Recent developments in genuine multi-sided surface representation and editing

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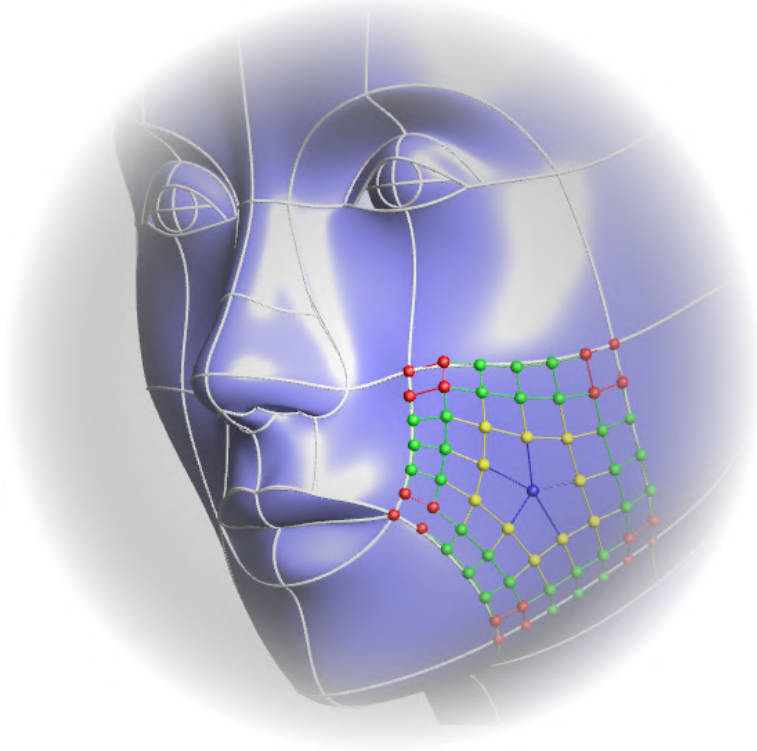
Outline

- Motivation
- Curved domain multi-sided patches
  - Generalized Bézier patches
  - Generalized B-spline patches
- Ribbon generation
  - Cross-derivatives by parameters
- Interior control
  - Parametric medial axis templates
  - Blending functions
- Tools for editing
  - Control vectors
  - Hierarchical editing
- Conclusion



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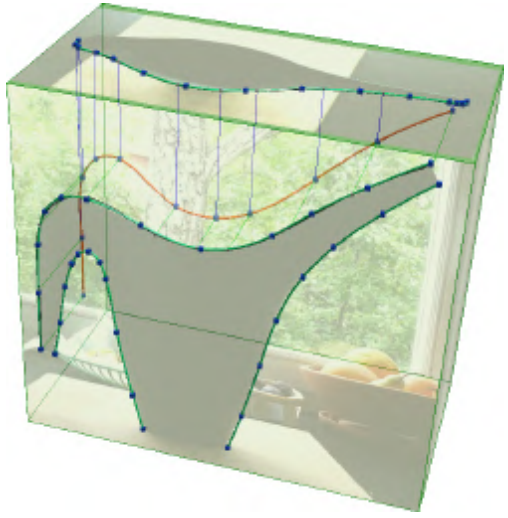
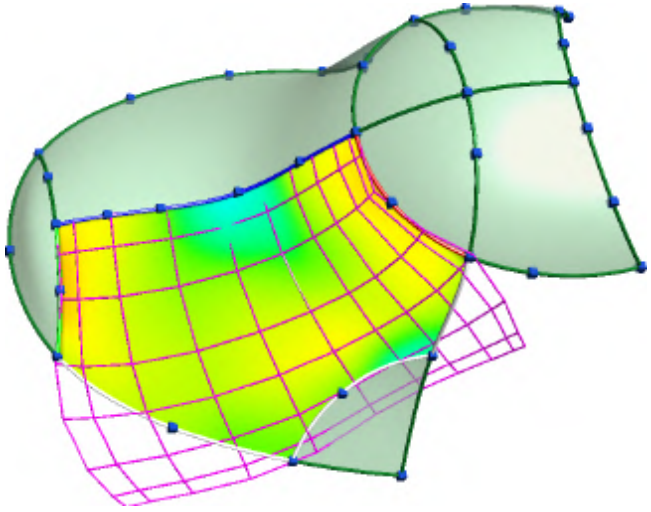
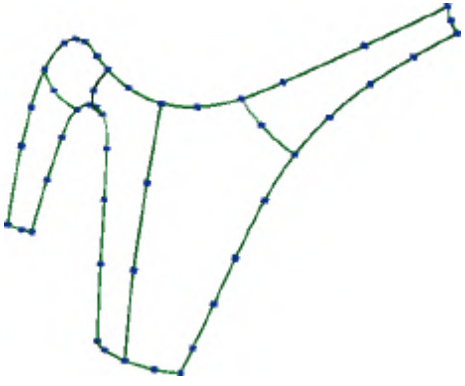
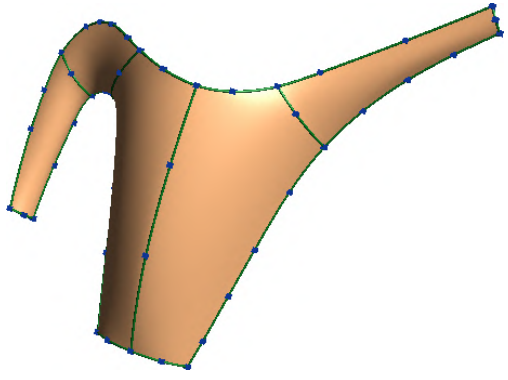
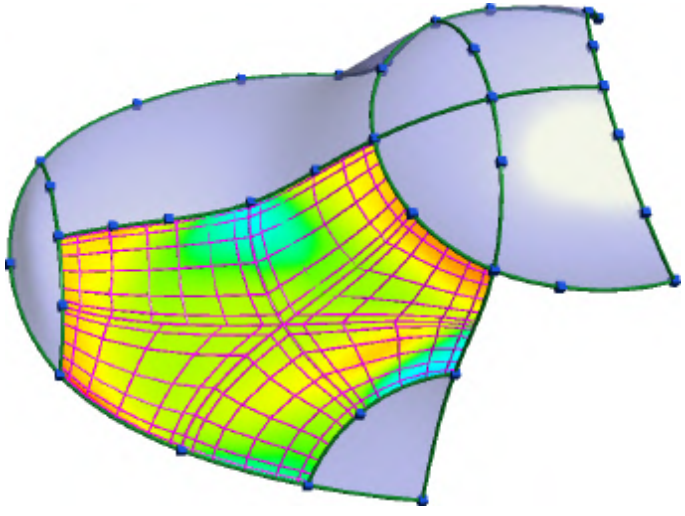
Multi-sided patches – Motivation from geometric design







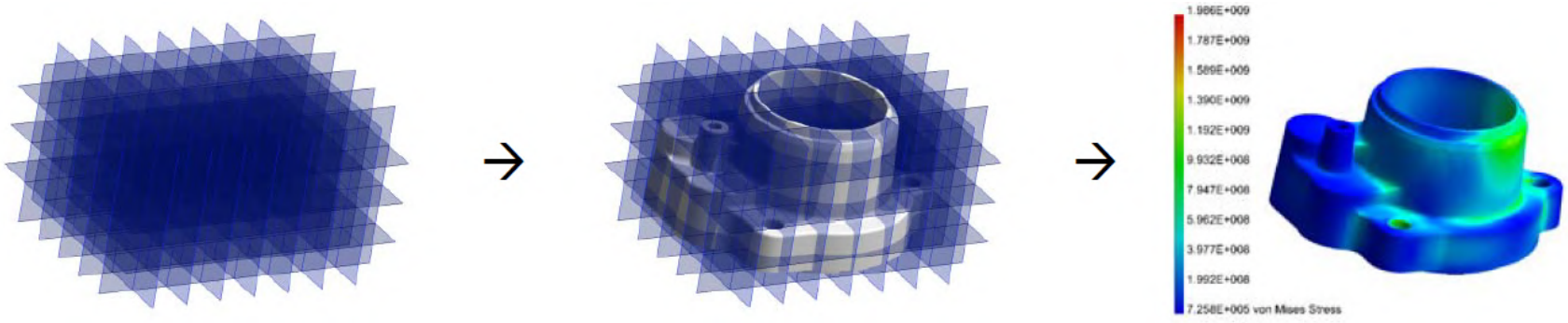




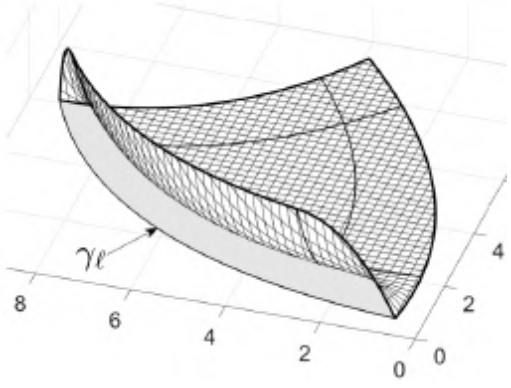
Demand for **genuine multi-sided** representations

[Gregory, Sabin, Loop-DeRose, Kato, Várady et al, Krasauskas, Karciauskas, Rockwood, Kosinka et al, Deng et al]

Multi-sided patches – Motivation from analysis



Immersed FEM [V. Shapiro]



[U. Reif]

- Trimmed IGA, WEB-spline, Finite Cell Method, meshless, PINN, ...
- Strong enforcement of boundary conditions $\rightarrow$ **Transfinite interpolation**
  [Kantorovich, Rvachev, Shapiro, Höllig, Reif, Cirak, Sukumar, ...]

NEW: Survey on multi-sided patches

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Genuine multi-sided parametric surface patches – A survey

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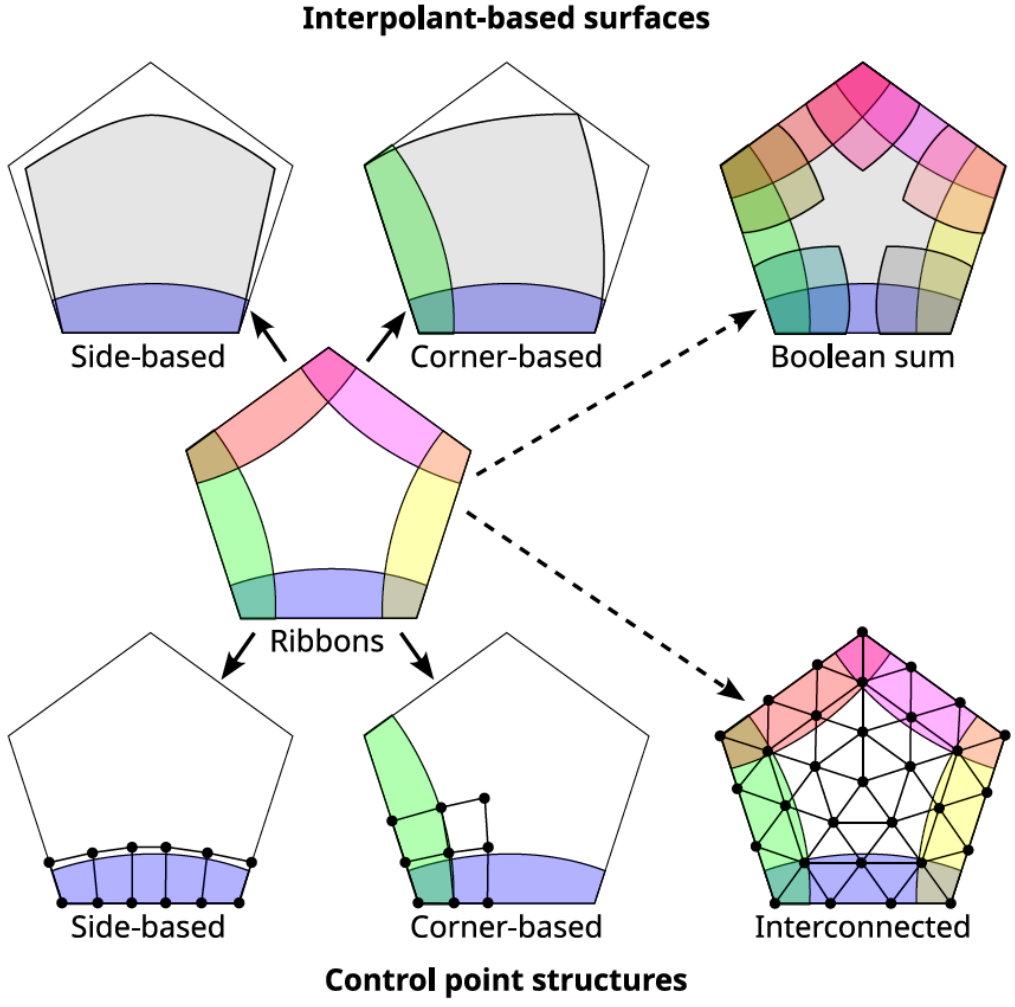
Ribbon-based surfaces
Transfinite interpolation
Control point patches
Surface modeling
General topology

ABSTRACT

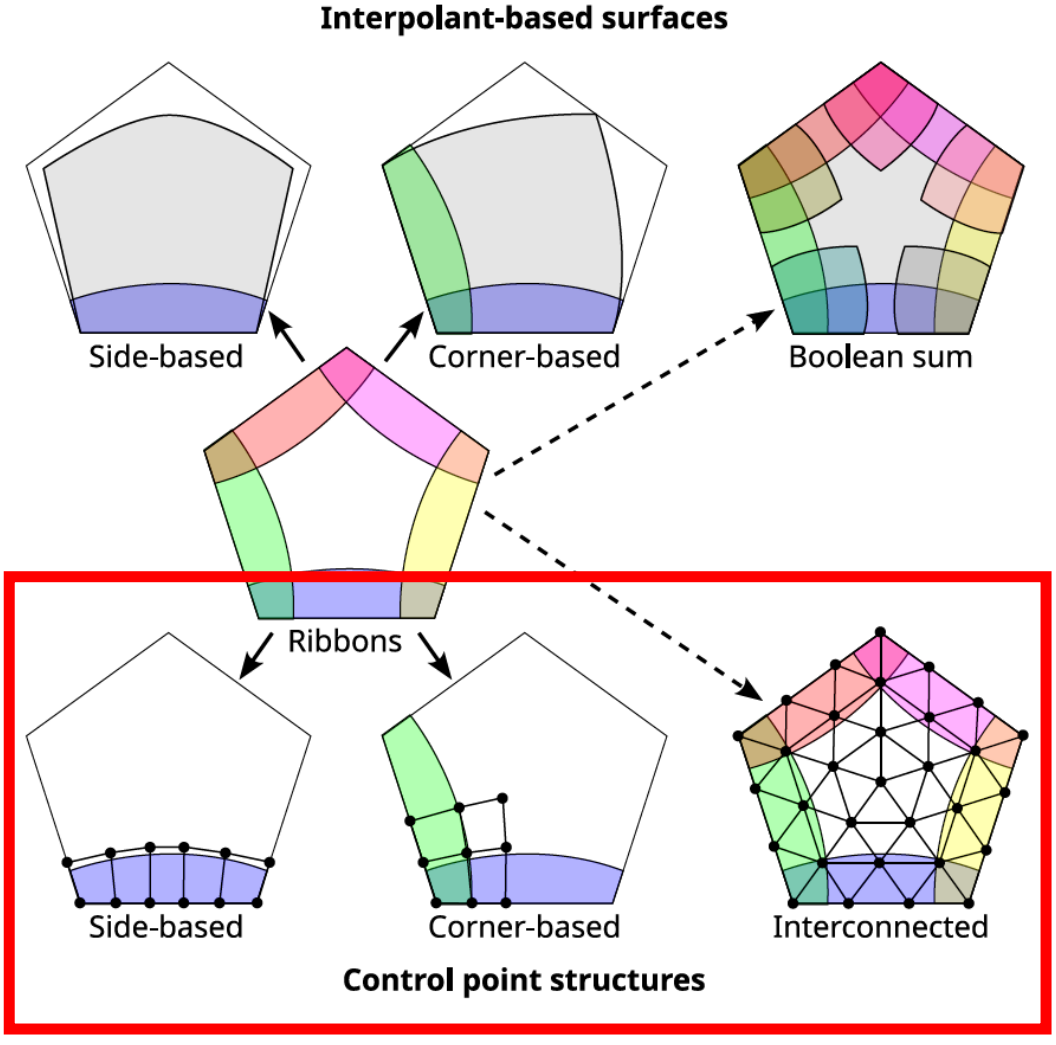
A state-of-the-art survey is presented on various formulations of multi-sided parametric surface patches, with a focus on methods that interpolate positional and cross-derivative information along boundaries.

Classification and constituents of multi-sided patches



Ribbon constraints → Patch equation → Domain → Parameterizations → Blending functions → Editing capabilities

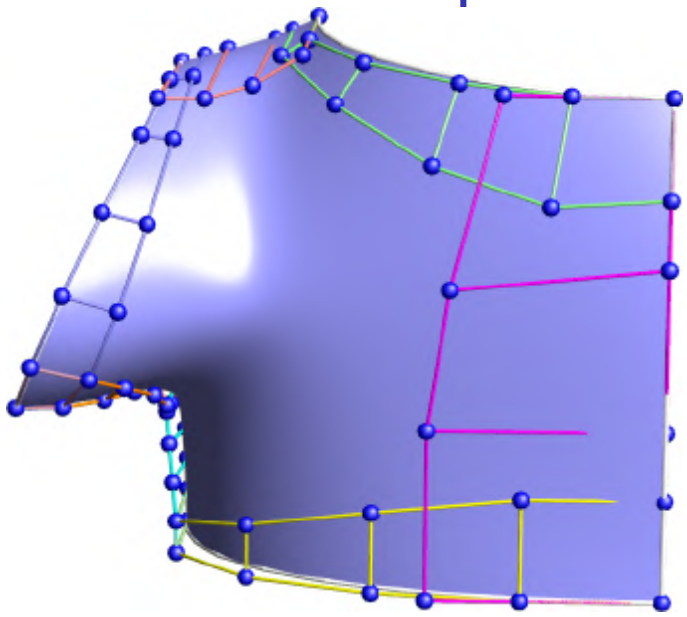
Classification and constituents of multi-sided patches



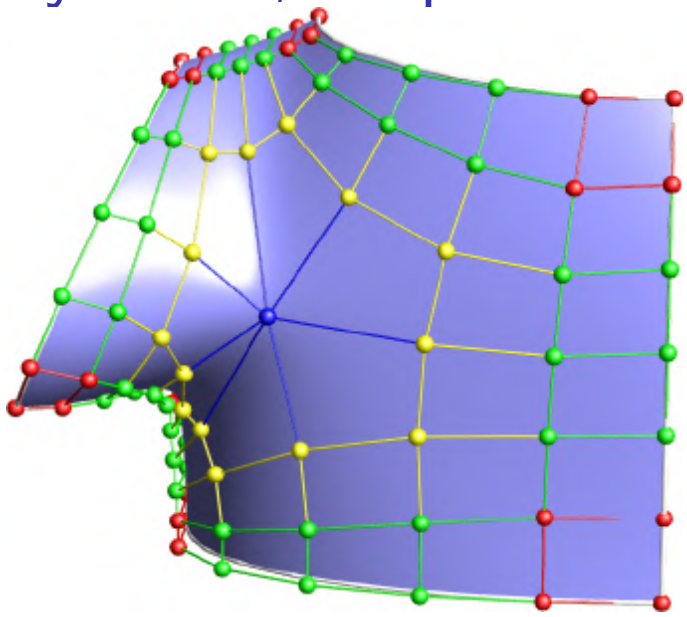
Ribbon constraints → Patch equation → Domain → Parameterizations → Blending functions → Editing capabilities

Outline

Generalized Bézier patches [Várady et al., '16]



Ribbon surfaces

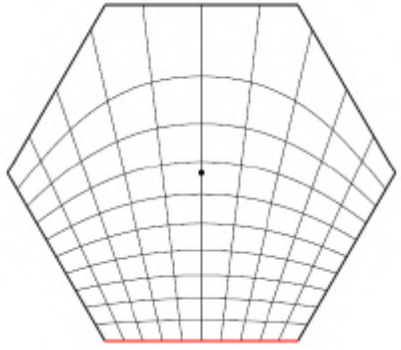


Interior control structure

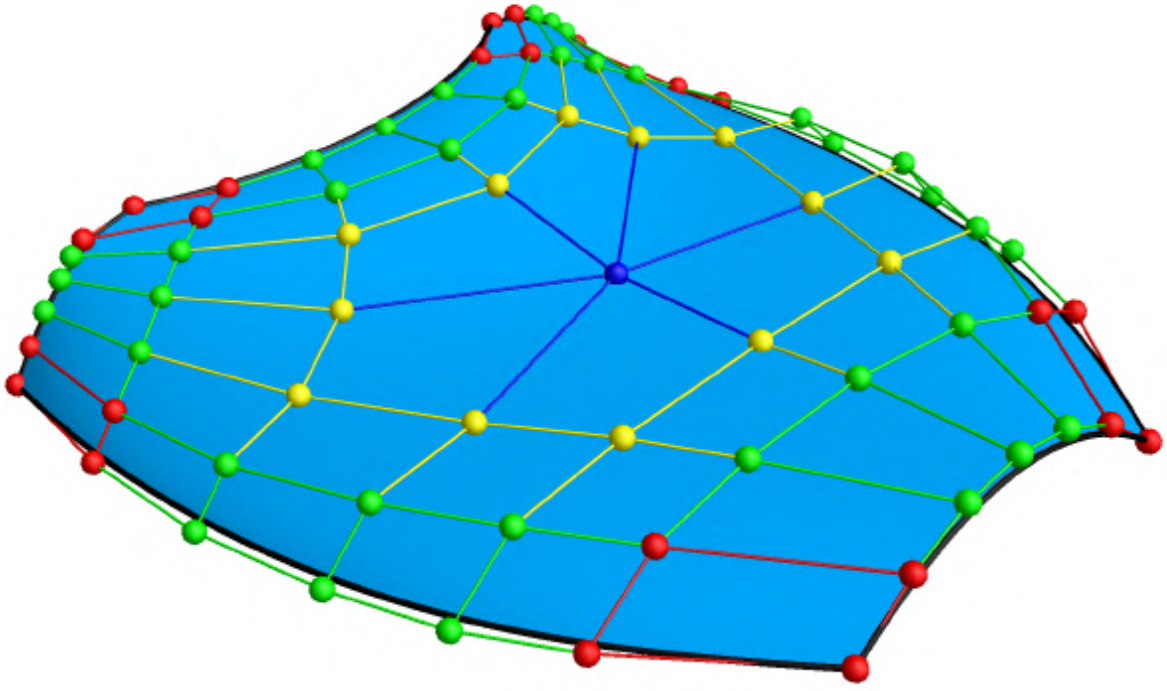
$$\mathbf{S}(u,v) = \sum_{i=1}^{n} \sum_{j=0}^{d} \sum_{k=0}^{(d-1)\div 2} \cdot \mathbf{C}_{ijk} \underline{\mu_{i,j,k}(u,v)} B^d_{i,j,k}(u,v) + \mathbf{C}_0 \cdot \underbrace{B_0(u,v)}_{1-\sum \mu B}$$

$B^d_{i,j,k}(u,v) := B^d_j(s_i(u,v)) \cdot B^d_k(h_i(u,v))$ with (s_i, h_i)

local parameters over <u>convex polygonal domain</u>.



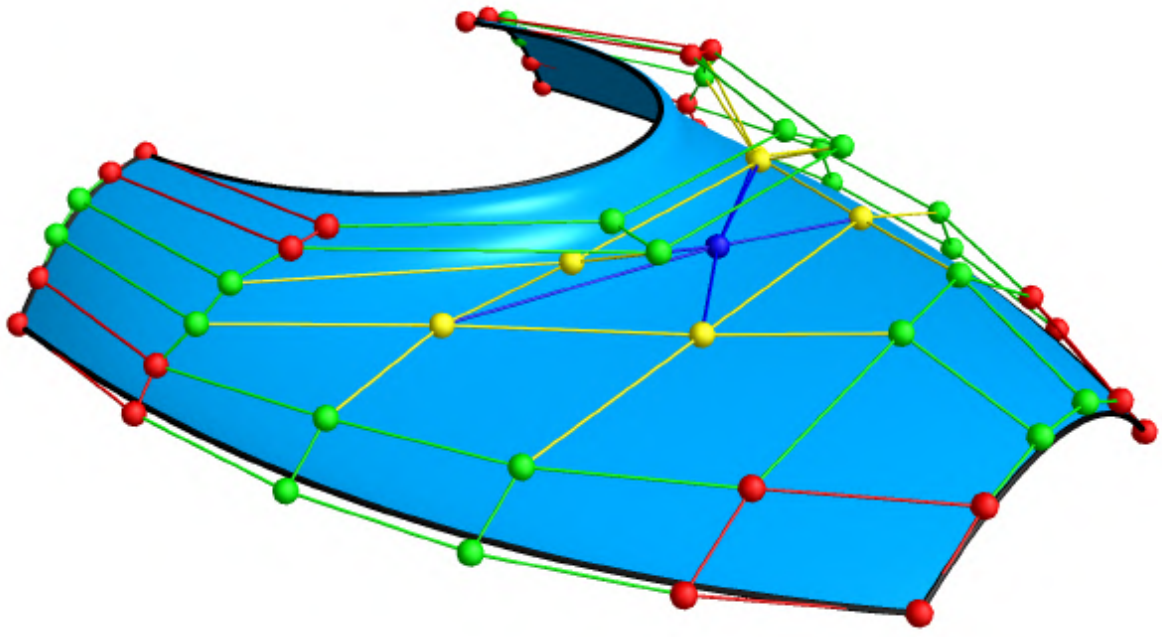
Generalized Bézier patches [Várady et al., '16]



Shape ✓

Interior control ✓

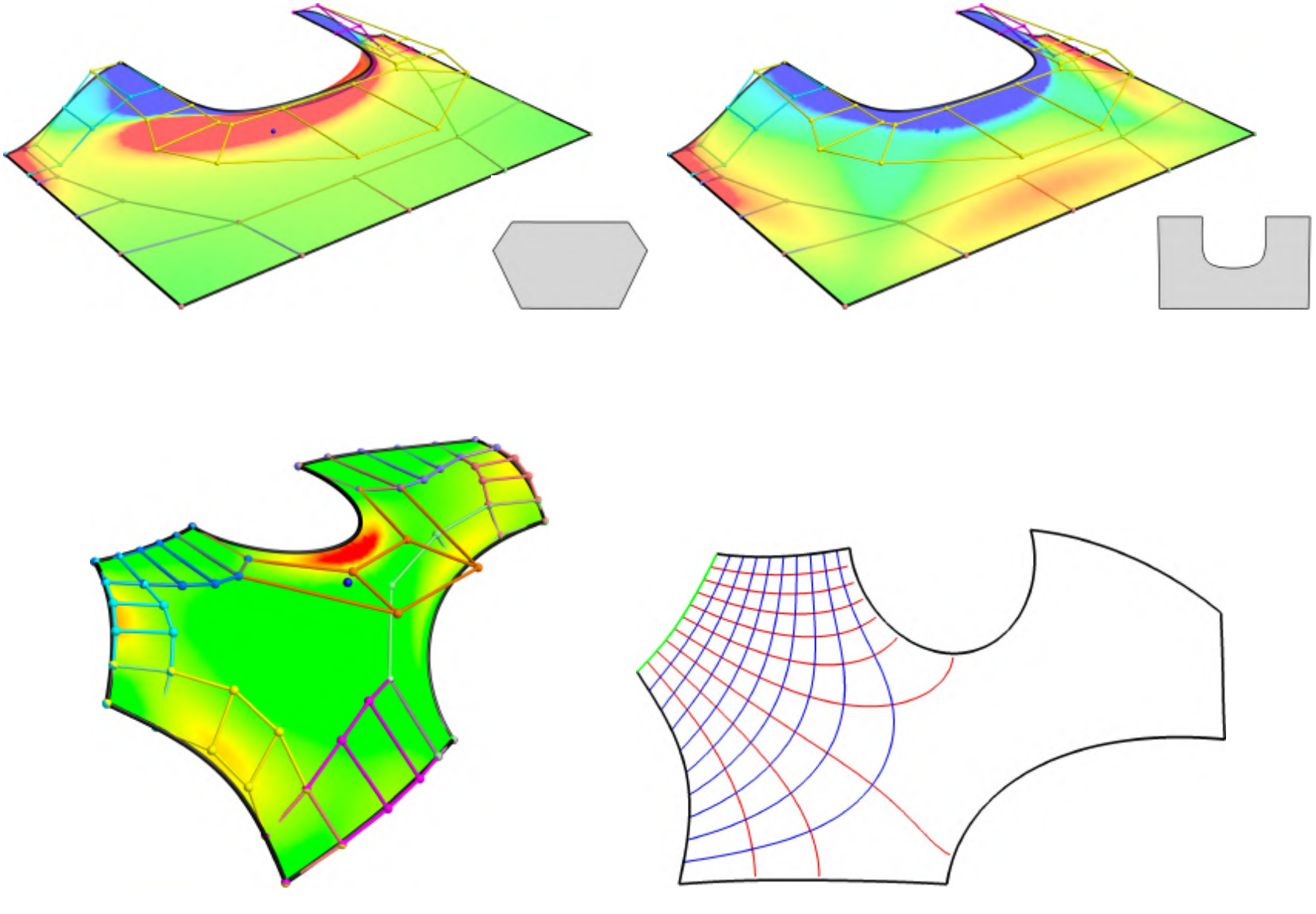
Generalized Bézier patches [Várady et al., '16]



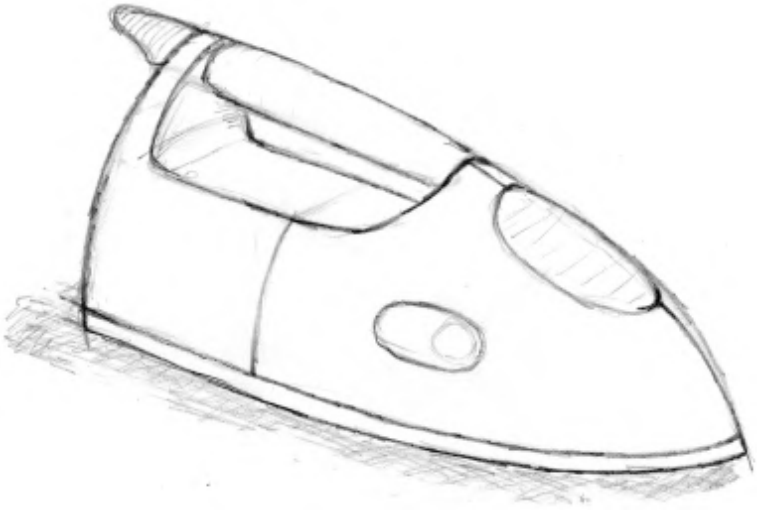
Shape ×

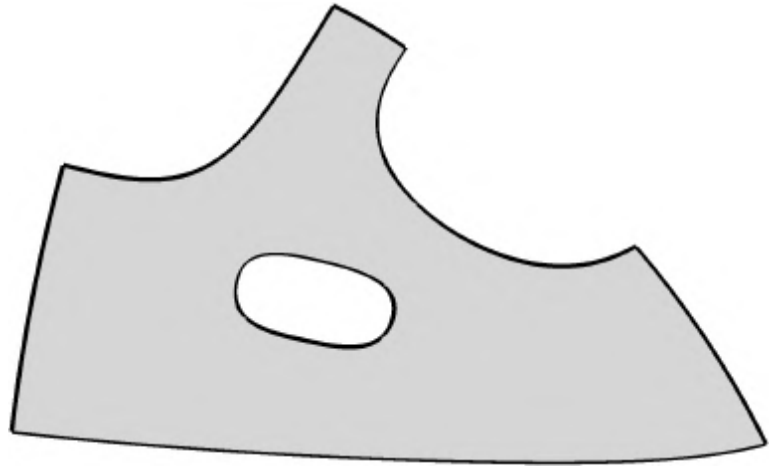
Interior control ✓

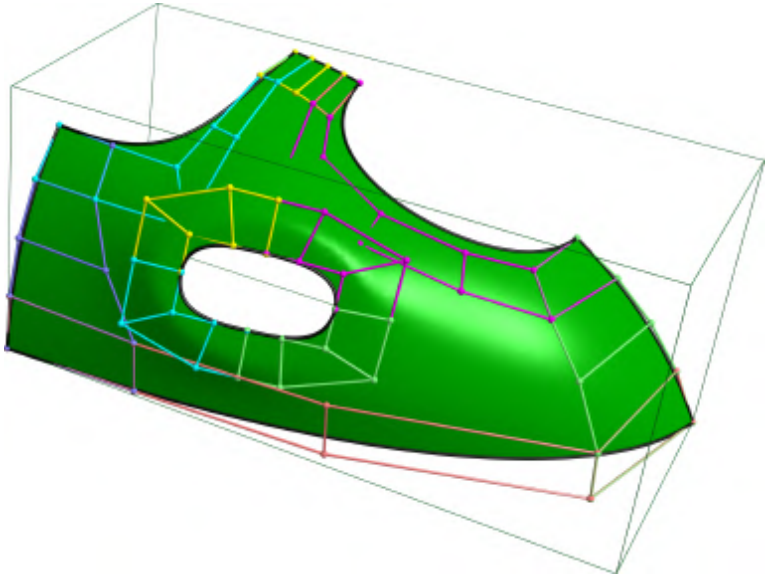
Curved domains [Várady et al., '20]

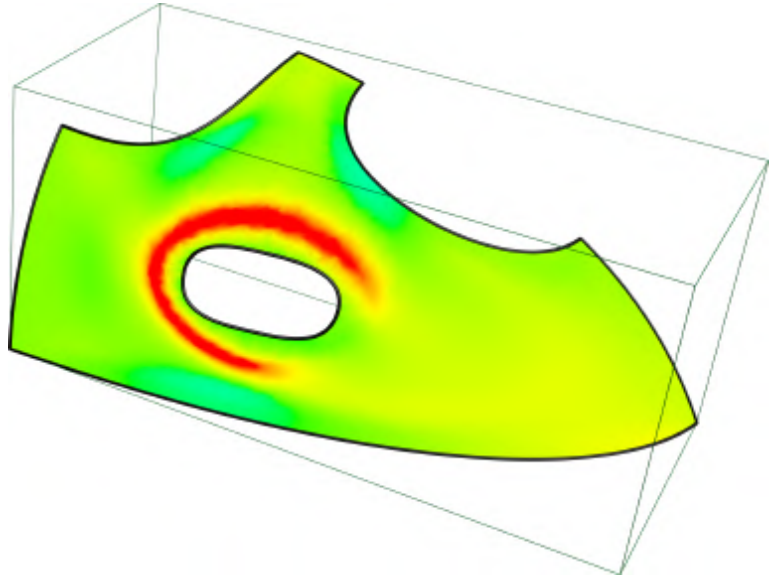


Multiply-connected domains [Várady et al., '20]

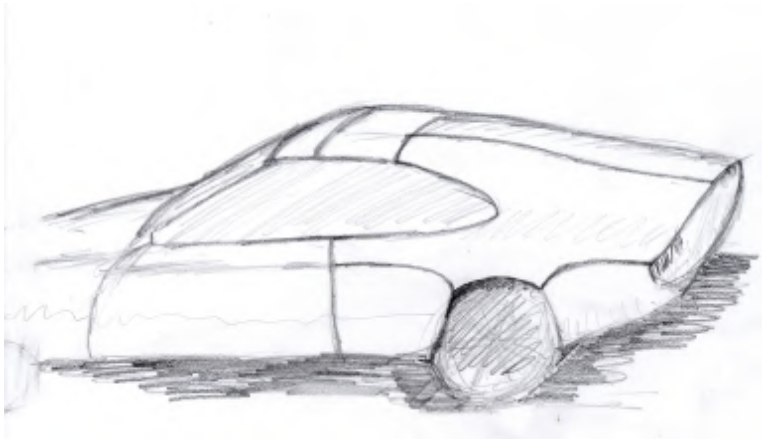


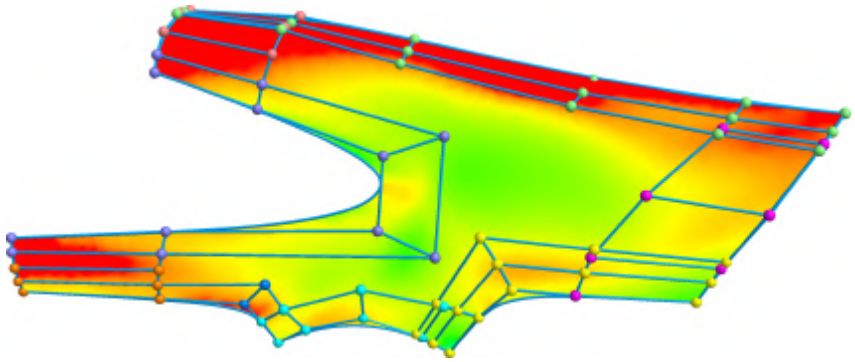


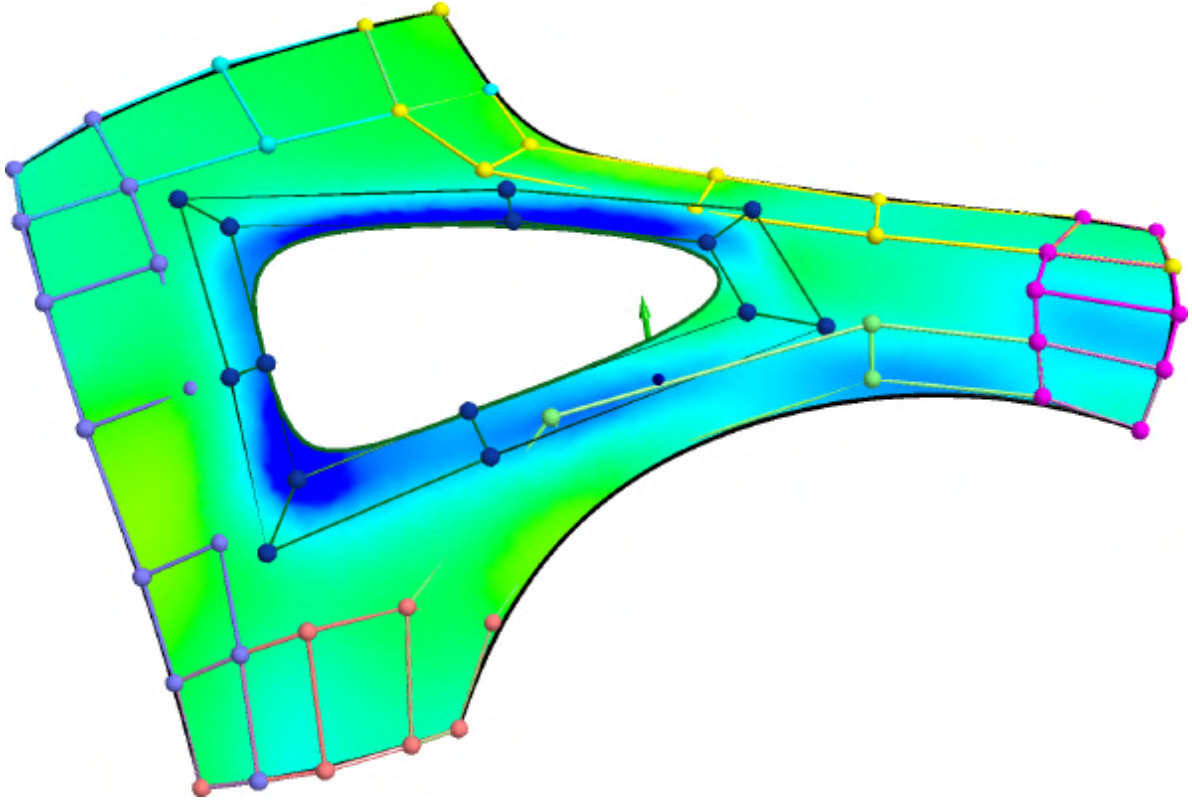




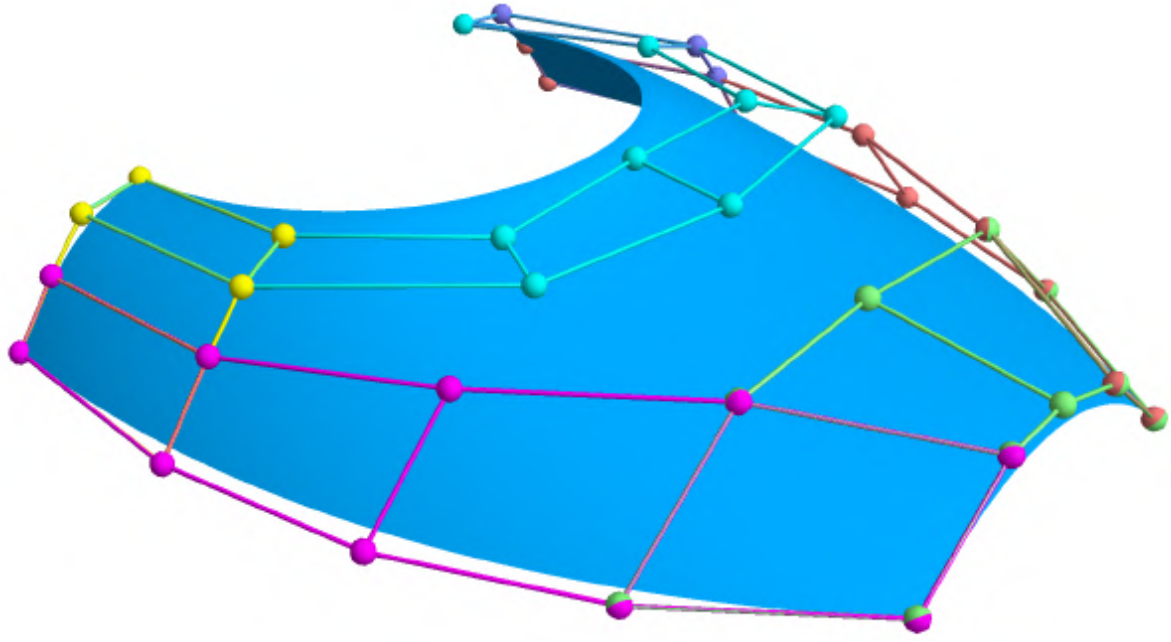
Generalized B-spline patches [Vaitkus et al., '21]







Curved domain Generalized Bézier patches



Shape ✓

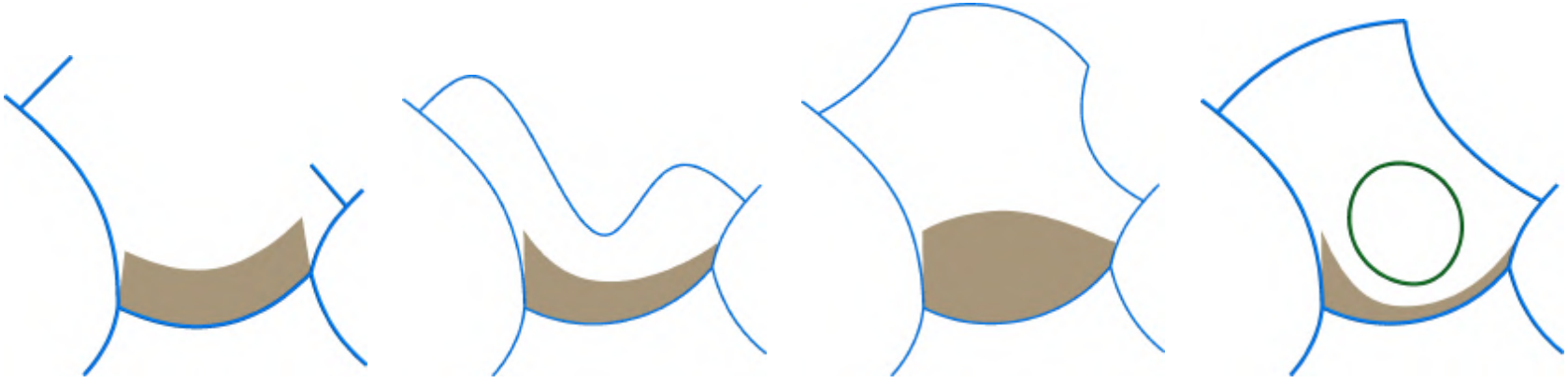
Interior control ×

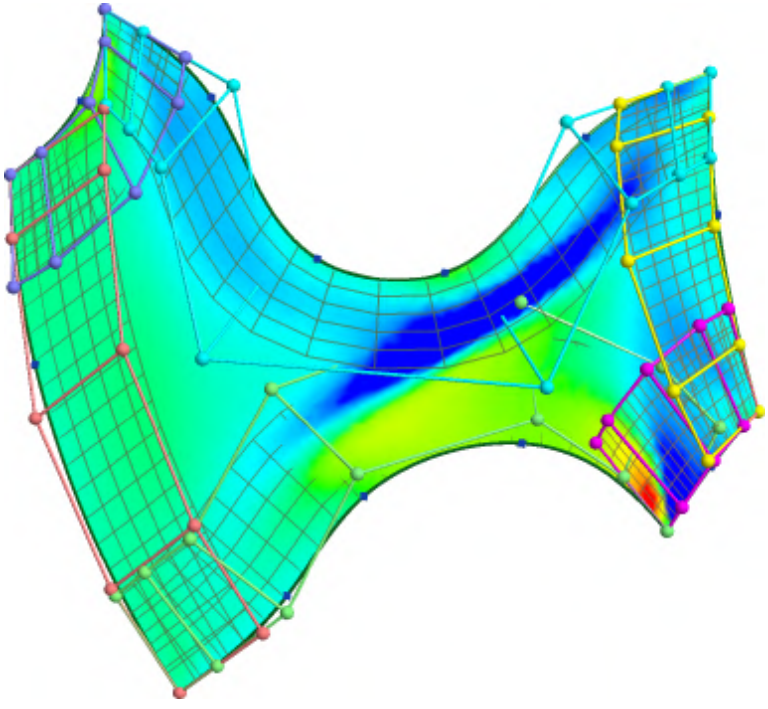
Ribbon setting?

Interior control?

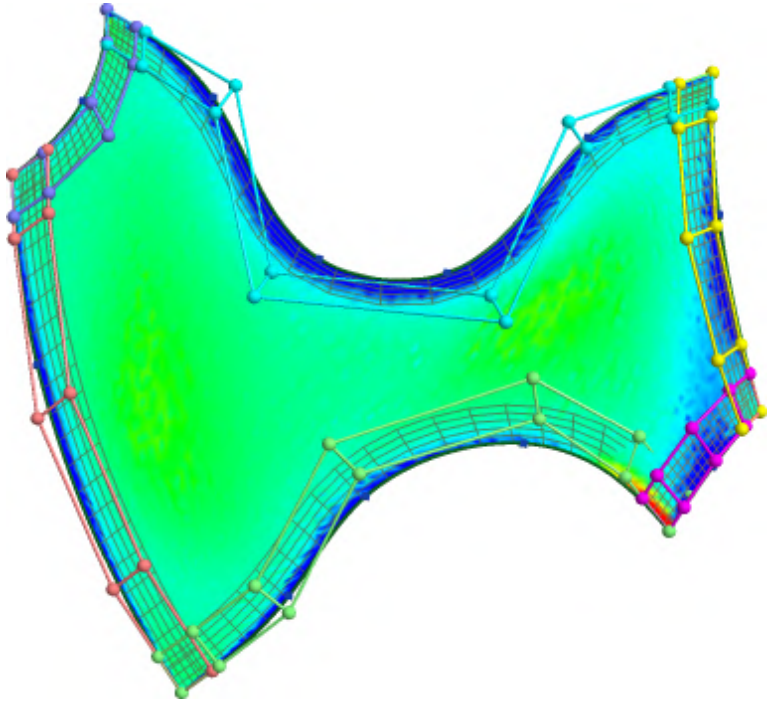
Outline

Importance of cross-derivative strength



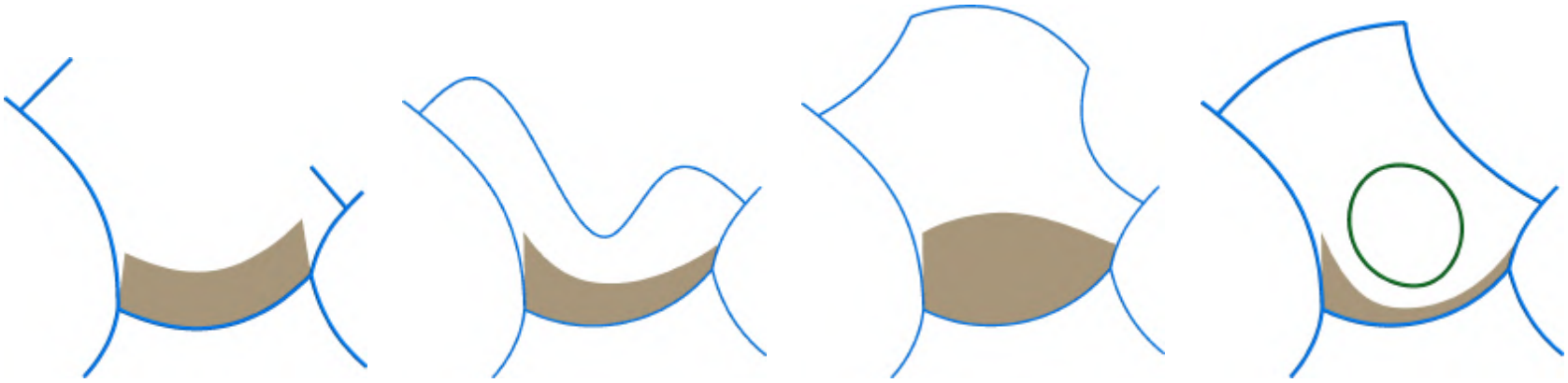


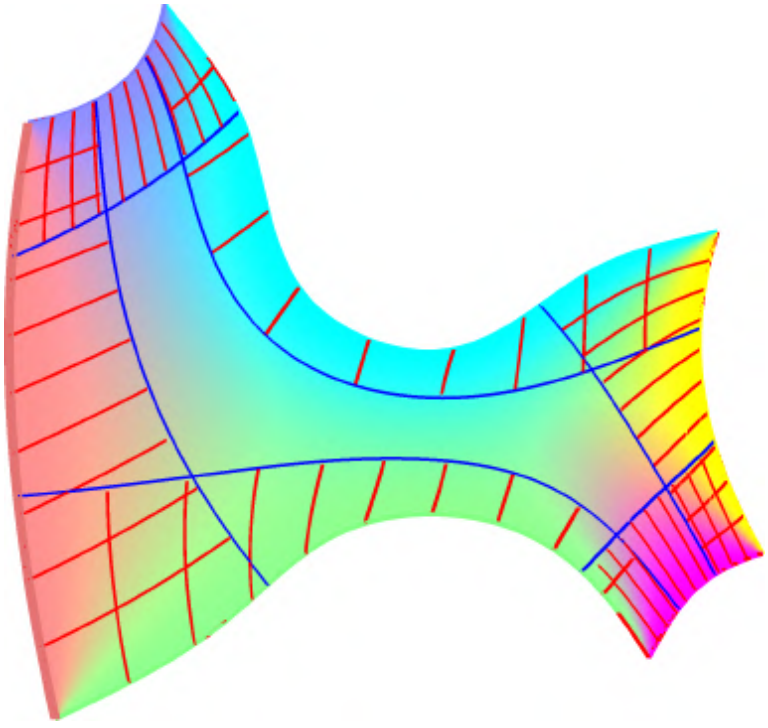
Too strong!



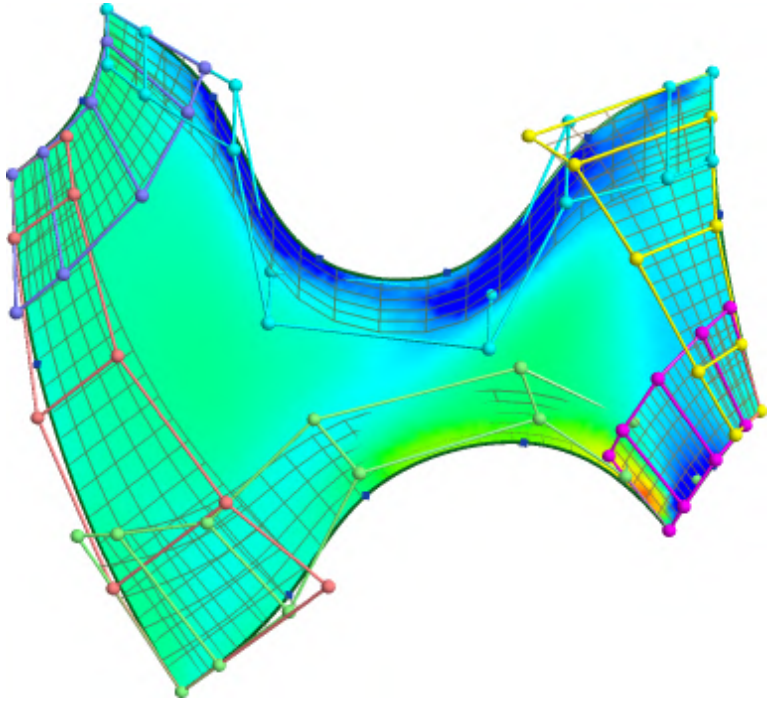
Too weak!

Cross-derivative setting [Salvi et al., '23]





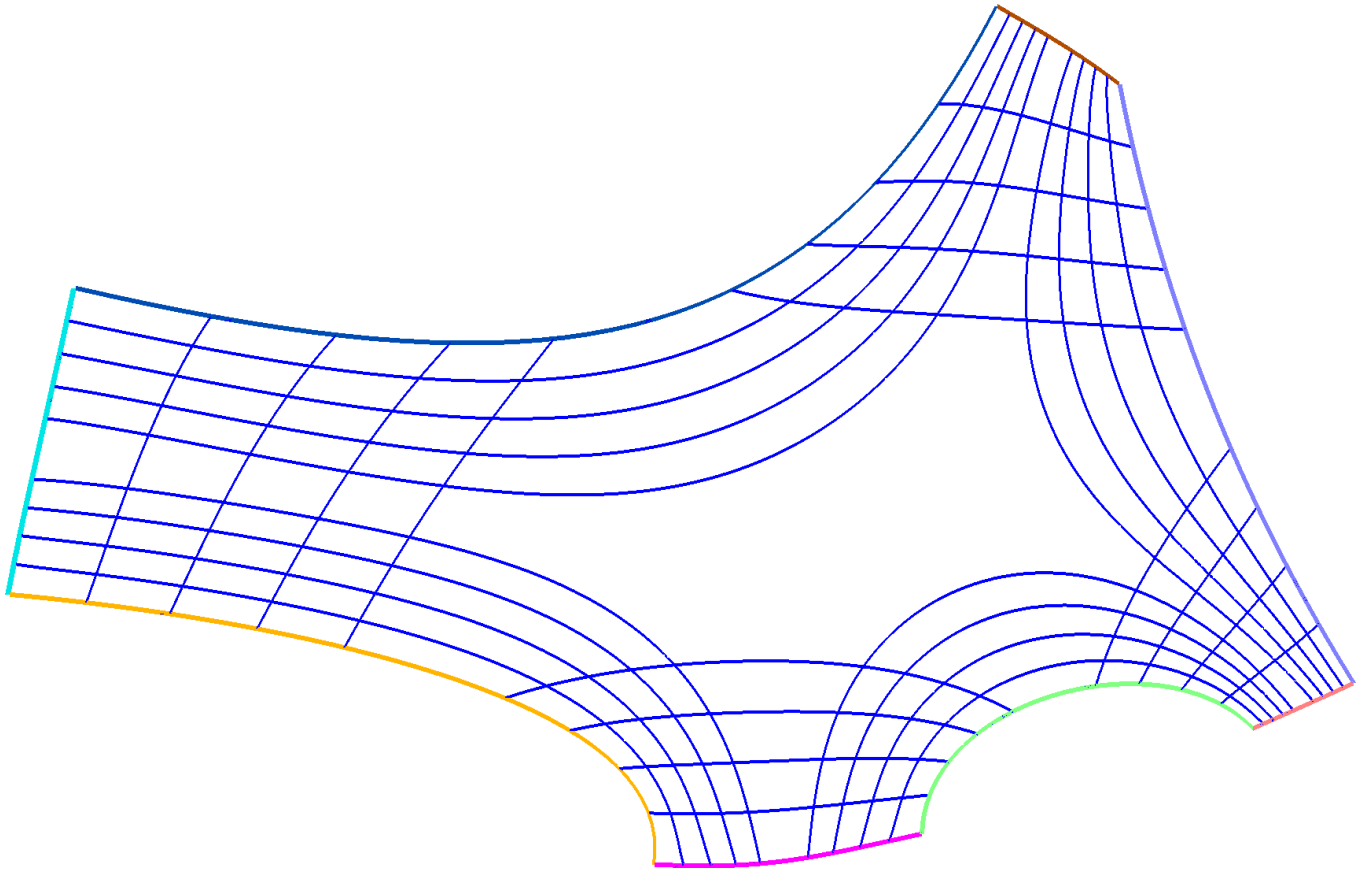
Domain & parameterizations



Good ribbon setting

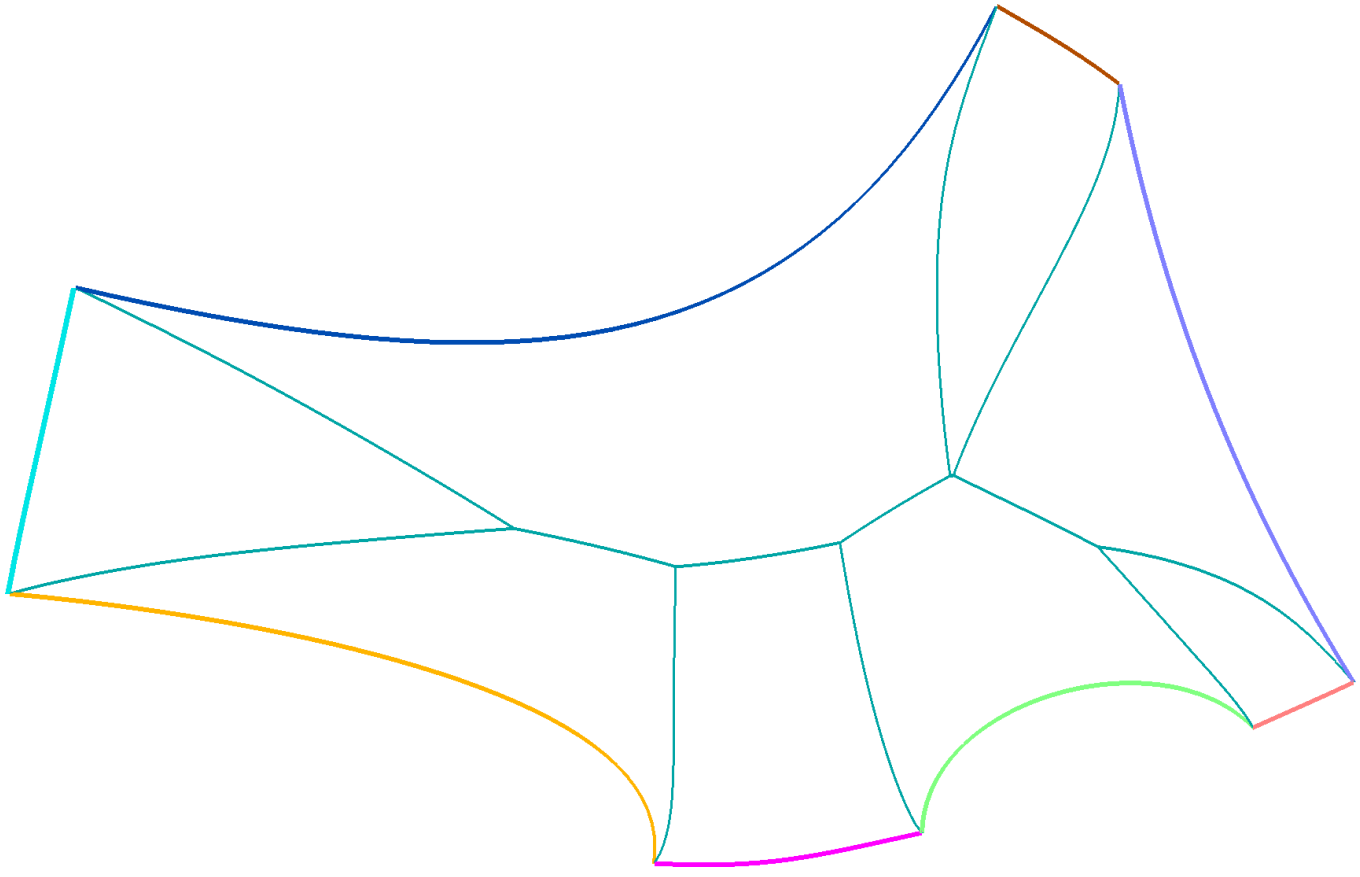
Outline

‘Templates’ from medial axis



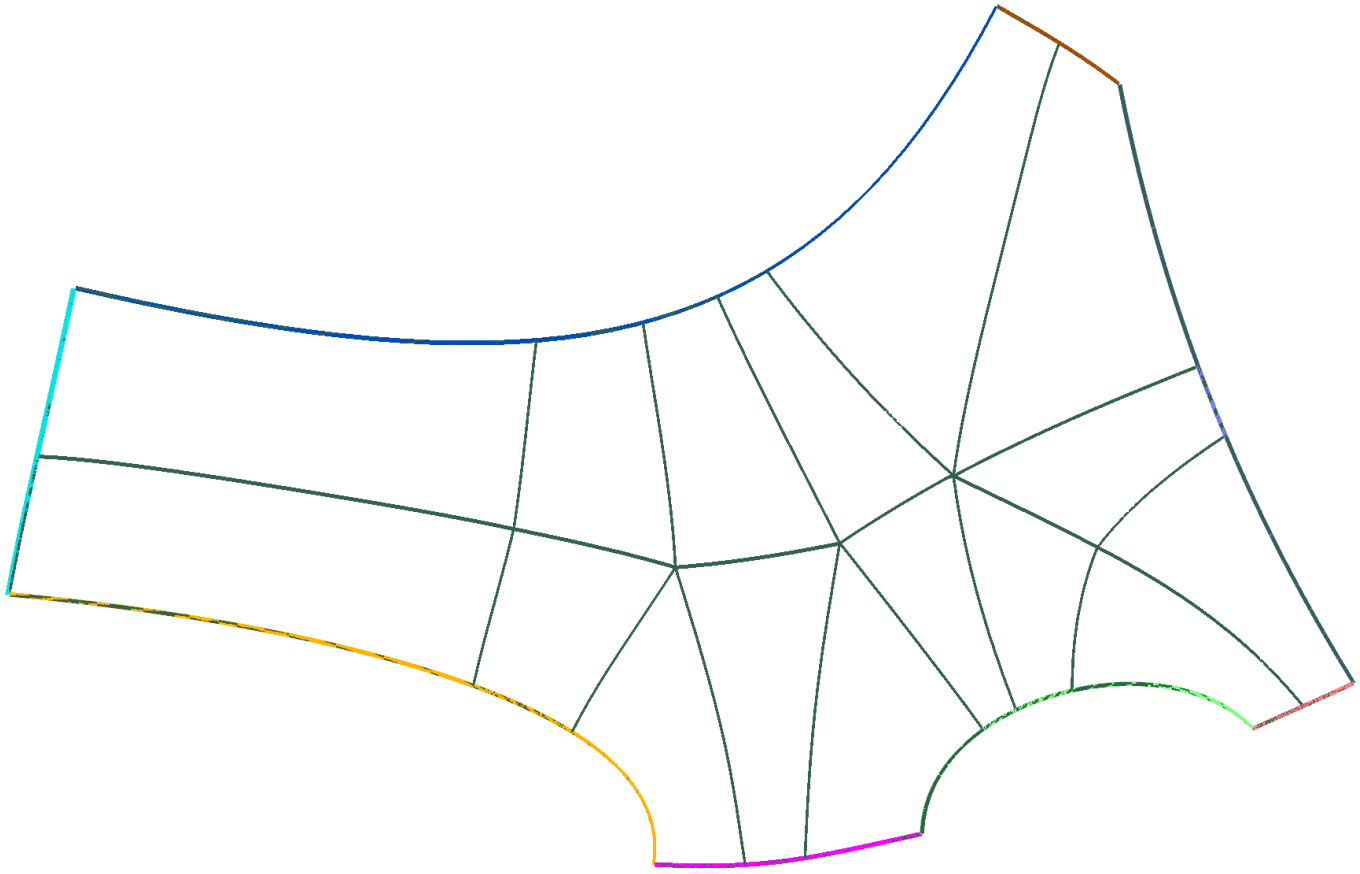
Distance parameters

‘Templates’ from medial axis



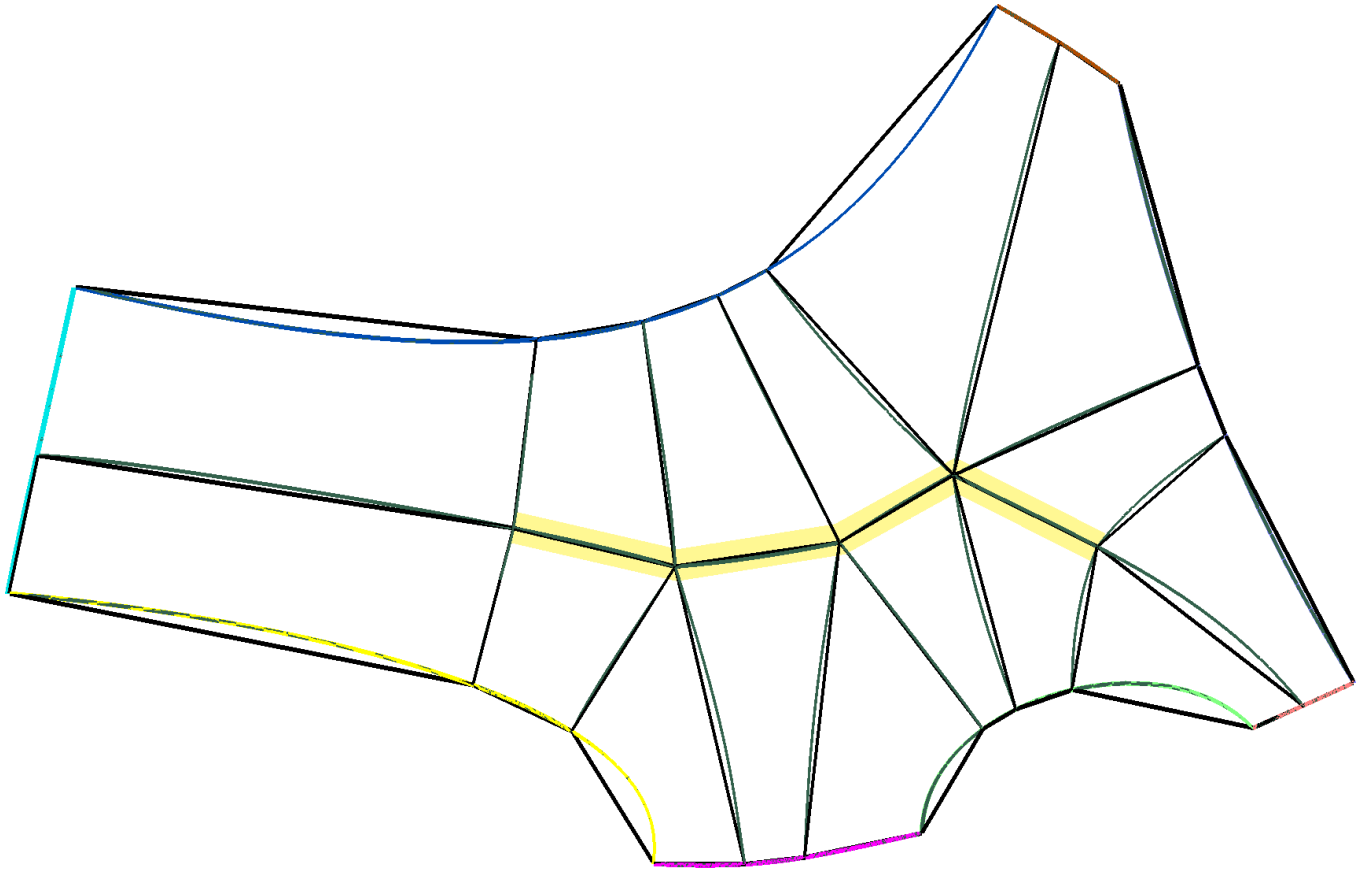
Parametric medial axis

‘Templates’ from medial axis



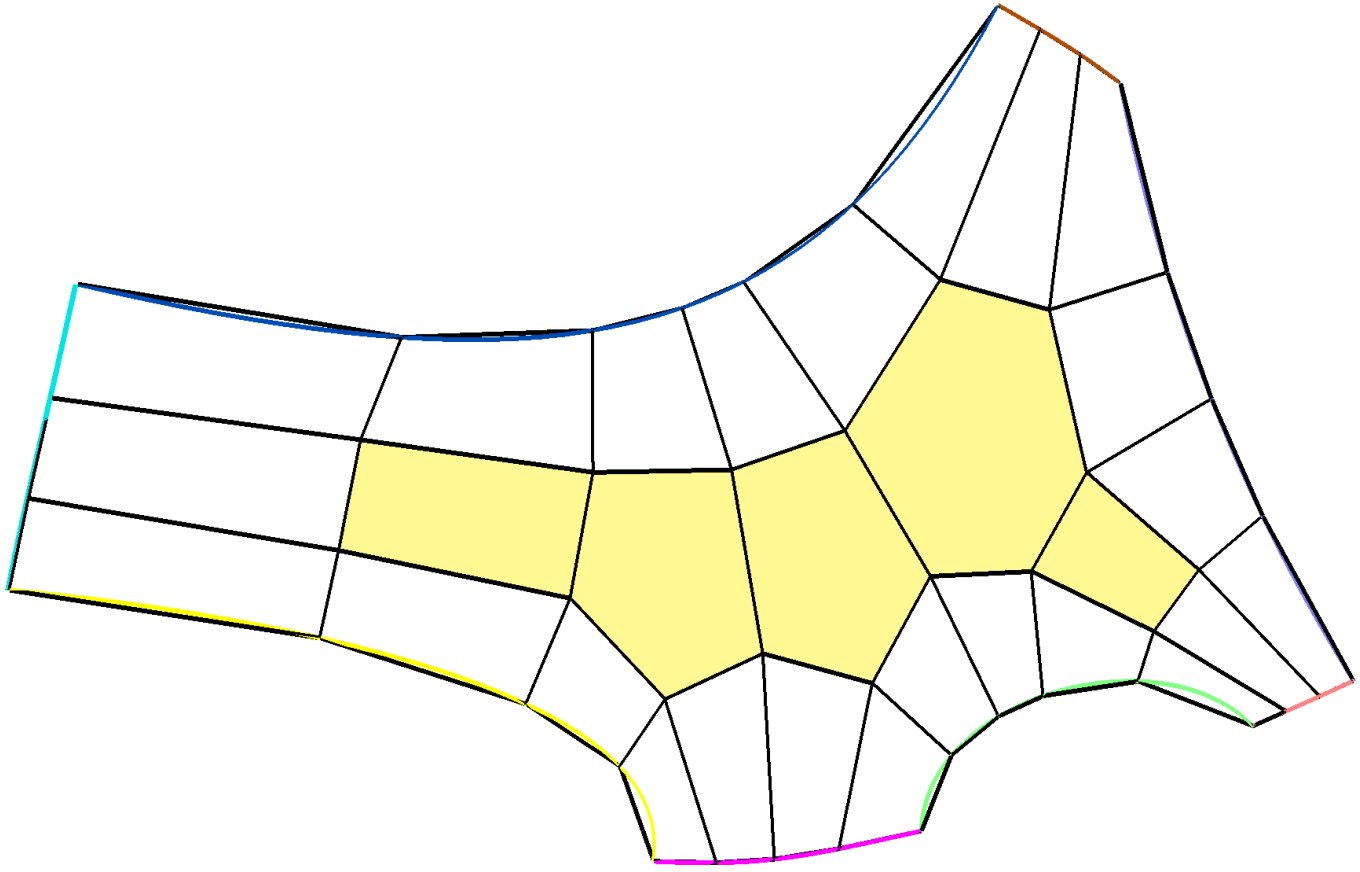
MAT-based quad structure

‘Templates’ from medial axis – $T2$



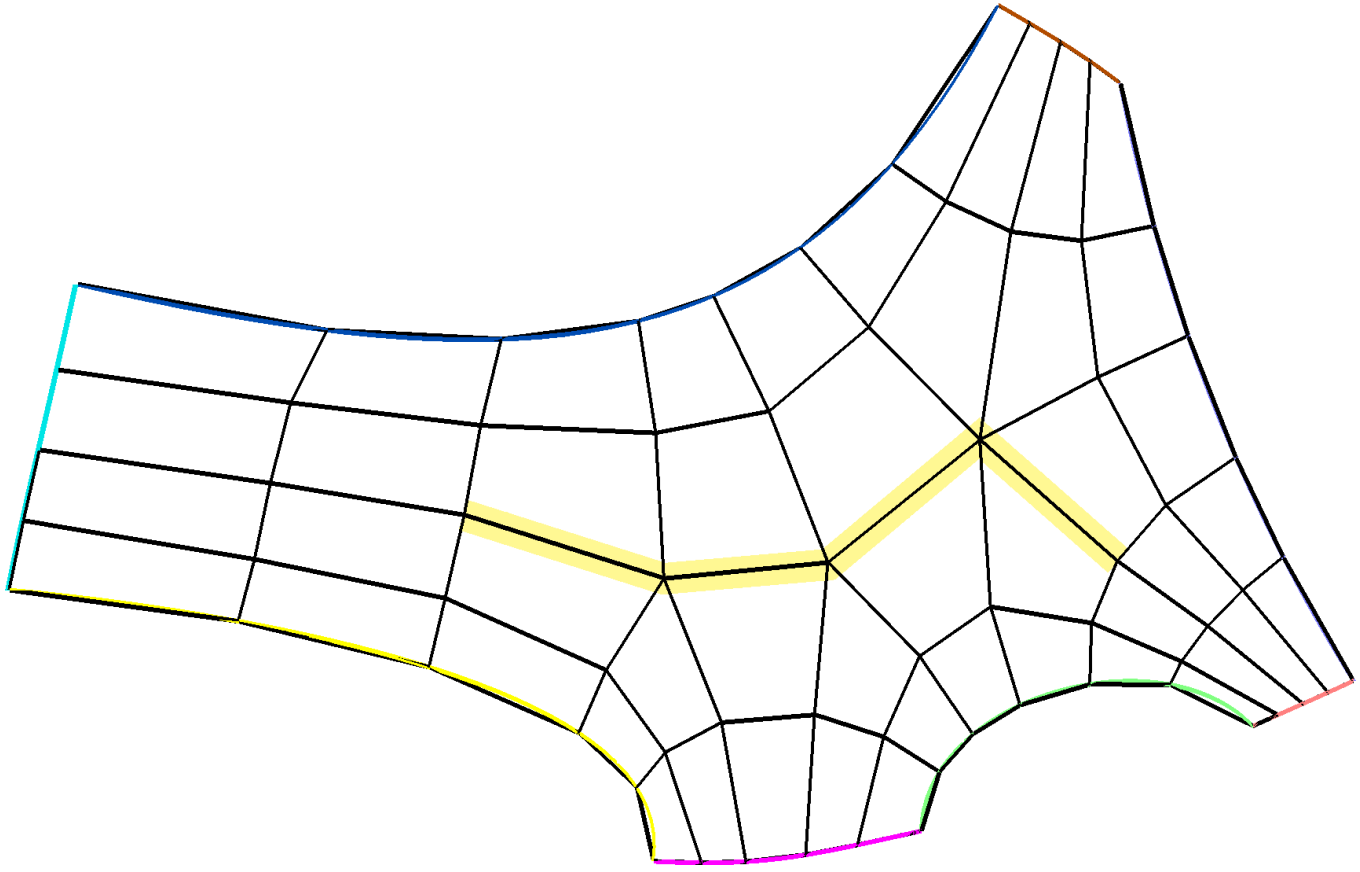
”Quadratic” Bézier template & skeleton

‘Templates’ from medial axis – $T3$



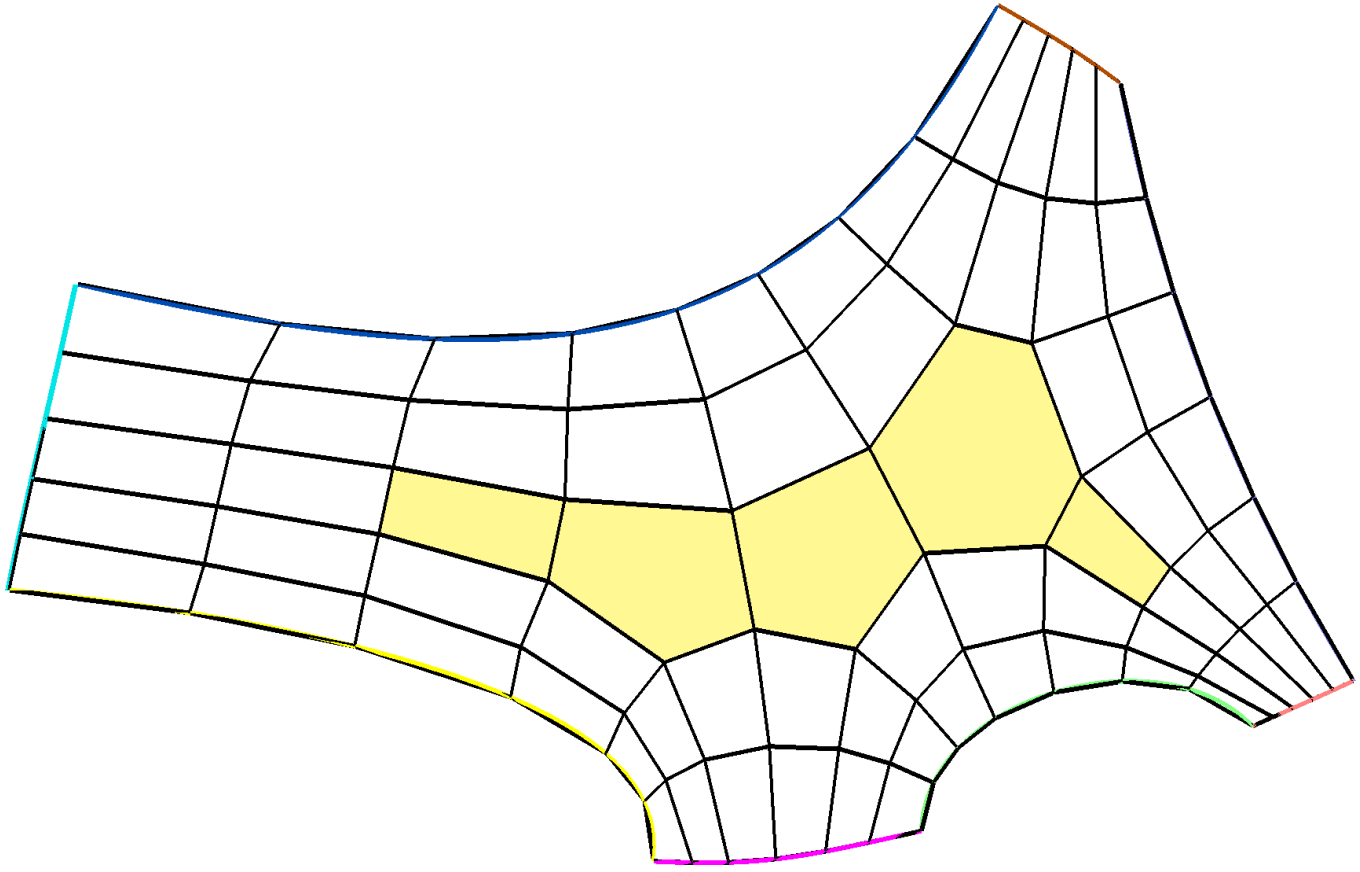
”Cubic” Bézier template & skeleton

‘Templates’ from medial axis – $T4$



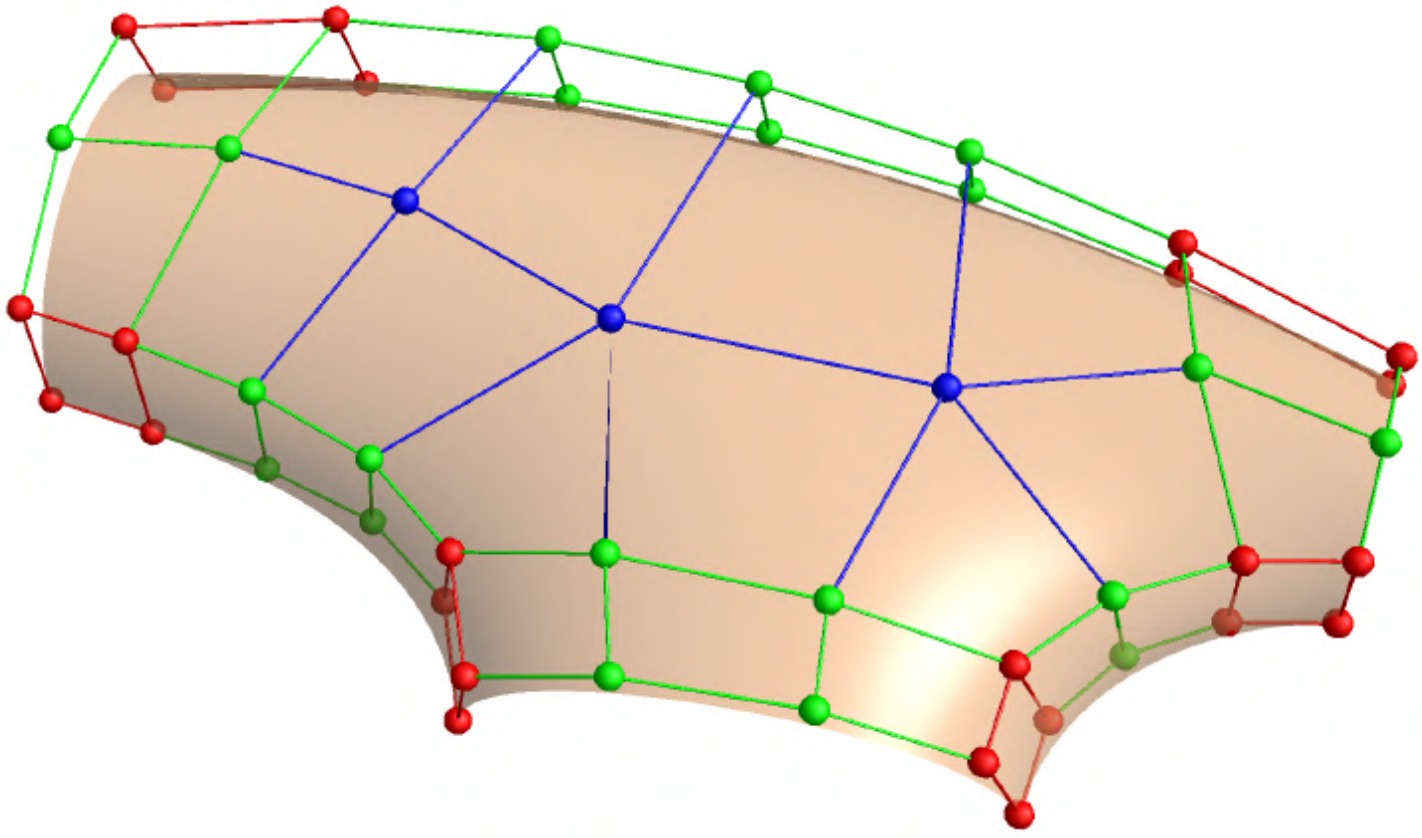
”Quartic” Bézier template & skeleton

‘Templates’ from medial axis – $T5$

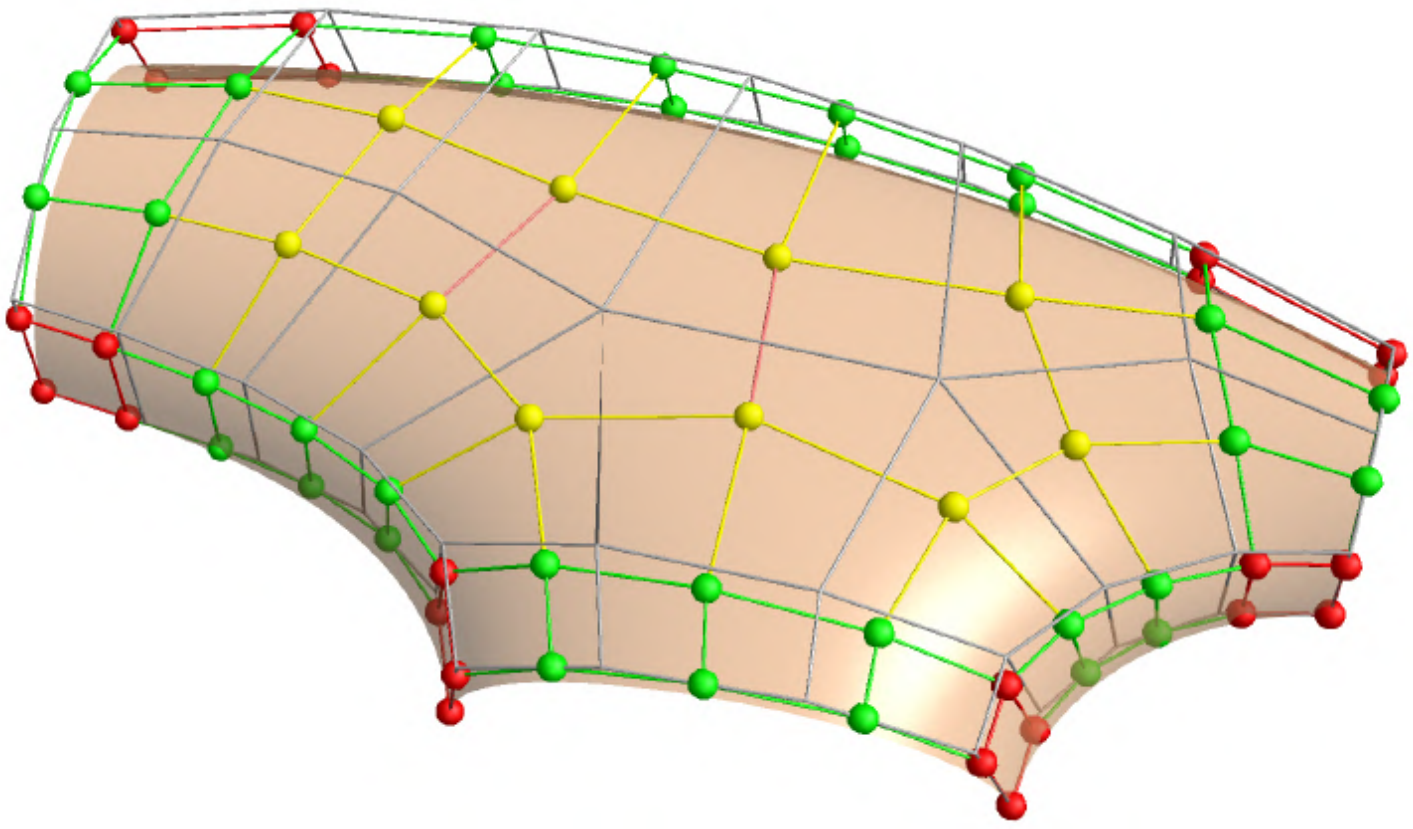


”Quintic” Bézier template & skeleton

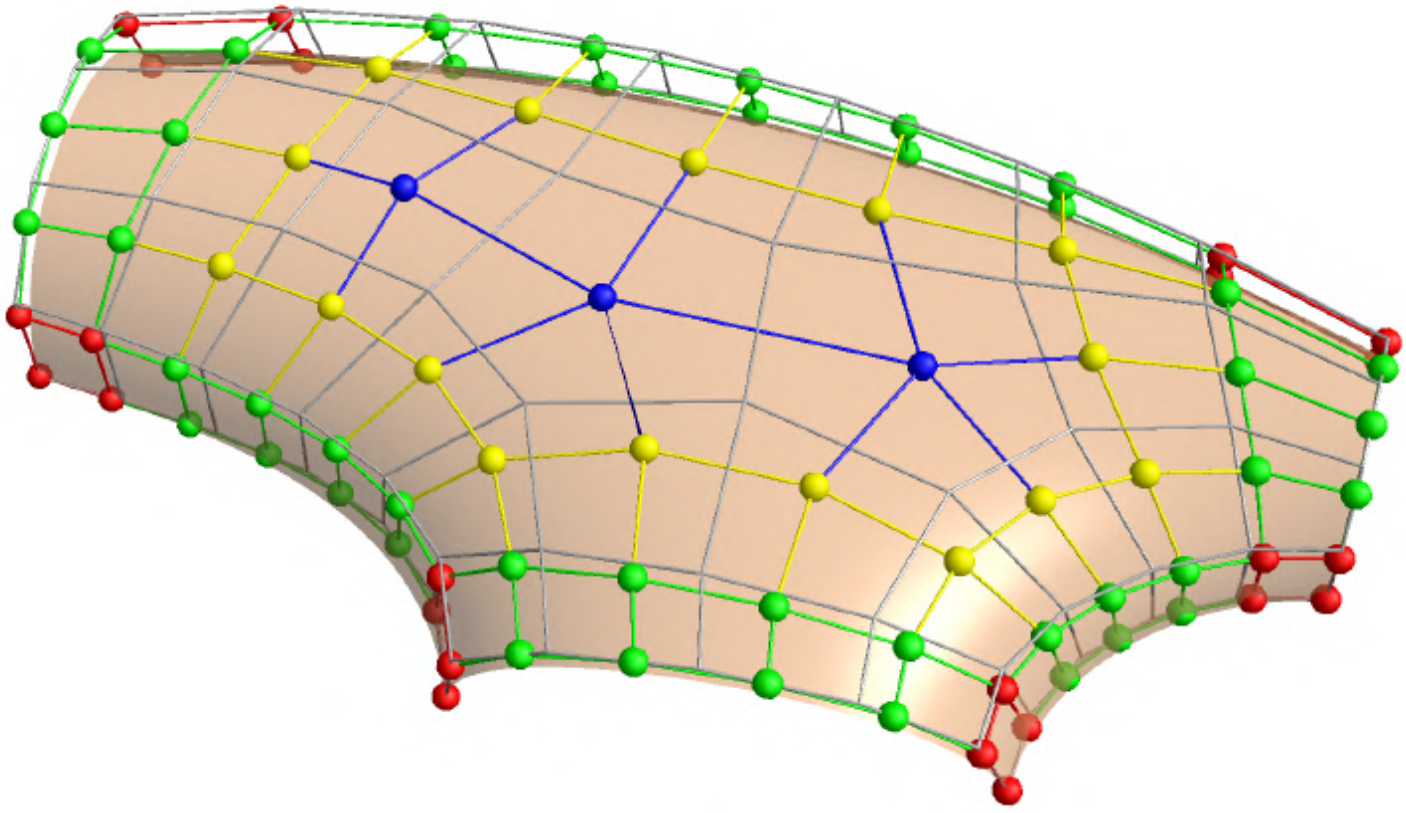
Control structure hierarchy



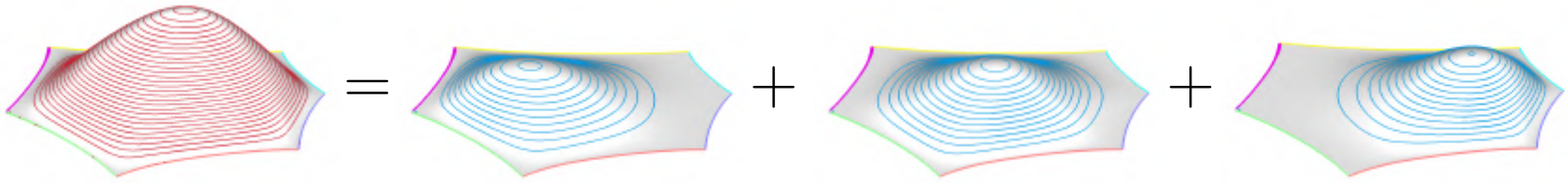
Control structure hierarchy

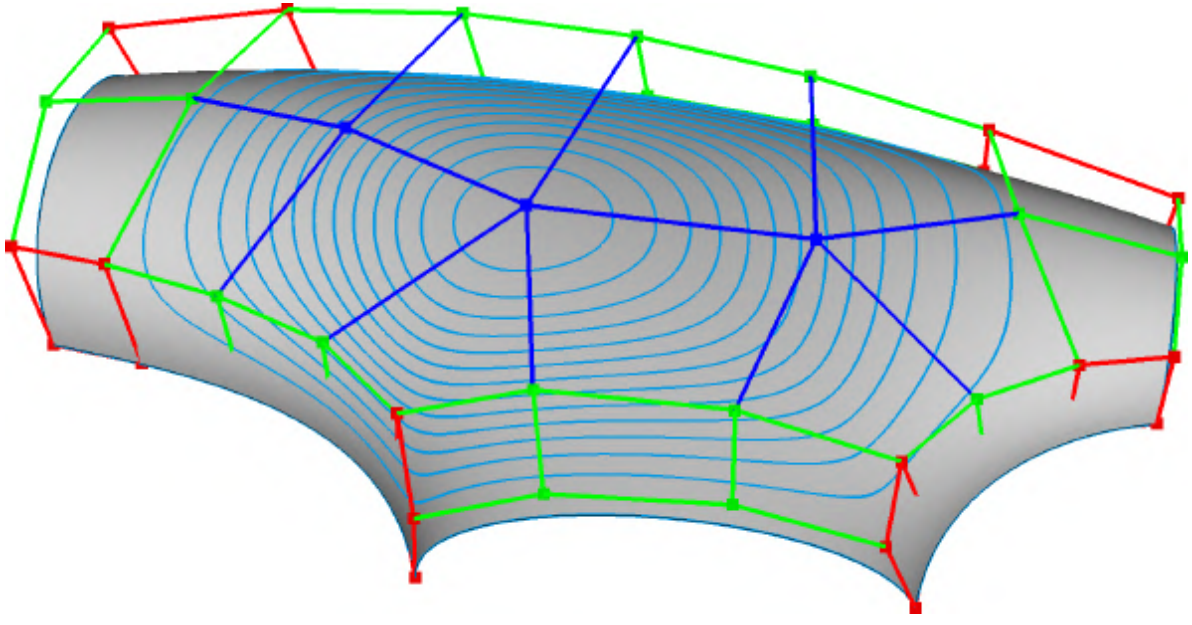


Control structure hierarchy



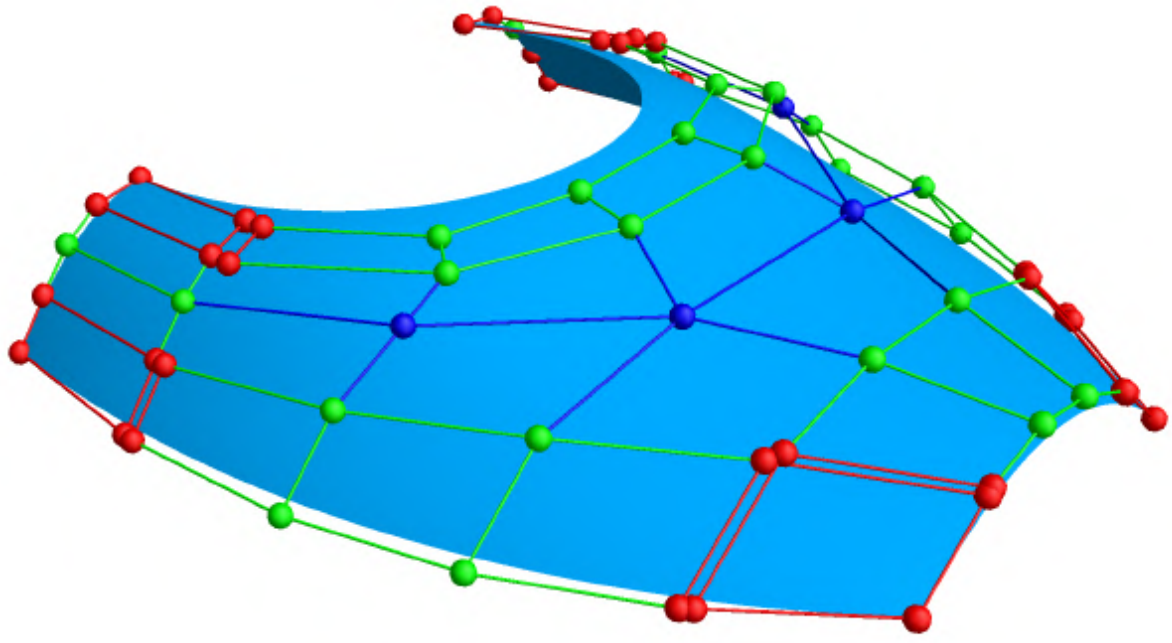
Blending functions





Assign to CPs a *combination* of Bernstein polynomials
Can also **distribute weight deficiency** proportionally

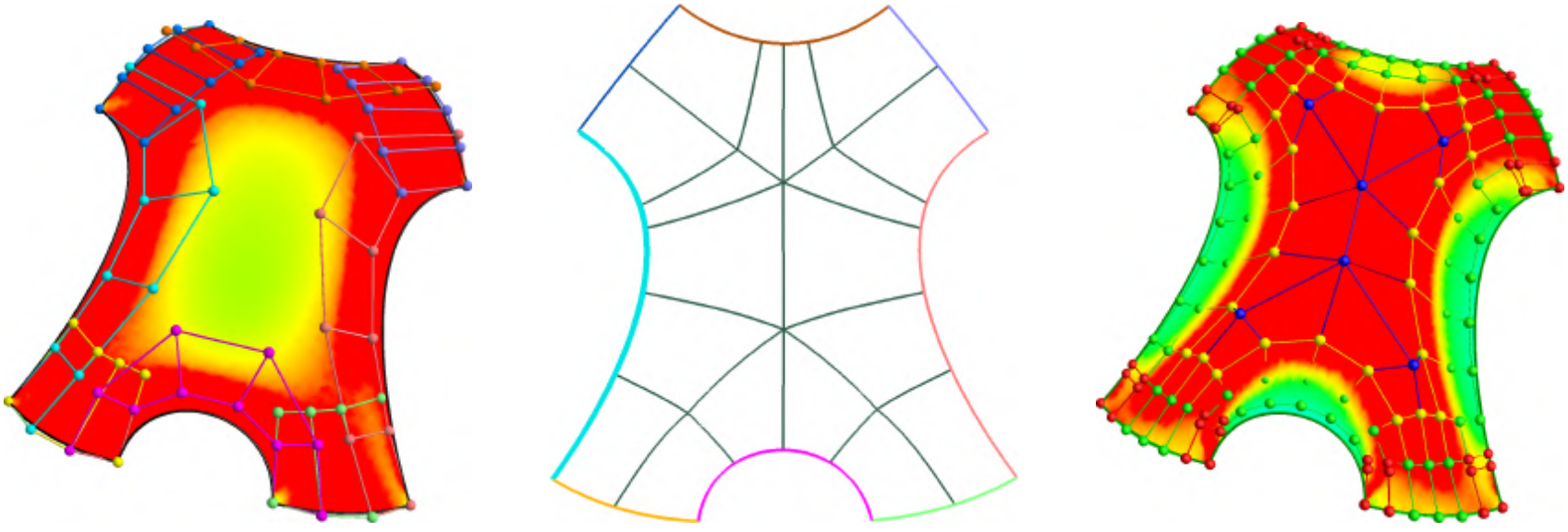
MAT-template Generalized Bézier patches



Shape ✓

Interior control ✓

MAT-template Generalized Bézier patches



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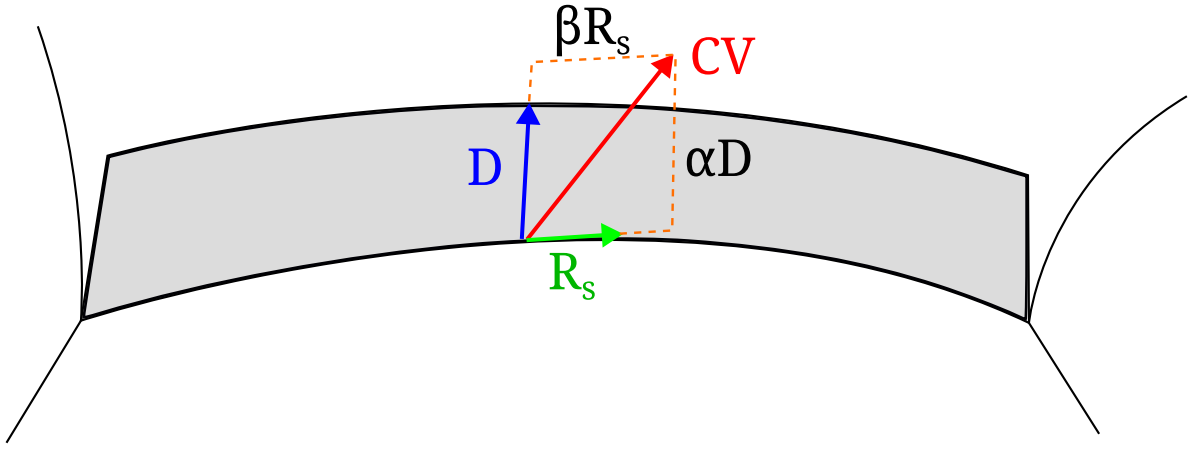
Special Section on SMI 2024

Interior control structure for Generalized Bézier patches over curved domains☆

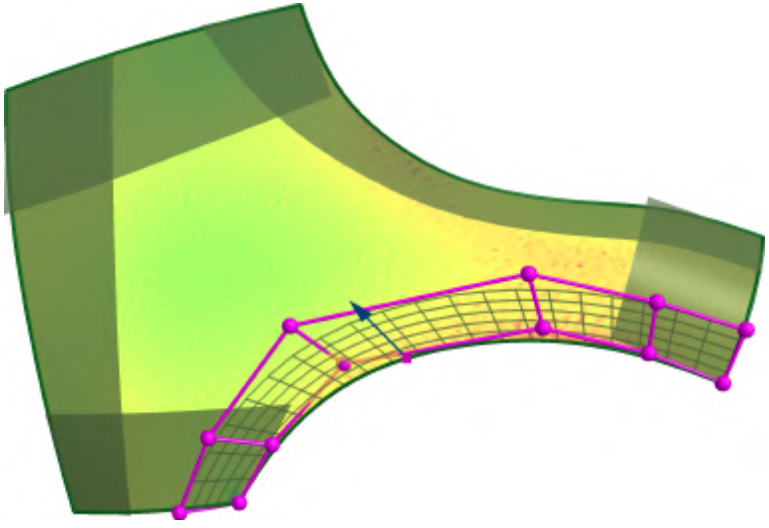
Márton Vaitkus *, Péter Salvi, Tamás Várady

Outline

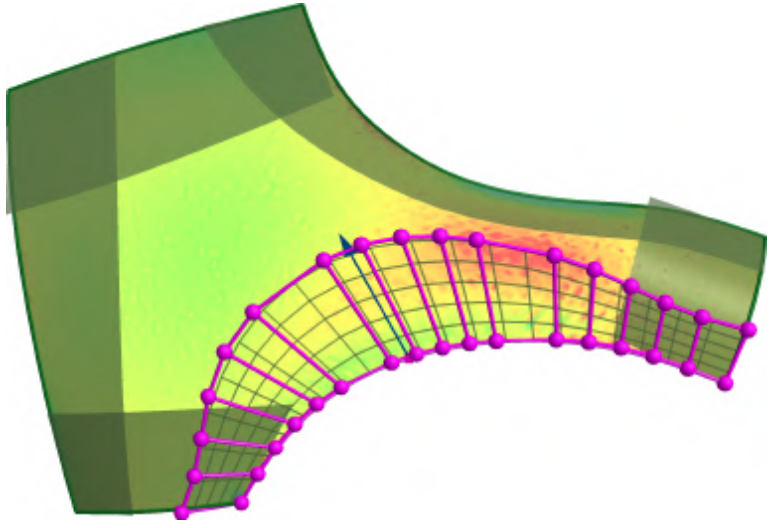
Editing ribbons by control vectors – Exact G^1



$$\mathbf{R}_h(s) = \alpha(s)\mathbf{D}(s) + \beta(s)\mathbf{R}_s(s)$$

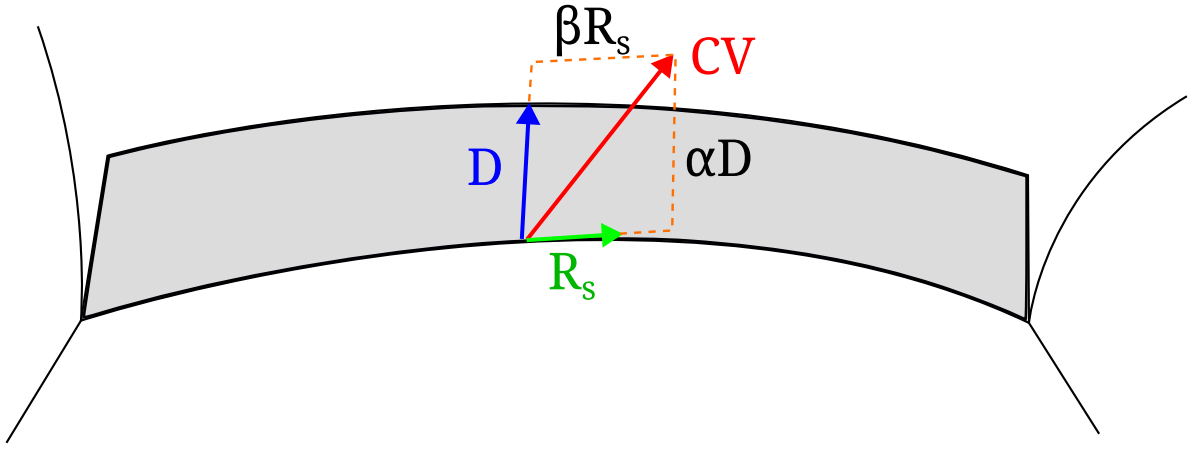


Before editing (deg-3)

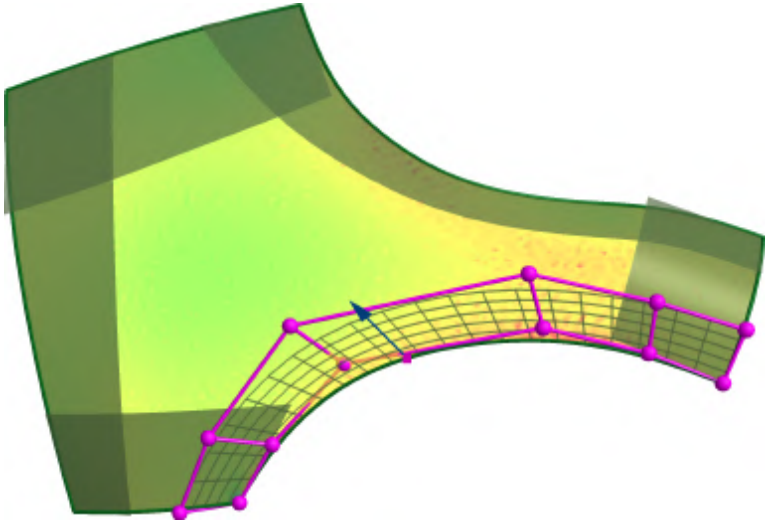


After editing (deg-6, G^1)

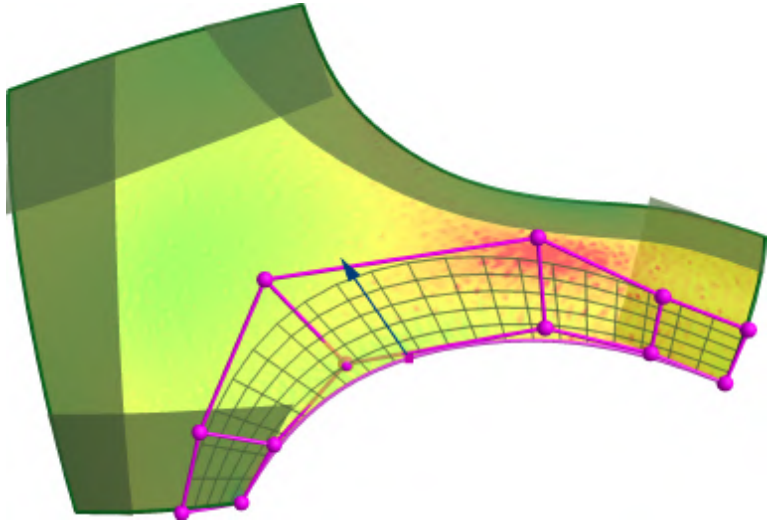
Editing ribbons by control vectors – Approximate G^1



$$\mathbf{R}_h(s) = \alpha(s)\mathbf{D}(s) + \beta(s)\mathbf{R}_s(s)$$

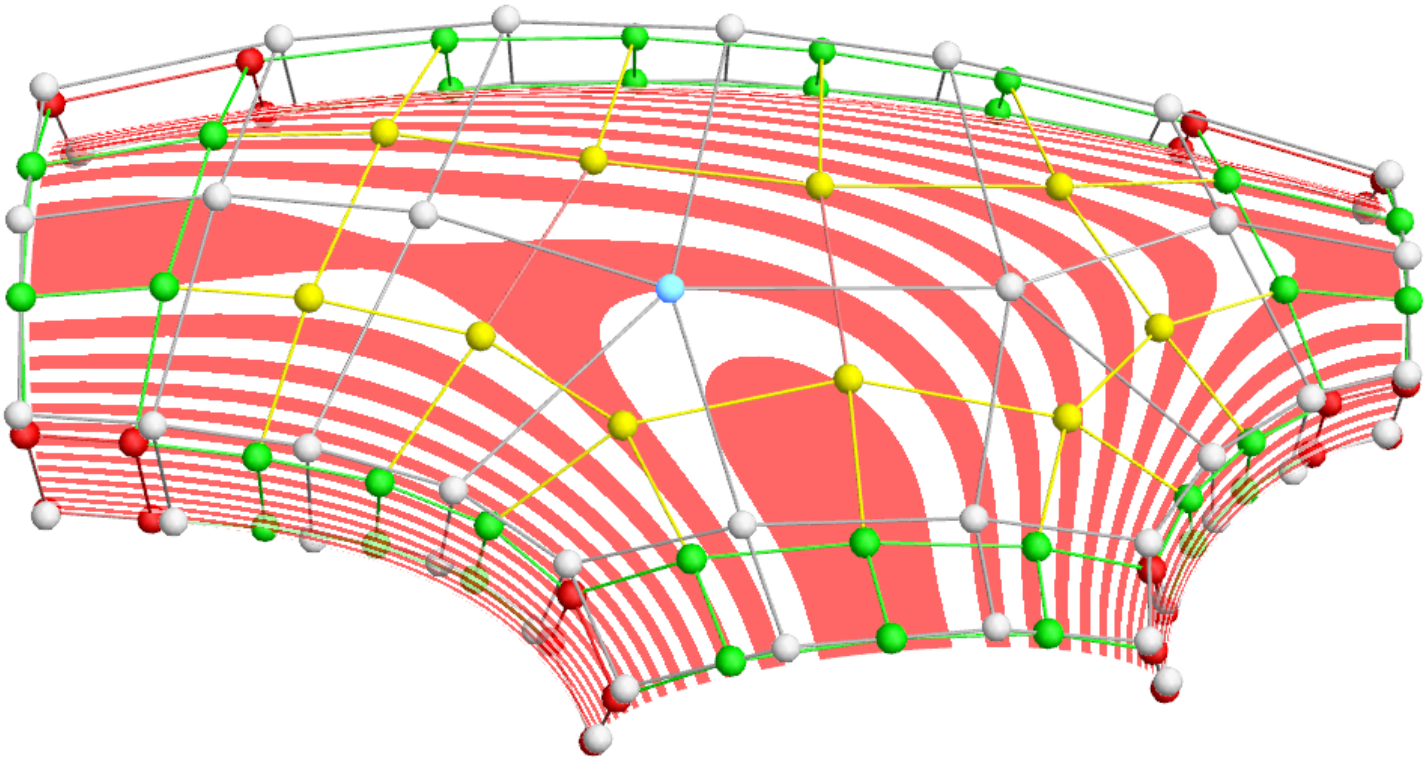


Before editing (deg-3)

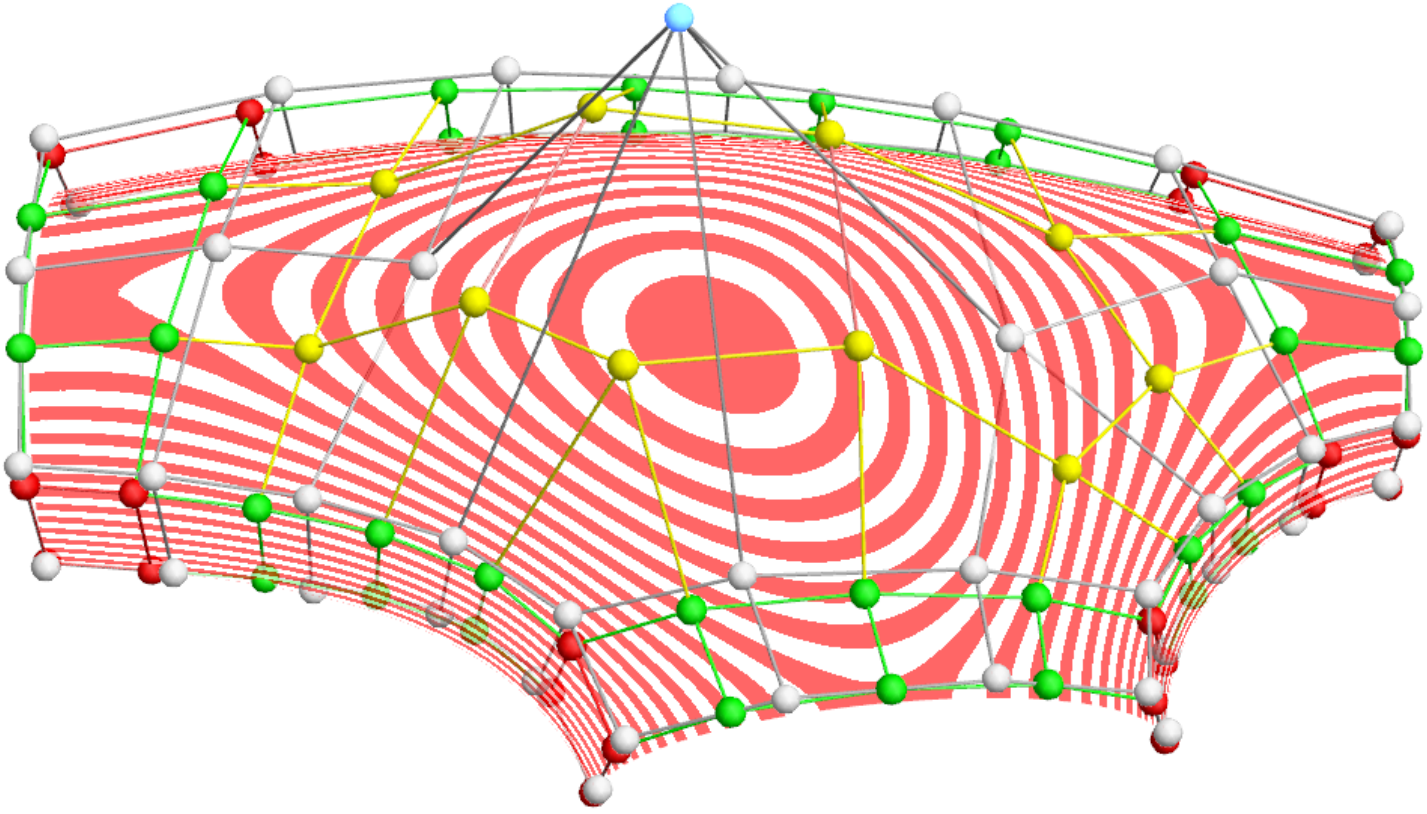


After editing (deg-3, NG^1)

Editing the interior hierarchically



Editing the interior hierarchically



Outline

Conclusion & future work

- Curved, multi-connected domains
  - Handling of highly curved boundaries
  - Natural cross-derivative lengths
  - MAT $\rightarrow$ interior control structure
- Interior blends
  - Proportional weight deficiency distribution
- Editing
  - Boundary CPs $\rightarrow$ implicitly by control vectors
  - Interior CPs $\rightarrow$ simultaneously with a falloff function

Work in progress:

- Interior control structure for Generalized B-splines
- Local interpolation for curve networks (a la Catmull-Rom)
- Shape optimization/fairing

Related papers

1. Multi-sided patch survey

   T. Várady, P. Salvi, M. Vaitkus:
   Genuine multi-sided parametric surface patches – a survey.
   Computer Aided Geometric Design, Vol. 110, #102286, 2024.

2. Modeling with control vectors

   P. Salvi, M. Vaitkus, T. Várady:
   Constrained modeling of multi-sided patches.
   Computers and Graphics, Vol. 114, pp. 86–95, 2023.

3. Independent interior controls

   P. Salvi:
   Intuitive interior control for multi-sided patches with arbitrary boundaries.
   Computer-Aided Design and Applications, Vol. 21(1), pp. 143–154, 2024.

4. MAT-based interior controls

   M. Vaitkus, P. Salvi, T. Várady:
   Interior control structure for Generalized Bézier patches over curved domains.
   Computers and Graphics, 2024. (accepted for SMI'24)

https://3dgeo.iit.bme.hu/