Information Theory and Secure Communications

Øyvind Ytrehus

Simula UiB

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Outline

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Cast: Main characters I

- *Alice*  , a talkative sender.

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- *Alice*  , a talkative sender.
- *Bob*  , an eager listener.

What do Alice and Bob want?

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- securely: secretly,privately, authenticated, stealthily
  - Cryptography
  - Information Theory
  - Other security techniques
  - Anonymizing networks, Private Information Retrieval

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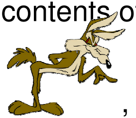
- *Willie*  , a wiley warden who wishes to determine with precision whether at all Alice transmits to Bob. Willie does not care about the content of transmitted messages.
- Fraudsters, impositors, active intruders, repudiators also exist...but beyond the scope

Requirements for secure effective communication (between Alice and Bob)

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- Better communication channel: Information theory
  - alternative/addition to cryptography
  - also applied to metadata
  - applied in 5G

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Information theory and noisy channels

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$$H(X|Y) = \sum_{y \in \mathcal{Y}} p(y) \Big(-\sum_{x \in \mathcal{X}} p(x|y) \log_2 p(x|y) \Big)$$

- The *mutual information* is

$$I(X;Y) = I(Y;X) = H(X) - H(X|Y) = H(Y) - H(Y|X). \tag{2}$$

Some remarks on information theory

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- Information versus psychology

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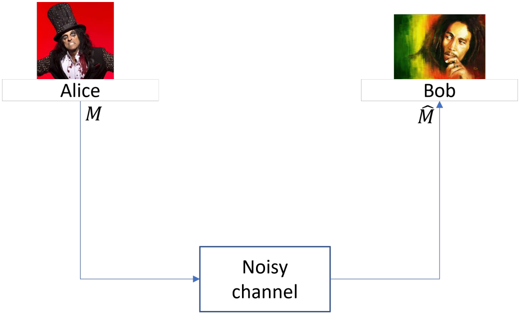
Communication between Alice and Bob



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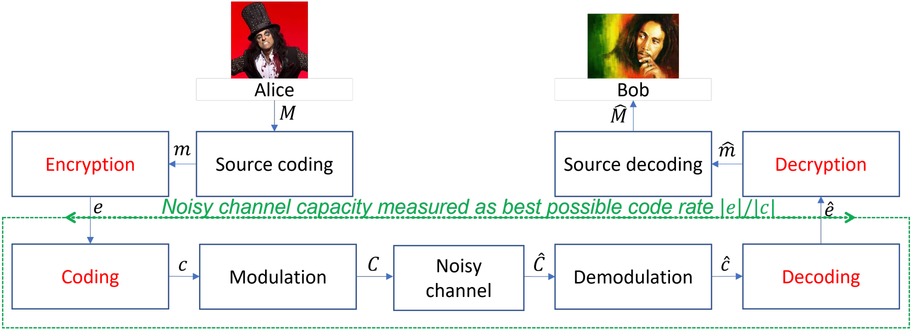
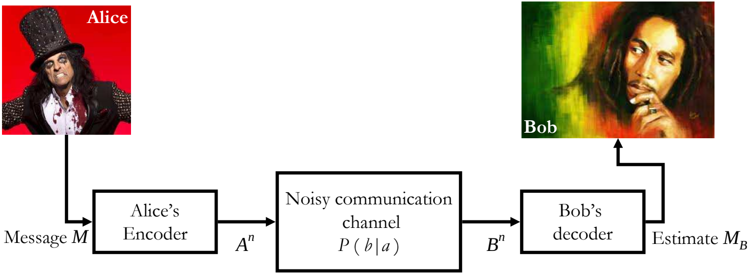


Shannon’s noisy channel



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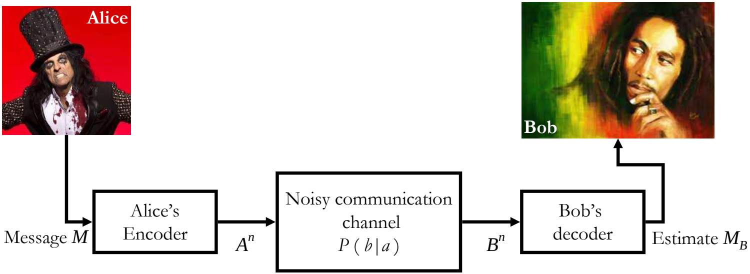

$$C_{Shannon} = \max_{p(a)} I(A;B),$$

Shannon's noisy channel: Tools used in proof

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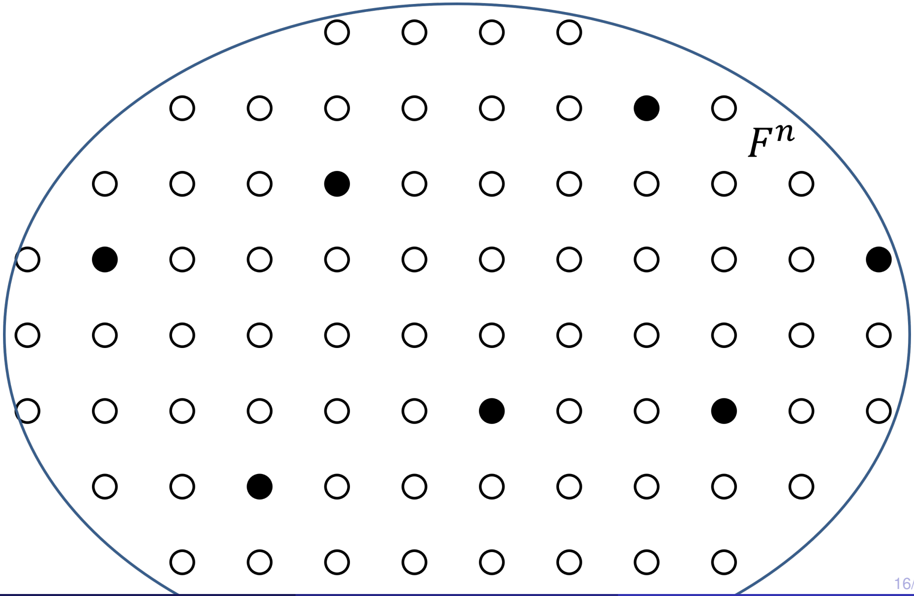
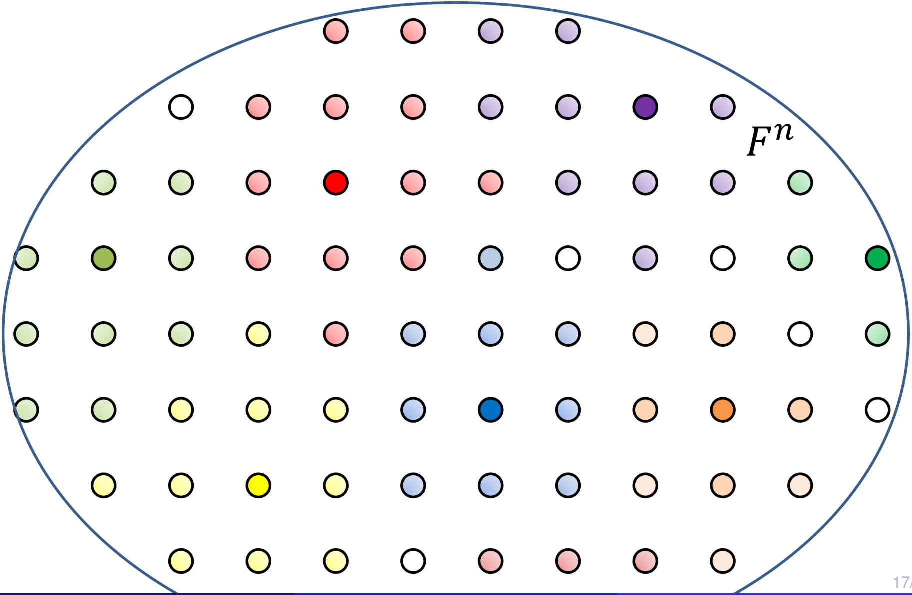
- Random coding

Error correcting code

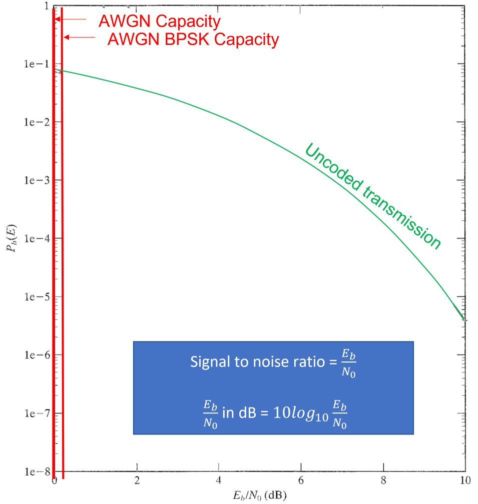


Error correcting code partitioning the space F^n



The Additive White Gaussian Noise (AWGN) channel

How?



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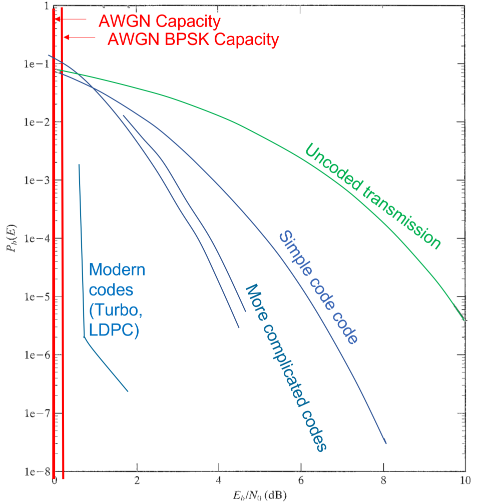

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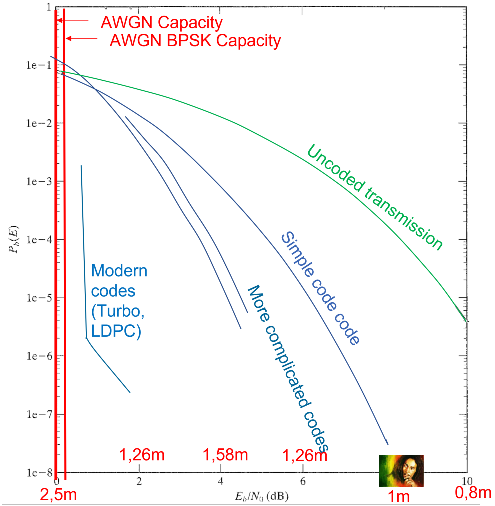
Received power = Transmitted power $\times d^{\alpha}, \alpha = 2..4$



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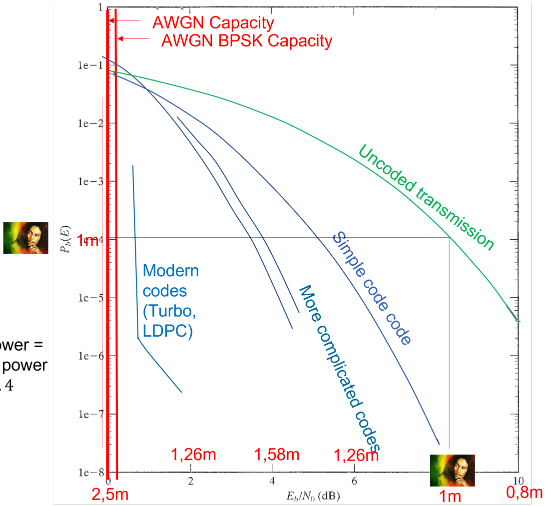

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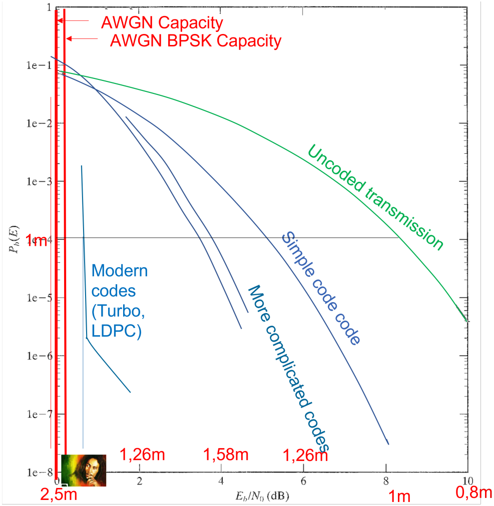

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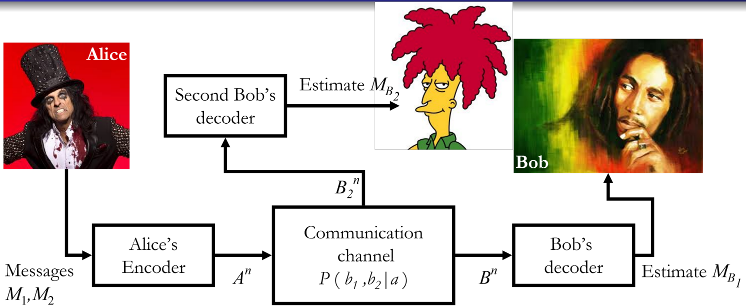

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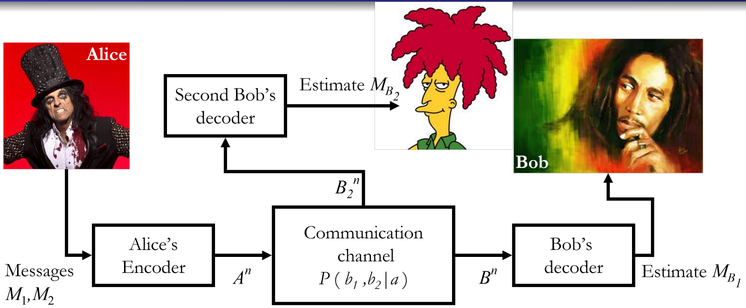

$$R_1 \leq C_1 = \max_{p_a} I(A; B_1), \; R_2 \leq C_2 = \max_{p_a} I(A; B_2),$$
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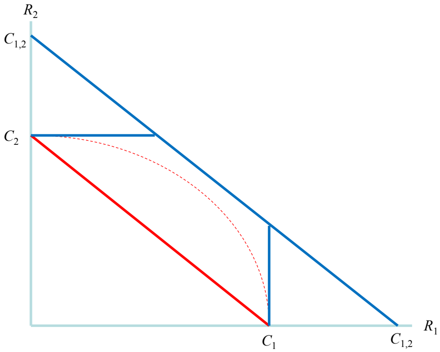

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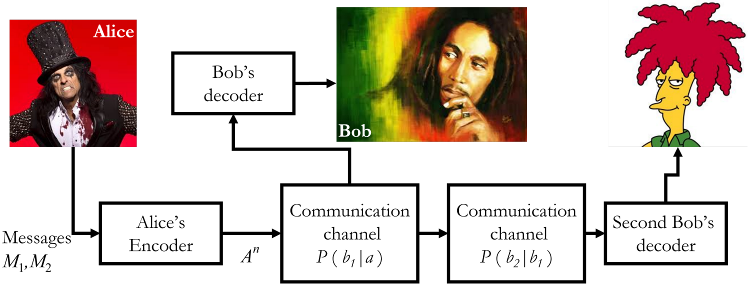

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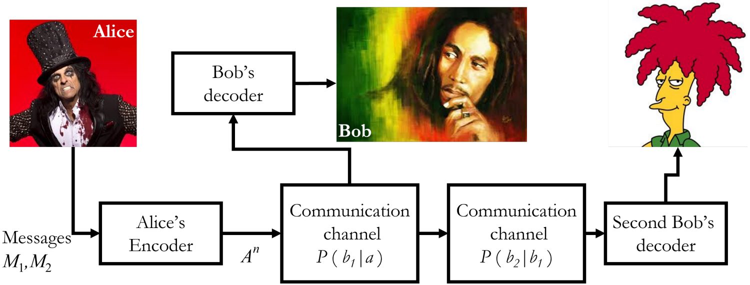


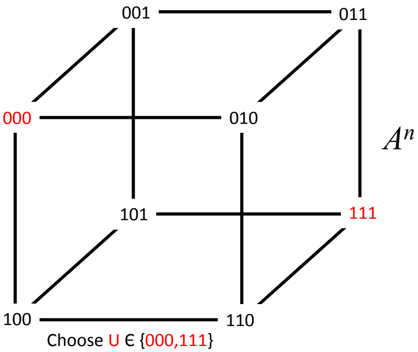
$$R_1 \leq I(A; B_1 | U),$$

$$R_2 \leq I(U; B_2),$$

for some pmf $p(u, a)$ and conditions on U.

What does this mean? What is U?

Assume $n = 3$ and that Bob_1 has an error free channel, while Bob_2 typically will see at most one bit error for each 3 sent.



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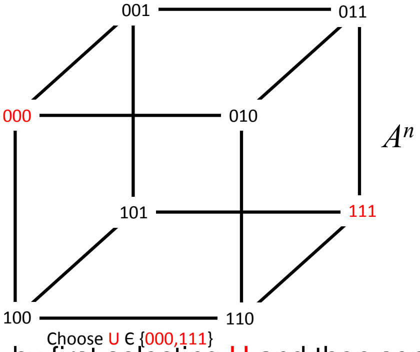
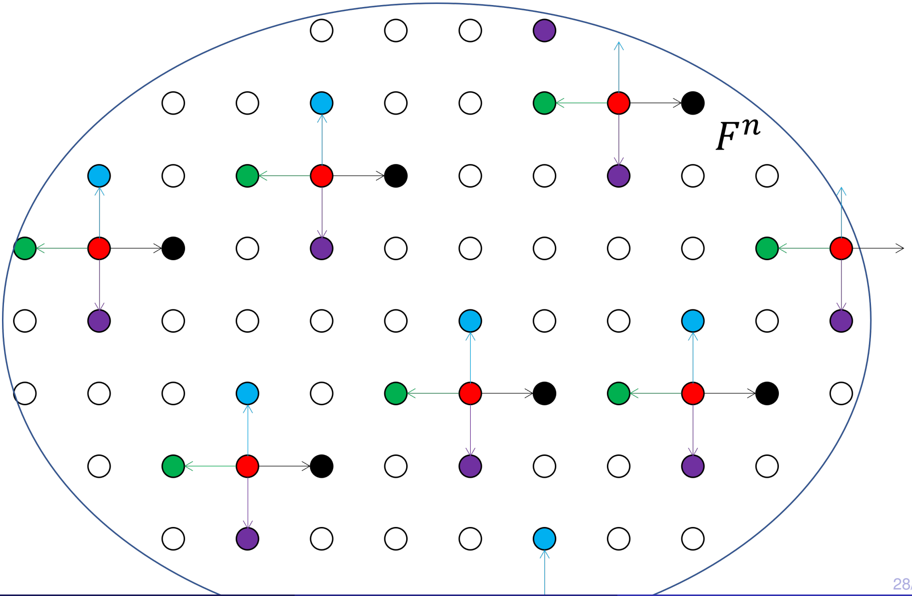


Then send three bits by first selecting U and then sending A as U plus one of the bit patterns $\{000, 001, 010, 100\}$.

Error correcting code and coset



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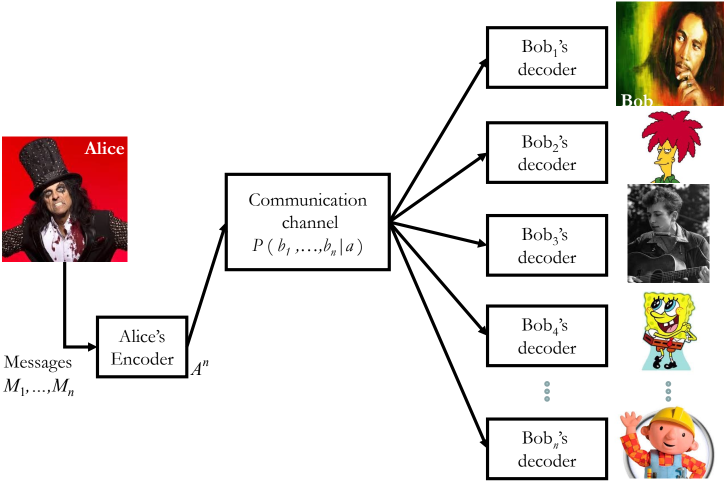

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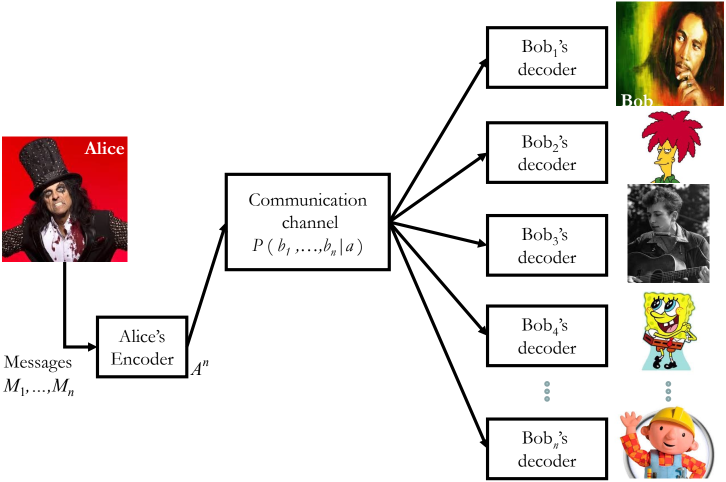
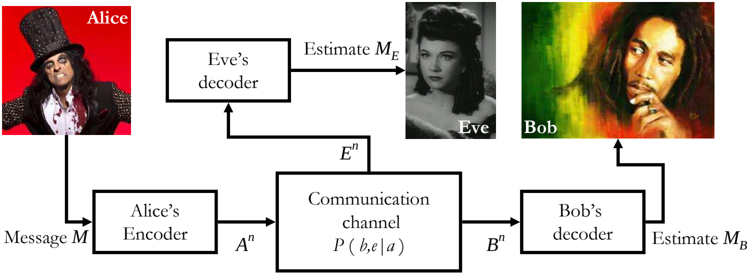


Outline

The wiretap channel (Type I)



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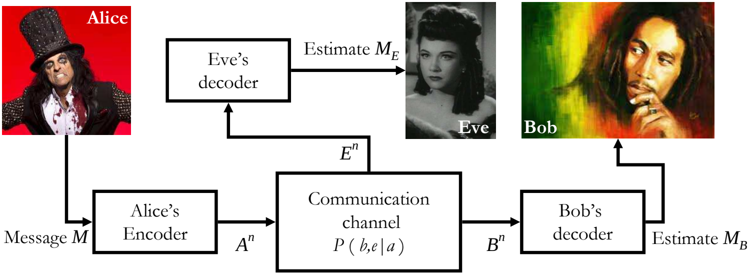

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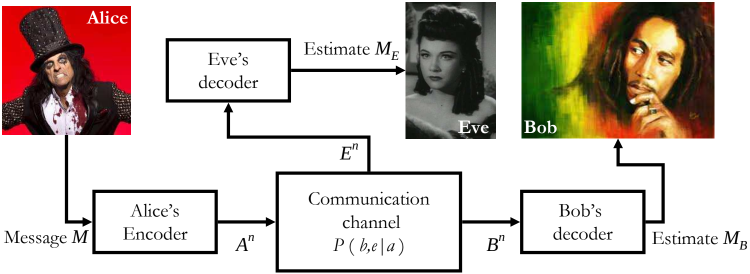

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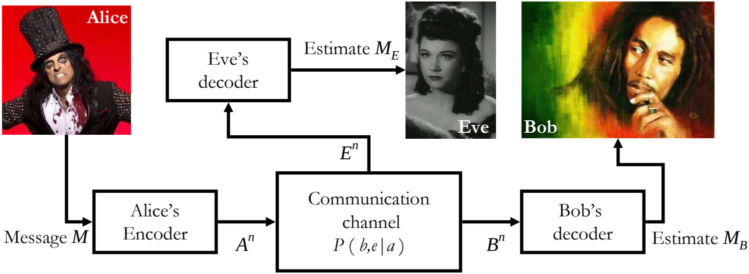

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The wiretap channel (Type I), Simple case

Simplest example: Degraded Bob's channel is noiseless, Eve's channel has noise. How to encode?

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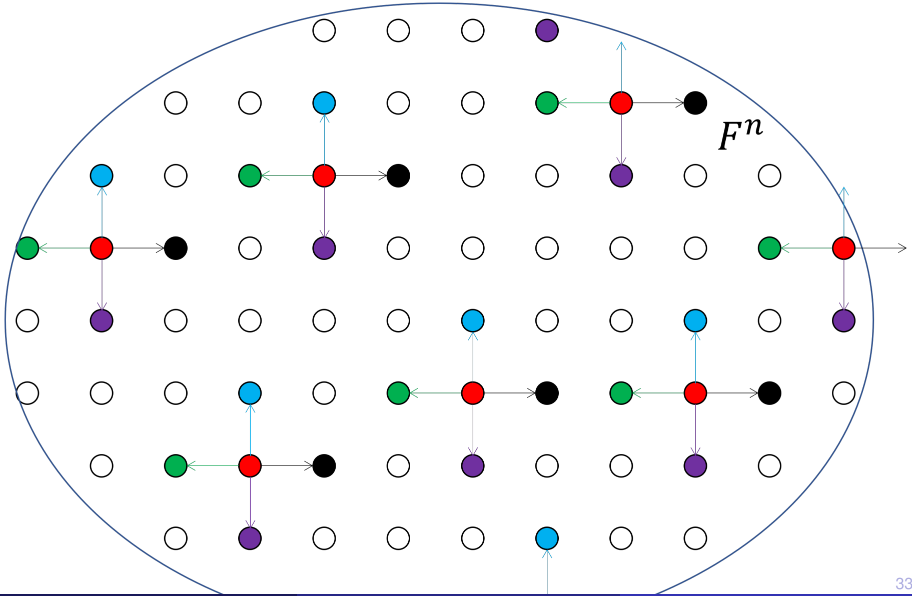
- select $[n, k]$ error correcting code according to $(n-k)/n < C_{DM-WTC}$
- Represent message by a *coset*
- Alice sends U = *random* codeword + corresponding coset leader

Error correcting code and coset



Degrees of Secrecy

Encoded message M is n-bit vector, Eve sees noisy n-bit E^n, n large

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Note: $I(* : *)$ can be replaced by Renyi information.
Note: Independent of Eve's computational resources.
Note: Category of secrecy may depend on code and coding scheme

Now, the practice...The Type I AWGN channel

- The different roles of Bob and Eve
  - Bob wants a simple and efficient decoding to get the best decoding solution: Bit error rate reasonable metric
  - Eve willing to spend more efforts, maybe try out different option: Mutual information

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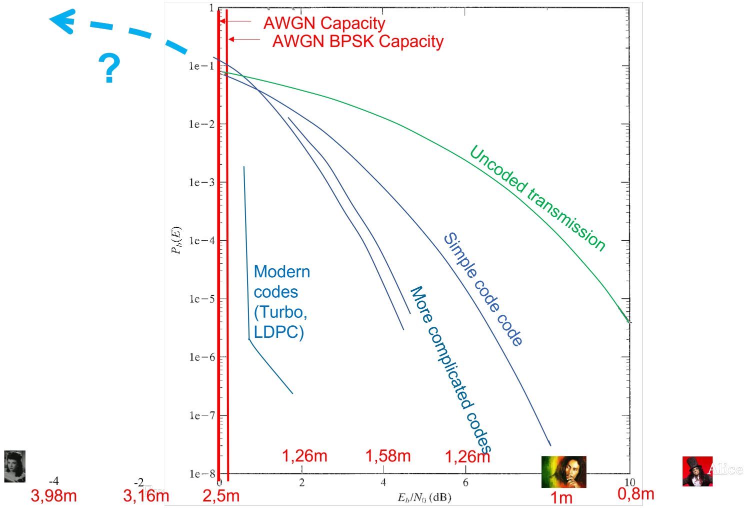

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  - Bounds exist (not very tight)
  - Exact computation: Trellis computation Complexit $\leq \min\{2^k, 2^{n-k}\}$
  - *Joakim Algrøy, Angela Isabel Barbero and Øyvind Ytrehus, "Determining the Equivocation in Coded Transmission Over a Noisy Channel", accepted for IEEE International Symposium on Information Theory, June 26-July 1, 2022*

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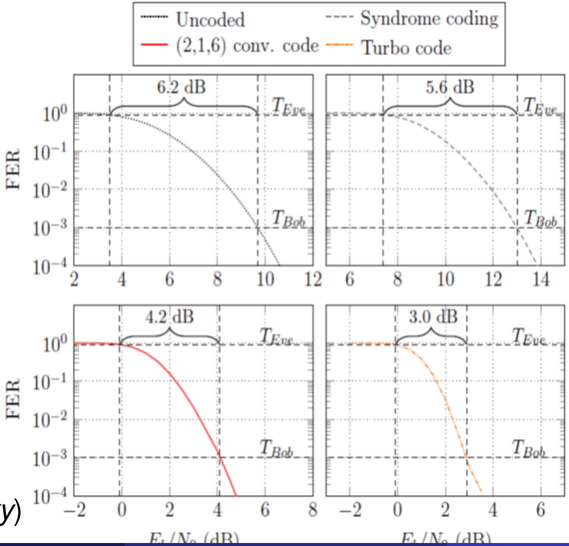
Now, the practice...The Type I AWGN channel

Comparison between syndrome coding and regular communication coding:



(*Figure: Joakim Algrøy*)

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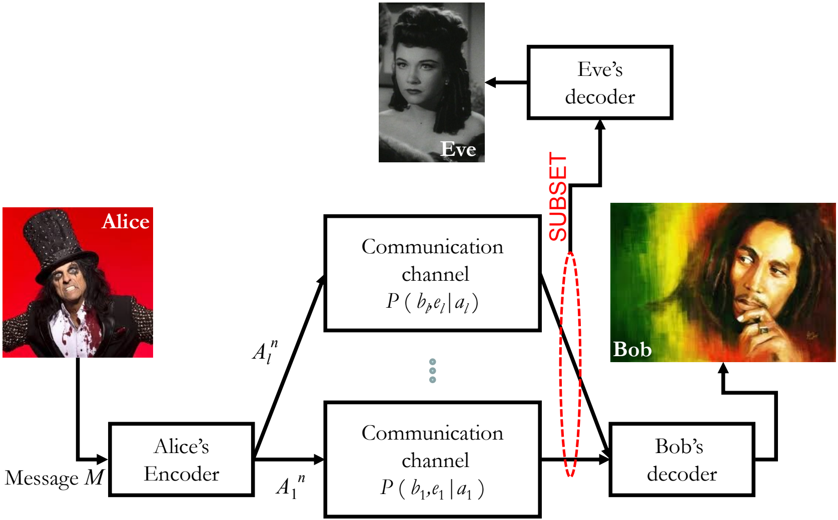
- How to deal with the tradeoff between secrecy, and power efficiency for Alice?
  That is, with a fixed redundancy r, devote r_B to help Bob and $r_E = r - r_B$ to confuse Eve. What is the optimum r_B?

The wiretap channel (Type II)



The wiretap channel (Type II)

How to encode?

- Choose a code with large generalized Hamming weights in the dual code
- Represent message by a syndrome vector
- Alice sends random codeword + corresponding coset leader

Outline

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Outline

"... as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns - the ones we don't know we don't know. ..."- *D.Rumsfeld*

Outline

Covert, deniable, subliminal, invisible, undetectable communication

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Steganography

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- It follows that Alice may send one bit to Bob by selecting a pre-agreed image for each of the three possible cover messages.

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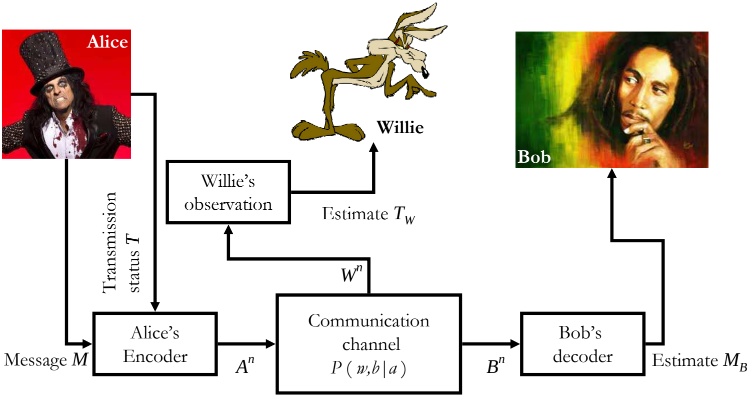
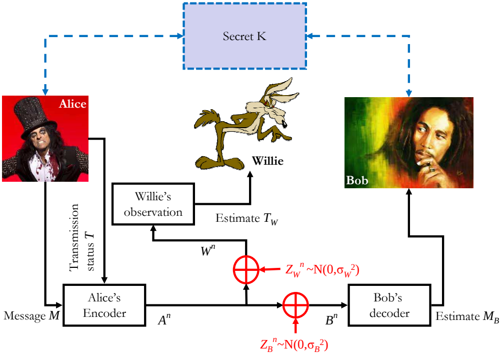
  - "Steganography", but hard for Willie to detect and prove
  - Can be blocked by zero-knowledge proofs etc, but still allows 1-bit subliminal channel (Desmedt)

Reliable deniable channels



Reliable deniable AWGN channels with randomness common to Alice and Bob



A^n, B^n, and W^n are real-valued n-dimensional vectors, and Z_B^n and Z_W^n are n-dimensional AWGN noise vectors. Alice and Bob need to share a secret key.

A reminder of complexity notation

- $f(n) = \mathcal{O}(g(n))$ if there exist constants $m, n_0 > 0$ such that $0 \leq f(n) \leq mg(n)$ for all $n \geq n_0$. This means that "$f(n)$ grows roughly at the same rate as $g(n)$".

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Reliable deniable AWGN channels with randomness common to Alice and Bob: Results

1. For any $\varepsilon > 0$ and *unknown* σ_W^2, Alice can reliably transmit $o(\sqrt{n})$ information bits to Bob in n channel uses while lower-bounding Willie's sum of the probabilities of detection errors $\alpha + \beta \geq 1 - \varepsilon$.

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2. If Alice knows a nontrivial lower bound $\hat{\sigma}_W^2 > 0$ on the noise power on Willie′s channel (*i.e.*, $\sigma_W^2 \geq \hat{\sigma}_W^2$), she can reliably transmit $\mathcal{O}(\sqrt{n})$ information bits to Bob in n channel uses while lower-bounding Willie′s sum of the probabilities of detection errors $\alpha + \beta \geq 1 - \varepsilon$.

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3. Conversely, if Alice attempts to transmit $\omega(\sqrt{n})$ bits in n channel uses, then, as $n \rightarrow \infty$, *either* $\alpha + \beta$ is arbitrarily close to zero *or* the communication to Bob is not reliable, regardless of the length of the shared secret.

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1. The capacity $\lim_{n\to\infty} \frac{\mathcal{O}(\sqrt{n})}{n} = 0$. But for finite codeword lengths n, a substantial amount $\mathcal{O}(\sqrt{n})$ of information may be reliably transmitted with low probability of detection.

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7. Quantum channel version

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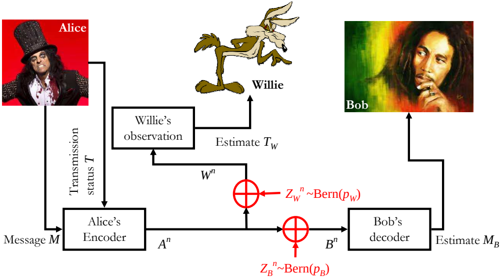

The binary symmetric subliminal channel. Here A^n, B^n, and W^n are binary n-dimensional vectors, and Z_B^n and Z_W^n are binary n-dimensional noise vectors in which elements are generated independently according to their respective Bernoulli distributions.

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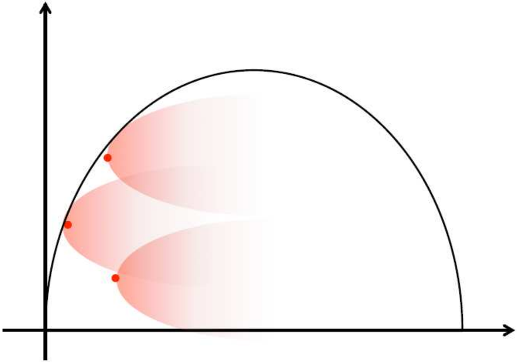
3. Bob faces a (modified) BSC decoding problem. When $T = 0$, such decoding will be unsuccessful with overwhelming probability. Thus the channel will not produce “false information” to Bob. When $T = 1$, such decoding will be successful with overwhelming probability, provided that the code is appropriately selected.

Reliable deniable BSC channels without randomness common to Alice and Bob: Bob's decoder



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  In this case Willie typically will have a better SNR than Bob.
- *Implementation in practice?* Random coding is merely a theoretical tool and has no practical usage. What *practical coding schemes* can be used?
  AWGN: possible to use a normal LDPC code?
  Noisy BSC subliminal channel: Need constant (low) weight codes; nonlinear.

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- From Willie's perspective, the assumption of knowing the code agreed between Alice and Bob is a best-case scenario. Reasonable approach in cryptanalysis, maybe less so in the context of deniable channels?
- For the previous issue, will a compressed sensing approach be sensible for Willie? That is, can we observe communication knowing that a code is used, but not which code is used?

Other issues

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Conclusion, Single-path communication

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2. For the warden Willie, there exist realistic scenarios that are worse than those assumed in the literature, and this creates extra problems.

Outline

More References

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