of the Rayleigh integral. This means that we excite with a measured approximation of the surface velocity.

The following four figures give examples of the output from the simulator. In addition it is possible to generate animations of travelling ultrasound pulses using the display of Fig. 2.

#### References

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ARRAY-RESPONSE Reference=38.96 [us] 6-FEB-1995 15:51 Theta=0 [deg] Phi=0 [deg] N=64 M=1 f=3.5 [MHz] pitch=0.5 osc=Inf Azimuth : no apodization Elevation : no apodization



FIGURE 1. Plot of beamwidth contours (-6, -12 and -20 dB) for a 64 element array with half lambda pitch at 3.5 MHz, focus = 60 mm.

 ARRAY-RESPONSE
 Reference=38.96 [us]
 6-FEB-1995 15:58

 Theta=0 [deg]
 Phi=0 [deg]
 N=64
 M=1
 f=3.5 [MHz]
 pitch=0.5
 osc=3

 Azimuth : no apodization
 Elevation : no apodization
 View: 3D default



FIGURE 2. Plot of pulse in focus as sent from the same array as in Fig. 1. Pulse form is 3 periods shaped with a cosine.



**FIGURE 3.** Beampattern obtained by summing energy over all time at a distance equal to geometric focus (60 mm).



FIGURE 4. Intensity plot along acoustic axis for continuous excitation for array of Fig. 1.