Exercise Sheet 5

Question 5.1 A concurrent dioid is a structure $(S, +, \cdot, \|, 0, 1)$ such that $(S, +, \cdot, 0, 1)$ is a dioid, $(S, +, \|, 0, 1)$ an abelian dioid $(x \parallel y = y \parallel x)$ and the following weak interchange law holds:

$$(w \parallel x) \cdot (y \parallel z) \leq (w \cdot y) \parallel (x \cdot z).$$

(a) Create a type class for concurrent dioids.

(b) Prove that the following small interchange laws are derivable.

(i) $x \cdot y \leq x \parallel y$,

(ii) $x \cdot (y \parallel z) \leq (x \cdot y) \parallel z$,

(iii) $(x \parallel y) \cdot z \leq x \parallel (y \cdot z)$.

Feel the joy!

(c) Give an interpretation statement to show how that shuffle languages form concurrent dioids. Start with the standard recursive function that computes the shuffle or interleaving of two words/lists. Then define a shuffle operation on languages. Finally prove the algebraic laws you need for the interpretation statement. Feel the pain!