Sums of Squares and Applications

Exercise 2

Let $\varphi = \varphi(x_1, \ldots, x_n)$ be a sentence (quantifier-free, if we want) that describes the semialgebraic set $M \subset \mathbf{R}^n$. Show that the closure $\overline{M}$ and the convex hull $conv(M)$ of M are again semialgebraic, by writing down a formula that describes $\overline{M}$, resp. $conv(M)$. For the convex hull you will need Carathéodory's theorem on convex hulls.