Abstracts for MMCS-2024

Fast cross validation and its theoretical validation

F. Bartel
Chemnitz University of Technology

Thursday 17.00, Auditorium 2

Many modern day algorithms utilize some form of regularization parameter. We will focus on least squares approximation where the regularization is done by using a finite-dimensional approximation space. Theoretically, we know the L2-error behavior and the resulting optimal truncation parameter. However, this is unknown in practice.

In this talk we have a look at the leave-one-out cross validation score. We reduce its tremendeous computational overhead to computing it as a byproduct of the approximation itself. Furthermore, we show a concentration inequality between the cross validation score and the seeked L2-error. This gives a novel theoretical guarantee for the use of cross validation with least squares approximation.

Counting lines in the Wang Shi split

A. Bressan and T. Takacs
IMATI, CNR Pavia

Thursday 10.40, Auditorium 2

The space of C^r piecewise polynomials on a general triangulation poses many theoretical challenges. It is known that the existence of local basis requires degree $d \geq 4r+1$, e.g., the C^1 Argyris element has degree 5. It is possible to lower the required degree by restricting to structured meshes. One way to achieve this is to split each triangle of a general triangulation in macro-elements. Classical examples are the Clough-Tocher and the Powell-Sabin splits. More recently the maximal regularity C^r splines of degree $d = r+1$ has been achieved on the Wang-Shi split for $r = 2, 3$. A small step towards a systematic description of C^{d-1}-splines of degree d on the Wang-Shi split is the simplification of the spline dimension formula. This is the topic of the talk and it reduces to geometric line counting problem.

HP-splines frequency parameter for MultiComponent Signals Interference Detection

V. Bruni, **R. Campagna** and D. Vitulano
Domenico Vitulano, University of Campania 'Luigi Vanvitelli', Caserta, Italy

Thursday 18.20, Auditorium 2

Hyperbolic-polynomial penalized splines (HP-splines) are a recent model for data regression. The spline space depends on a frequency parameter and HP-splines reduce to the widely used P-splines when setting this parameter to zero. HP-splines are suitable for data with an exponential trend, which are frequently encountered in applications. Existence, uniqueness, and reproduction properties are available, together with an efficient algorithm for data-driven frequency parameter selection. We propose a strategy to use HP-splines in the context of signal analysis, to predict the presence of interference regions of MultiComponent Signals (MCSs). These last play a key role in many fields such as biology, audio processing, seismology, air traffic control, and security. They have effective representations in the time-frequency plane where special curves, called ridges, carry information about the instantaneous frequencies of the signal components. However, identifying the ridges is usually a difficult task for signals with interfering components. Therefore, the automatic identification of interference regions is an attractive topic. We prove that the frequency parameter of the HP-splines acts as a *litmus paper*, detecting the interactions between signal components in the spectrogram. In addition, second-order constraints related to the model stiffness, improve the instantaneous frequency recovery.

Fast evaluation of derivatives of Bézier curves

F. Chudy and P. Woźny
University of Wrocław

Wednesday 14.20, Auditorium 1

New geometric methods for fast evaluation of derivatives of polynomial and rational Bézier curves are proposed. They apply an algorithm for evaluating polynomial or rational Bézier curves, which was recently given by the authors. Numerical tests show that the new approach is more efficient than the methods which use the famous de Casteljau algorithm. The algorithms work well even for high-order derivatives of rational Bézier curves of high degrees.

Optimal linear and non-linear dimensionality reduction

A. Cohen
Sorbonne University

Thursday 09.00, Auditorium 1

Understanding how to optimally approximate general compact sets by finite dimensional spaces is of central interest for designing efficient numerical methods in forward simulation or inverse problems. The concept of n-width, introduced in 1936 by Kolmogorov, is well taylored to linear approximation methods. The interest for n-width has recently been revived by the approximation of parametrized/stochastic PDEs, and the development of reduced basis methods. We will survey some of these recent results. We then focus on analogous concept for nonlinear approximation which are still the object of current research, motivated in particular by the development of neural networks, and possible application to hyperbolic parametrized PDEs for which linear methods are not effective. We shall discuss a general framework that allows to embrace various concepts of linear and nonlinear widths, and present some recent results and relevant open problems.

Error Bounds for Meshless Quadrature Formulas

O. Davydov and B. Degli Esposti
University of Giessen, Germany

Wednesday 16.40, Auditorium 1

We present a somewhat different view on the family of meshless quadrature formulas introduced in the talk by Bruno Degli Esposti, by relating them to the compatibility conditions for certain boundary value problems. This approach allows us to derive *a priori* error bounds for the quadrature, based on the analysis of numerical methods for the corresponding boundary value problems. In this way we obtain error bounds for curved 2D domains relying on classical techniques of the finite difference method.

Meshless Quadrature Formulas Arising from Numerical Differentiation

O. Davydov and **B. Degli Esposti**
University of Florence, Viale Morgagni 67, Firenze, Italy

Wednesday 16.20, Auditorium 1

We present a new family of quadrature formulas on scattered nodes, and demonstrate its effectiveness on smooth and piecewise smooth 2D and 3D integration domains Ω. The most distinguishing aspect of our method is that stable quadrature weights are found as a minimum-norm solution to a sparse linear system obtained as a discretization of the divergence theorem by numerical differentiation. The method simultaneously produces quadrature formulas for Ω and for its boundary $\partial\Omega$.

Unlike classical approaches based on interpolation, precomputation of integrals of monomials or other basis functions over Ω is not required. Also, in contrast to composite quadrature, no partition of either Ω or $\partial\Omega$ is needed. Numerical experiments and theoretical arguments show that the convergence order of the quadrature formulas matches that of the numerical differentiation formulas.

A New 3D Subdivision Algorithm

A. Dietz
Technical University of Darmstadt

Friday 11.00, Auditorium 2

The realm of 3D subdivision algorithms is largely uncharted, and so far, only a few primal schemes generalizing cubic B-spline subdivision have been proposed. In this talk, we present the development of a novel dual subdivision algorithm generalizing quadratic B-spline subdivision. It begins with the determination of suitable subdominant eigenvectors of the subdivision matrix based on circle packing, followed by the derivation of the matrix itself using normalization and the exponential of a Colin de Verdière matrix. Our algorithm is applicable to the dual of any hex mesh, has positive weights, respects symmetries, and has a triple subdominant eigenvalue 1/2. Further, when applied to a prism, it reduces naturally to the tensor product of a 2D Doo-Sabin-type scheme and 1D corner cutting.

Bringing B-splines to Industry

T. Dokken
SINTEF Digital

Thursday 18.40, Auditorium 1

The first course on spline technology was lectured at the University of Oslo (UiO) by Prof. Tom Lyche in 1976. The author of this presentation, Tor Dokken, was one of the master's students following these lectures. After the course Tom was Tor's master thesis supervisor. In 1978 Tor was employed as a researcher in the CAD group of the Central Institute for Industrial Research (SI that merged with SINTEF in 1993). The CAD-group addressed industrial CAD/CAM-challenges, many of which were related to sculptured surfaces. Spline technology was our preferred choice for addressing these. In 1980, industrial use of spline technology was in its infancy and there was a strong need for further algorithmic and theoretical work to fulfill industrial needs. Building on the contacts established when Tom was Tor's master's advisor, a close cooperation was established between UiO and SINTEF for extending and improving spline technology. Tom continuously made SINTEF aware of new developments of spline technology, while SINTEF identified the needs for improvement of the theory behind splines for better addressing industrial challenges. One of the more recent topics for cooperation was the theory for Locally Refined B-splines published in 2013 and developed in cooperation between Tom and Tor.

Quadrature rules for C^1 quadratic spline finite elements on the Powell-Sabin 12-split

S. Eddargani, T. Lyche, C. Manni and H. Speleers
Department of Mathematics, University of Rome Tor Vergata, Italy

Friday 10.20, Auditorium 1

Smooth splines on triangulations are favoured for approximation and numerical simulation, in particular in isogeometric analysis, thanks to their inherent support for local refinements and precise representation of complex geometries. A notable instance includes C^1 quadratic splines on Powell-Sabin (PS) 12-split triangulations. To take advantage of the flexibility of these splines in isogeometric Galerkin methods, efficient quadrature rules for this space are essential. In this talk, we discuss quadrature rules for the space of C^1 quadratic splines on the PS12-split. In particular, we present an optimal 4-node quadrature rule that exactly integrates the space on a single triangle, with positive weights and nodes symmetric with respect to the barycentre. Additionally, we present a family of weighted quadrature rules that accurately assemble system matrices in isogeometric Galerkin methods.

Stackable surface rationalization for freeform architectural design

Konstantinos Gavriil
SINTEF, Norway

Thursday 12.20, Auditorium 2

Surface stackability is the measure of how well a freeform surface can be decomposed into stackable components. Under this view, a freeform surface is treated as one modality of a single bimodal geometric object which adheres to two configurations: the deployed surface state and the stacked volume state. This dual configuration introduces a geometric link between freeform surfaces and volume foliations.

Stackability has applications in freeform architecture and digital fabrication methods, such as hot blade cutting and conformal 3D printing, and allows for efficiency in material use, packing, storage and transportation.

I will present work-in-progress results and discuss possible future research questions.

A unified framework for advanced spline constructions in (iso)geometric modeling

C. Giannelli and H.M. Verhelst
University of Florence

Thursday 11.00, Auditorium 1

We present a unified framework for advanced spline constructions for geometric design and isogeometric analysis. Our framework enables both spline adaptivity as well as spline constructions over multi-patch configurations. In particular, different adaptive spline bases on structured mesh configurations can be considered, enabling the possibility of varying refinement capabilities in (iso)geometric modeling and processing. A selection of examples will be presented.

A rational C^1 cubic B-spline form over a Powell–Sabin refined triangulation

J. Grošelj and A. Šadl Praprotnik
University of Ljubljana

Thursday 11.00, Auditorium 2

A standard approach to the construction of smooth low degree polynomial splines over an unstructured triangulation is based on splitting of triangles in such a way that the refined triangulation allows the imposition of smoothness constraints without dependence on geometry. A well-established splitting technique is the Powell–Sabin 6-refinement, which can be used to define C^1 quadratic splines as well as splines of higher degree and smoothness.

In this talk we review the construction of C^1 cubic splines over a Powell–Sabin 6-refinement and show how these splines can be represented in a B-spline-like basis possessing crucial properties for geometric design, namely local support, nonnegativity, and a partition of unity. This in turn enables the definition of a rational B-spline form by following the common CAGD recipe of assigning a positive weight to each of the basis functions. We discuss properties of the introduced form and present some applications.

On the consistent reasoning paradox of intelligence and optimal trust in AI: The power of 'I don't know'

A. Hansen
Cambridge University and University of Oslo

Friday 09.00, Auditorium 1

We present the Consistent Reasoning Paradox (CRP). Consistent reasoning, which lies at the core of human intelligence, is the ability to handle tasks that are equivalent, yet described by different sentences ('Tell me the time!'and 'Could you please tell me the time?'). CRP: There are collections of problems for which there exists an AI that does not hallucinate and is correct on all the problems, given that they are described by specific sentences. However, if such an AI would emulate human intelligence and attempt to be consistently reasoning, it would hallucinate (wrong yet plausible answers) infinitely often on the same problems. Moreover, it is strictly harder to detect if the AI was wrong than solving the original problem. In fact, one cannot detect the hallucinations (with probability $p > 1/2$) even with access to an oracle providing a correct solution to the problem. Also, there are cases where the AI will be correct, but it cannot explain logically why. This implies that any trustworthy, consistently reasoning and explainable AI must be able to say 'I don't know', and that this is the strongest form of trust possible. The CRP is not exotic, but widespread in the sciences. We demonstrate the classes of problems for which it is possible to design consistently reasoning and explainable AI that will say 'I know'and be correct, and say 'I don't know'on problems it will never be able to solve or explain. As a consequence, an AI may avoid hallucinations, however, if such an AI will emulate human intelligence, it becomes fallible. Moreover, it may not be able to logically explain itself. Thus, to maintain trustworthiness it must be able to say 'I don't know'.

A practical implementation of lofted surfaces using Lagrange interpolation and blending

T.E. Henriksen and J.J Ågotnes
UiT The Arctic University of Norway

Wednesday 15.40, Auditorium 1

Lofting, also known as skinning, is an operation in geometric modelling used to create a surface between a set of pre-defined base curves. We present an alternative method for the existing lofting methods. Our method is based on Lagrange interpolation and sub-surface blending techniques, rather than the traditional usage of B-spline curve interpolation or the recently popular T- splines surface construction. The combination of Lagrange interpolation and sub-surface blending reduce problems related to knot insertion, unnecessary control points and incompatibility between base curves. This alternative method also gives the opportunity for live editing of the surface and Ck smoothness between the base curves, where k is the order of the blending function. A prototype showcasing its usages for several example sets of different basis curves is demonstrated.

On the spherical clothoid

A. Ionut
independent researcher Costa Rica

Thursday 18.00, Auditorium 2

We revisit a nonlinear spline primitive for 3-space first studied by Even Mehlum. It is the spherical clothoid, the spherical curve with geodesic curvature a linear function of arc length. We present its Cartesian coordinate functions using confluent hypergeometric functions (the Kummer functions) and its stereographic projection onto the complex plane. New Humbert series results are also presented along with generating function formulas related to the associated Meixner-Pollaczek polynomials.

Data-driven parameterization for adaptive spline fitting

C. Giannelli, **S. Imperatore**, A. Mantzaflaris and F. Scholz
University of Florence

Friday 10.20, Auditorium 2

Geometric model reconstruction from scattered or unorganized 3D point clouds is a ubiquitous problem in several applications. To be approximated using spline geometric models, the input data must be suitably parametrized. In this talk, we present novel geometric deep learning techniques for parameterizing a scattered point cloud in $\mathbb{R}^3$ on a planar domain by exploiting graph convolutional neural networks. Specifically, we first introduce a data-driven parameterization model that predicts the parametric values of the input point cloud from the proximity information of its 3D items and its dual line graph. Secondly, we present an alternative learning model, characterized by a novel boundary informed message-passing input layer that avoids line-graph computation. Finally, we show the effectiveness of these parameterization learning models for adaptive spline fitting with (truncated) hierarchical B-spline constructions.

Complex plane rational Bézier curves

L. Fernández-Jambrina
Universidad Politécnica de Madrid, Spain

Thursday 18.00, Auditorium 1

The real plane can be described using the complex structure as a complex line. Many properties remain the same on using this complex description, but other ones arise from the fact that the projective group of the real plane is not the same and is neither included in nor comprises the projective group of the complex line. The obvious example is that the complex group includes inversion, whereas the real group does not. This amounts to new geometry when the complex structure is considered. In the case of rational Béezier curves the key feature is that weights are no longer real, but complex. This possibility has been explored in several papers in the literature. For instance, circumferences arcs are of degree one as complex curves, but of degree two as real curves. In this talk we would like to emphasize the differences between both representations of rational plane curves, the relation between them and the possibilities provided by the use of the complex projective group.

Local Polynomial Reproduction for Manifold Spline Projectors

B. Jüttler
Johannes Kepler University, Linz/Austria

Thursday 10.20, Auditorium 2

This talk focuses on the property of polynomial reproduction, which is es- sential for quantifying the approximation power, in the context of splines on manifolds. More specifically, we consider the extension of the spline con- structions to domains that are manifolds, possibly with boundaries. This extension is frequently used to gain additional flexibility for applications in geometric modeling and isogeometric analysis. On the one hand, we observe that the derivation of the error bounds carries over to the manifold setting. On the other hand we describe the property of local polynomial reprroduc- tion needed in order to adapt the arguments to the manifold setting. We also plan to discuss a concrete construction that realizes this framework for the two-dimensional (i.e., surface) case. Special attention will be paid to the effect of linearly dependent functions spanning the manifold spline space, which are needed to guarantee certain symmetries of the construction. Joint work with P. Langgruber.

Rectifying control polygons for Minkowski Pythagorean hodograph curves

S. H. Kim
Pusan National University

Thursday 17.40, Auditorium 1

This presentation introduces an advancement in Minkowski Pythagorean hodograph (MPH) curves by extending the notion of rectifying control polygons from Euclidean Pythagorean hodograph (PH) curves. We construct the Gauss-Legendre (GL) polygon using Gauss-Legendre quadrature applied to the hodograph. Since the parametric speed of the curve can be represented by a polynomial, the GL polygon with sufficient edges possesses the rectifying property, ensuring its length matches that of the MPH curve. Moreover, we address the inverse problem, offering a methodological approach for deriving MPH curves from a given GL polygon. This improvement holds promise for facilitating smoother deformation of MPH curve shapes and for widening the scope of MPH curves' applicability across diverse mathematical and practical domains.

Least squares approximation with planar Pythagorean–hodograph curves

R.T. Farouki, **M. Knez**, V. Vitrih and E. Žagar
Faculty of Mathematics and Physics, University of Ljubljana,
Jadranska 19, Ljubljana, Slovenia

Thursday 15.40, Auditorium 1

Polynomial Pythagorean–hodograph (PH) curves, characterized by the property that their unit tangent is rational, have many important features for practical applications. The property of having a polynomial parametric speed significantly simplifies the computation of the curve's length and the reparameterization to the arc-length parameter. The construction and analysis of planar PH curves may be greatly facilitated using the complex variable model, in which points in the Euclidean plane are identified with complex numbers. The typical way to compute PH curves is through the interpolation of prescribed geometric data, and a wide number of different interpolation schemes can be found in the literature.

In this talk, we present a different approach. By defining a suitable metric function for planar curves, based on the complex representation, we define the PH least squares approximants to a given planar curve. Because of the non-linear nature of PH curves, constructing least squares approximants involves solving a non-linear optimization problem. However, a simplified method that requires only the solution of a linear system and is developed by formulating the approximation in the complex preimage space will be presented. Several computed examples will illustrate its implementation and performance. The extension of the methodology to approximation by B-spline PH curves will also be addressed, and issues regarding the generalization to spatial curves will be mentioned.

A Point-Normal Interpolatory Subdivision Scheme Preserving Conics

N. Bügel, L. Romani and **J. Kosinka**
University of Groningen

Friday 10.40, Auditorium 2

In the domain of interpolatory subdivision schemes, there is a demand for developing an algorithm capable of (i) reproducing all types of conic sections whenever the input (point-normal) data are arbitrarily sampled from them, (ii) generating a visually pleasing limit curve without creating unwanted oscillations, and (iii) having the potential to be naturally and easily extended to the bivariate case. In this paper we focus on the construction of an interpolatory subdivision scheme that meets all these conditions simultaneously. At the center of our construction lies a conic fitting algorithm that requires as few as four point-normal pairs for finding new edge points in a subdivision step. Several numerical results are included to showcase the validity of our algorithm.

Isogeometric collocation for solving the biharmonic equation over planar multi-patch domains

M. Kapl, **A. Kosmač** and V. Vitrih
University of Primorska

Thursday 10.20, Auditorium 1

We present an isogeometric collocation method for solving the biharmonic equation over planar bilinearly parameterized multi-patch domains. The developed approach is based on the use of the globally C^4-smooth isogeometric spline space [Kapl, Vitrih, 2021] to approximate the solution of the considered partial differential equation, and proposes as collocation points two different choices, namely on the one hand the Greville points and on the other hand the so-called superconvergent points. Several examples demonstrate the potential of our collocation method for solving the biharmonic equation over planar multi-patch domains, and numerically study the convergence behavior of the two types of collocation points with respect to the L^2-norm as well as to equivalents of the H^s-seminorms for $1 \leq s \leq 4$.

In the studied case of spline degree $p = 9$, the numerical results indicate in case of the Greville points a convergence of order $\mathcal{O}(h^{p-3})$ independent of the considered (semi)norm, and show in case of the superconvergent points an improved convergence of order $\mathcal{O}(h^{p-2})$ for all (semi)norms except for the equivalent of the H^4-seminorm, where the order $\mathcal{O}(h^{p-3})$ is anyway optimal.

Volume blending type spline constructions and their application to isogeometric analysis

T. Kravetc
Department of Computer Science and Computational Engineering, Faculty of Engineering Science and Technology, UiT – The Arctic University of Norway

Thursday 12.00, Auditorium 1

Modeling solid objects is essential for simulating real-world phenomena. Iso- geometric analysis is a powerful method for simulation that utilizes the same type of basis functions for both modeling and analysis. The blending type spline construction is grounded in the blending of local geometry using un- derlying expo-rational basis functions. For IGA representation, we use so- called combined expo-rational basis functions, which are a combination of expo-rational local basis functions and overlapping Bernstein polynomials. As a grid for basis evaluation, we employ 3D tensor product construction, which extends naturally from tensor product surfaces. This talk demonstrates the evaluation of the combined expo-rational basis functions on a 3D tensor product parametric domain. Solid models generated using volumetric combined expo-rational basis functions are also presented. Alongside manual modeling, mapping techniques are employed to project the parametric domain onto given boundary face parameterization as well as onto objects modeled in CAD software. The resulting solid object serves as the computational domain for solving PDE problems. Blending type spline constructions offer unique modeling options due to the presence of local geometry. Furthermore, the basis locality provides specific properties to the solutions of governing equations.

A novel method for manipulating polynomial curves by the Gauss–Legendre control polygon with points interpolating property

H.P. Moon, S.H. Kim and **S-H Kwon**
The Catholic University of Korea

Thursday 18.00, Auditorium 1

Authors have recently introduced Gauss–Legendre curves [1], marking a significant departure from traditional Bézier curves. Of particular interest is their behavior when dealing with high-degree curves, where the Gauss–Legendre control polygon closely mimics the resulting curve, in stark contrast to the significantly larger Bézier counterpart, thereby hindering practical curve design. Although Gauss–Legendre curves possess the endpoint interpolation property, direct control over intermediate curve points remains limited due to the control polygon's influence solely on the hodograph at nodes.

Addressing the need for fine-tuning high-degree polynomial curves to pass through specific points, we introduce a novel method employing Gauss–Legendre control polygons for additional point interpolation. This scheme facilitates simultaneous control over the hodograph and interpolation, ensuring the curve passes through designated points. We elaborate on the construction of appropriate polynomials to be used as weights for expressing the curve as a barycentric combination of control points. Leveraging these polynomials, we propose a methodology enabling the direct manipulation of the hodograph and curve points at nodes simultaneously, offering enhanced control over polynomial curve design.

A deformation technique for curves, surfaces and volume

A. Lakså
UiT The Arctic University of Norway

Wednesday 17.00, Auditorium 2

Any parametric curve, surface or volume where the formula is based on poly- nomials, rational functions, trigonometric functions, exponential functions, logarithmic functions, etc. can be made ”editable” by adding a ”B-spline structure”. This means that we can easily change the shape of a curve, surface or volume by adding knot vectors corresponding to polynomial B- splines of degree 1. In each knot value that has a knot interval on its sides, we introduce a ”point” that interpolates the geometric object, which has an orientation and a size, that is, it can be rotated and scaled. The ”points” are actually homogeneous matrices. In each internal knot value we have, for a curve, a knot interval on each side. These two intervals form the domain of the matrix in the knot. At each knot interval we then have two matrices that are active. We then blend these two matrices with a blending function and get a matrix function with one parameter, which we then multiply by the original curve. For a tensor product surface we have knot vectors in two directions and thus 4 matrices and 2 blending functions to generate a matrix function with two parameters, and for a volume 8 matrices and 3 blending functions to generate a matrix function with three parameters. The result is a curve, surface or volume with control points that interpolate and can be rotated and scaled. The continuity depends on the blending function we choose. With a proper choice, the continuity will be the same as for the original.

Approximation of Functions from Korobov Spaces by Shallow ReLU Neural Networks

Y. Liu
City University of Hong Kong

Wednesday 17.20, Auditorium 1

The work is the first innovative approach and result of the approximability of shallow neural networks on Korobov spaces. During the talk, a dimensional independent rate of approximating functions from the Korobov space by ReLU shallow neural networks will be established. Following the first main result, a generalization error will be emphasized. Fi- nally, a discussion with a specific example will be given as a justification for the novelty and sufficiency of the main results.

Geometrically continuous spline constructions based on *a priori* gluing data

M. Marsala, **A. Mantzaflaris** and B. Mourrain
Inria at Université Côte d'Azur,
2004 route des Lucioles, 06902 Sophia Antipolis, France

Friday 10.20, Auditorium 2

In this talk we pursue geometrically smooth spline constructions over quadrangular meshes, which are based on gluing functions that are chosen *a priori*. We aim at globally G^1 or G^2 smooth families of surfaces, defined locally by explicit masks, which approximate the classical Catmull-Clark subdivision surface scheme. The resulting surfaces are represented as a collection of biquintic Bézier or B-Spline patches around extraordinary vertices and join with the prescribed smoothness. Moreover, we describe sets of basis functions generating the space of biquintic G^1 or G^2 splines over a quadrangular mesh. Starting from the defining geometric continuity relations, which are induced with the help of quadratic gluing functions, we explore the space of solutions and obtain explcit values for the control points on each quadrilateral. We present optimal strategies for choosing free parameters during the construction and demonstrate experimentally that the resulting surfaces and bases are suitable for isogeometric analysis.

Maximally smooth cubic quasi-interpolation operators on arbitrary triangulations

M. Marsala, C. Manni and H. Speleers
University of Florence

Friday 10.40, Auditorium 1

Splines on triangulations, and in particular spline quasi-interpolants (QIs), are widely used tools both in approximation theory and applications. In general, highly smooth splines are often preferred. In this talk we consider C2 cubic splines on arbitrary triangulations equipped with a special refinement, the so-called cubic Wang-Shi split, and we present the construction of related QIs for approximating sufficiently smooth functions. The QIs are locally described on each (macro) triangle in terms of a simplex spline basis recently provided in the literature and the corresponding coefficients are defined through linear functionals obtained by solving suitable local Hermite interpolation problems. The approximation quality of the presented constructions is analyzed with several numerical tests.

A super-resolution approach to classification

H. N. Mhaskar and R.O. Dowd
Claremont Graduate University

Wednesday 17.40, Auditorium 1

The traditional approach in machine learning is to treat classification problem as a problem of function approximation. This creates a gap between the theory, where one requires the target function to be smooth, and the practice, where the class boundaries may be non-smooth, even touching each other. We propose a novel paradigm, where we consider each class k appearing with a probability distribution μ_k, $k = 1, \ldots, K$. The data consists of samples $\{xj_j\}$ drawn from an unknown convex combination of these distributions. We then use our localized kernels developed for solving super-resolution signal separation problems to separate the supports of the measures μ_k. With these supports identified accurately, one can then seek the label of one of the points in each of the supports, and extend it to the entire support. In this "cautious active learning" manner, one can solve the problem with a small if not minimal number of labels queried at judiciously chosen points x_j

Tractable and asymptotic behaviour of mixed Wiener spaces

M. Moeller, S. Stasyuk and T. Ullrich
University of Technology Chemnitz

Thursday 16.20, Auditorium 2

The best m-term approximation has been a rather theoretical subject of study in approximation theory since its inception by Stechkin in 1955. Re- cently however Jahn, T.Ullrich and Voigtlaender have found some practical application for it by using it in a new bound on the sampling numbers. One important class of spaces where this bound can give an improvement over existing ones are weighted Wiener spaces. And resulting from that also cer- tain classes of dominating mixed smoothness Besov spaces. Motivated by this, a new bound for the best m-term approximation in these spaces will be shown in this talk as well as a sharp asymptotic bound on the Gelfand widths of these spaces will also be provided, which are a natural lower would on the sampling widths. A second main focus of our research was in the development of tractable results that give good upper bounds in the preasymptotic setting. These were again considerd for Gelfand numbers, sampling numbers and best m- term approximation.

Alternating and joint surface approximation with THB-splines: results and industrial perspective

C. Giannelli, S. Imperatore, A. Mantzaflaris and **D. Mokriš**
MTU Aero Engines AG, Munich, Germany

Wednesday 15.00, Auditorium 2

We will have a closer look at three surface approximation techniques: the *alternating point distance minimisation* (A-PDM), an alternating method that is a *hybrid between the PDM and the tangent-distance minimization* (A-HDM) and the *joint optimization method*. A common feature of these three methods is that they approximate a parametrised point cloud by not only searching for suitable control points but also by adapting the parameters of the data points. While the literature provides considerable experience with these methods in the curve case, relatively little is known about their performance when constructing surfaces. We have successfully combined the three methods with the truncated hierarchical B-splines (THB-splines) and our examples demonstrate a significant reduction of degrees of freedom necessary to construct a satisfying approximation.

The second focus of the talk will be the industrial viewpoint. Introducing MTU and its activities in developing, manufacturing and repairing aircraft engines will enable putting the methods into an application context. We will review our team's past experience in industrialising geometric methods; based on this, we will discuss the "boring" and technical part of what the next steps and hurdles towards productive use would be. While the problematics of the geometric modeling at MTU is fairly specific, the principles discussed could be interesting for anyone intent on seeing their methods applied in practice.

Hermite interpolation methods for (M)PH over PH curves

H.P. Moon
Dongguk University, Seoul, Republic of Korea

Thursday 16.00, Auditorium 1

Polynomial Pythagorean hodograph(PH) curves are classified into three primary categories: planar PH, spatial PH, and Minkowski PH curves. In this study, we address the polynomial PH curves that satisfy two PH conditions simultaneously. Specifically, a polynomial curve that meets both spatial and planar PH conditions is called a PH over PH (PHoPH) curve, as the spatial PH curve traverses over its PH planar projection. Similarly, a polynomial curve satisfying both Minkowski and planar PH conditions is denoted as a MPH over PH (MPHoPH) curve. We conduct an analysis of the algebraic structures for PHoPH and MPHoPH curves using quaternion and Clifford algebra frameworks, respectively. Leveraging these algebraic structures, we propose G^1 and C^1 Hermite interpolation methods for (M)PHoPH curves of degrees 5 and 7, correspondingly.

Ten years of collaboration in geometric modeling and approximation theory

G. Muntingh
Sintef, Oslo, Norway

Thursday 11.20, I/

In this talk, we take a personal tour of results in geometric modelling and approximation theory, resulting from a decade-long inspiration and collaboration with Tom Lyche. If there is time, we end with some recent results..

A new class of flexible nets in isotropic geometry

P. Olimjoni
KAUST, Thuwal, Saudi Arabia

Wednesday 16.20, Auditorium 2

Motivated by the design of flexible nets, we classify all nets of arbitrary size $m \times n$ that admit a continuous family of area-preserving Combescure transformations. Two $m \times n$ nets are called Combescure transformations of each other if their corresponding edges are parallel. There are just two different classes. The nets in the first class are special cases of cone nets that have been recently studied by Kilian, Müller, and Tervooren. The second class consists of Königs nets having a Christoffel dual with the same areas of corresponding faces. We apply isotropic metric duality to get a new class of flexible nets in isotropic geometry. We also study the smooth analogs of the introduced classes.

Enhancing Surface Reconstructions from Scattered Datasets: Jump Estimates and Quasi-Interpolation with LR B-splines

C. Bracco, C. Giannelli, **F. Patrizi** and A. Sestini
University of Florence

Wednesday 14.20, Auditorium 2

Surface reconstruction from scattered datasets remains a significant challenge, necessitating advanced methodologies to handle irregular data distributions effectively. In this talk, we introduce novel fault jump estimates designed to inform local refinement strategies within surface approximation schemes employing adaptive spline constructions. Our approach takes advantage of the inherent relationship between data discontinuities and sharp surface variations by exploiting the detected and estimated jumps in the input point cloud to guide mesh refinement algorithms. We thus propose a quasi-interpolation (QI) framework based on locally refined B-splines (LR B-splines) able of incorporating local meshlines along the detected faults. In this setting, the jump estimates provide an upper bound indication of the necessary refinement steps to attain an appropriate approximation, thereby preventing excessive refinement and unnecessary of degrees of freedom.

A central focus lies in developing a tailored local operator for the LR B-spline QI scheme, ensuring adaptive spline approximations that align with the nature and density of the scattered data configuration. Through a selection of numerical examples spanning synthetic and real datasets with diverse geographical features, we demonstrate the efficacy of our methodology. The computational efficiency of our approach is further enhanced through anisotropic refinement of the spline spaces in presence of axis-aligned faults.

Control point modifications of the Pythagorean hodograph curves

R.T. Farouki, **F. Pelosi**, and M.L. Sampoli
University of Siena

Thursday 16.20, Auditorium 1

Pythagorean-hodograph (PH) curves incorporate special algebraic structures that facilitate an exact determination of many derived properties, such as arc lengths and offset (parallel) curves, [1]. The planar PH curves usually employ a representation based on complex polynomials, and the spatial PH curves are based on a quaternion polynomial representation. Although PH curves are compatible with the Bézier/B-spline form, their non-linear nature makes the construction of PH curves, subject to specified geometrical requirements, more challenging.

Apart from some cubic cases, attempts to characterize PH curves in terms of constraints on their control polygons are typically cumbersome and non-intuitive. Moreover, any modification of the Bézier/B-spline control points of a given PH curve will, in general, compromise its PH nature. Consequently, the focus has primarily been on *ab initio* constructions of PH curves matching specified geometrical data, rather than *a posteriori* modification of existing PH curves.

The intent of the present study is to address this inability to modify PH curve segments by displacements of their control points, extending the results of [2]. The focus is on the complex-variable model for planar PH curves, although the approach may also be adapted to the quaternion form of spatial PH curves and readily extended to general degree.

References

[1] Rida T. Farouki, (2008), Pythagorean-Hodograph Curves: Algebra and Geometry Inseparable, Springer, Berlin.

[2] R. T. Farouki, F. Pelosi and M. L. Sampoli (2023), Construction of planar quintic Pythagorean hodograph curves by control polygon constraints, *Comput. Aided Geom. Design* 103, article 102192.

Smooth Surfaces of Low Degree

Jorg Peters
University of Florida

Wednesday 16.40,II

Free-form surfaces of low degree walk the fine line between a lack of expressiveness and the potential elegance of a simple shape preventing surface oscillations. Downstream processes benefit from simplicity. This talk will discuss some of the trade-offs and some recent better understanding of algorithms and lower bounds, both related to some of Tom Lyche's research.

The Multichannel Blind Deconvolution Problem in Parallel MRI

G. Plonka
University of Göttingen

Thursday 14.00, Auditorium 1

One of the biggest innovations in magnetic resonance imaging (MRI) within the last years was the concept of parallel MRI. In this setting, the use of multiple receiver coils allows the reconstruction of high-resolution images from undersampled Fourier data such that the acquisition time can be substantially reduced. Mathematically, the parallel MRI reconstruction problem can be seen as a multi-channel blind deconvolution problem, where the coil sensitivity functions and the magnetization image have to be recovered simultaneously from the acquired data. In this talk, we will give a short survey on existing reconstruction methods in parallel MRI so far. Further, we propose a new algorithm, called MOCCA, to recover the coil sensitvities and the magnetization image from incomplete Fourier measurements. Our approach is based on suitable parameter models for both, the magnetization image and the sensitivities. The derived MOCCA algorithm provides perfect reconstruction results if the model assumptions are satisfied. Moreover, it has a low computational complexity and fits real MRI data sufficiently well such that it is applicable in practice. The results presented in this talk have been obtained jointly with Yannick Riebe and Benjamin Kocurov..

Sampling projections in the uniform norm and optimal function recovery

K. Pozharska
Institute of Mathematics of the NAS of Ukraine

Thursday 15.40, Auditorium 2

We show that there are sampling projections onto arbitrary n-dimensional subspaces of the space of bounded functions with at most $2n$ samples and norm of order $\sqrt{n}$. This gives a more explicit form of the Kadets-Snobar theorem for the uniform norm. The result is based on a specific type of discretization of the uniform norm, connected to the Marcinkiewicz–Zygmund inequalities.

Further, we discuss consequences for optimal recovery in L_p and new sharp bounds for the n-th linear sampling numbers.

This is a joint work with D. Krieg, M. Ullrich and T. Ullrich.

Hermite interpolation with G^1 curve and surface splines with rational offsets

C. Manni, **H. Prautzsch** and H. Speleers
Karlsruhe Institute of Technology (KIT)

Thursday 18.20, Auditorium 1

We derive the dual representation of rational quadratic curves and surfaces in their cubic representations with de Casteljau's poles and use it for cubic circles and spheres to show that one can interpolate points with normals by piecewise rational G^1 spline curves and surfaces with piecewise rational offsets of class 3, where the curves are planar.

Approximation of piecewise smooth functions by nonuniform nonlinear quadratic and cubic spline quasi-interpolants

F. Aràndiga and **S. Remogna**
University of Torino

Thursday 17.40, Auditorium 2

Spline approximation methods based on quasi-interpolation are used in many mathematical and scientific applications. If the function to be approximated is smooth, a spline quasi-interpolant is able to well reconstruct it, but if the function has jump discontinuities, we observe that the Gibbs phenomenon appears when approximating near discontinuities. Therefore, in this talk we apply Weighted Essentially Non-Oscillatory (WENO) techniques to modify classical quasi-interpolants in the space of C^1 quadratic and C^2 cubic splines on nonuniform partition of the real line, in order to avoid such oscillations and, at the same time, maintain the high-order accuracy in smooth regions. We study the convergence properties of the proposed quasi-interpolants and we provide some numerical and graphical tests confirming the theoretical results.

Algebraic characterization of planar cubic and quintic Pythagorean-Hodograph B-spline curves

L. Romani and A. Viscardi
University of Bologna, Italy

Thursday 15.20, Auditorium 1

Pythagorean-Hodograph (PH for short) B-spline curves are piecewise-polynomial parametric curves of odd-degree for which the Euclidean norm of the hodograph, that is its first derivative with respect to the parameter, is a polynomial B-spline. They provide a generalization of Pythagorean-Hodograph Bézier curves and, with respect to general B-spline curves of the same degree, have fewer degrees of freedom. However, while it is well-known how to distinguish planar PH cubics and quintics from general Béezier curves of degree three and five, respectively, so far an algebraic characterization of planar low-degree PH B-spline curves is missing. Our novel contribution thus consists in providing a unified algebraic characterization of clamped and closed planar PH B-spline curves of degree three and five.

Interpolation of 3D data streams via rational rotation-minimizing quintic splines

C Giannelli, **L. Sacco** and A. Sestini, and Z Šír
Università degli Studi di Firenze

Thursday 16.40, Auditorium 1

Effective trajectory design is essential in the field of robotics, as it influences the efficiency, accuracy, and safety of autonomous motions. Advances in the state of the art have led to high-performance vehicles, opening up the possibility of moving on complex trajectories. One possible choice is to rely on polynomial Pythagorean-Hodograph (PH) curves, a specific class of curves with interesting computational properties. The first part of the presentation will center on fundamental principles concerning PH curves, with particular emphasis on their subclass having rational rotation minimizing adapted frames (RRMFs). Afterwards I will show a novel geometric approach to the characterization of a set of quintic RRMF curves useful for applications. Finally, a related algorithm designed to address 3D data interpolation, which ensures the construction of PH spline paths with G^1 continuity, will then be introduced.

Optimal one-sided approximants of circular arc

A. Šadl Praprotnik, A. Vavpetič and E. Žagar
Institute of Mathematics, Physics and Mechanics

Thursday 16.40, Auditorium 2

Circular arcs are one of the most fundamental curve shapes used in CAGD. It is well known that they posses a rational parametrization, but no polynomial parametrization exists. In case a polynomial model is required, one must resort to the polynomial approximation. Several optimal approximants of circular arcs by polynomials of different degrees have already been derived.

In this talk we consider the problem of optimal one sided circular arc approximants, i.e. polynomials that best approximate a given circular arc from the inside or from the outside. Thus, we are looking for a polynomial $\mathbf{p}$ which interpolates the two boundary points of a circular arc and minimizes the infinity norm of the simplified radial error ψ, i.e, $\psi(t) = ||\mathbf{p}(t)||_2^2 - 1, \quad t \in [-1, 1]$, where $\mathbf{p}(t) = \sum_{j=0}^{n} B_j^n(t)\mathbf{b}_j$ is the reparameterized Bernstein-Bézier representation of $\mathbf{p}$. Since we are dealing with one sided approximants, the function ψ must be either non-negative or non-positive on the interval $[-1, 1]$. We mainly focus on finding the optimal one sided cubic $\mathcal{G}^0$ and quartic $\mathcal{G}^1$ approximant of a circular arc with its inner angle $\leq \pi$, but we also consider several other cases, depending on the geometric continuity and the degree of polynomial $\mathbf{p}$.

Outlier removal strategies in isogeometric analysis

E. Sande
EPFL

Thursday 12.40, Auditorium 1

Smooth isogeometric spline discretizations obtain an extremely good spectral approximation of differential operators compared to classical finite element methods, however they still present a small number of outliers, i.e., poorly approximated eigenvalues and eigenfunctions. This superior spectral behaviour translate into improved numerical simulations, especially for explicit dynamics, but the presence of outliers dampens the possible gain. In this talk we first present a theoretical understanding of outliers and explain how their removal is related to the classical n-width problem. We then further describe how these theoretical insights can lead to outlier removal strategies for various elliptic differential operators and for explicit dynamics on nontrivial geometries.

Improving ANOVA Approximation with anisotropy Parameters

F. Bartel and **P. Schröter**
University of Technology Chemnitz

Wednesday 14.40, Auditorium 2

This talk is concerned with learning the anisotropic smoothness of a function based on scattered data. We use this smoothness information in our approximation algorithm improving the convergence rate. In particular, we use the least squares approximation with trigonometric polynomials and frequency boxes with optimized side ratio. Here the NFFT (Nonequispaced Fast Fourier Transform) is applicable to accelerate the computation time of the approximation. We combine these findings with the truncated ANOVA (analysis of variances) decomposition. This method makes high-dimensional problems feasible. The optimal choice of frequency boxes from above occurs here multiple times for every ANOVA term. With our approach we are able to optimize hundreds of parameters in order to gain approximation accuracy with minimal overhead. Numerical experiments indicate the applicability of our results. This talk based on a joint work with Felix Bartel.

Closed Pythagorean Hodograph curves and associated framing motions

H-P Schröcker and **Z. Šír**
Charles University in Prague

Thursday 17.00, Auditorium 1

We present a method for the construction of regular, closed, C^∞, rational PH curves or, equivalently, closed bounded framing motions. The usual construction of rational PH curves, using the preimage quaternion polynomial and a rational speed function, is extended to the projective line.

We provide a sufficient and necessary condition for the existence of bounded and cusps-free curves having a prescribed direction field. The practical aspect of the construction is ensured via suitable convex optimization. We also consider the case of closed rotation minimizing motions and the related problem of parallel transport.

Numerical methods for optimal representation of deforming surfaces with spherical topology

C. Sorgentone
Sapienza University of Rome

Thursday 12.40, Auditorium 2

Applications in fluid mechanics, video games and image processing often involve simulations of 3D objects with spherical topology. When the geometry is changing, high distortions of the surface point distribution may arise, especially for long time simulations; this could cause aliasing and then stability problems. We present a spectral reparametrization procedure able to maintain an optimal representation of the objects also for long time simulations, preserving also area and volume. We will also show how this technique is fundamental when dealing with surface PDEs.

Spline approximation: The blessing of smoothness and outlier-free isogeometric analysis

H. Speleers
University of Rome Tor Vergata

Wednesday 13.00, Auditorium 1

Isogeometric analysis is a well-established paradigm to improve interoperability between geometric modeling and numerical simulation. It is based on smooth spline representations and shows important advantages over classical C^0 finite element analysis. In particular, the higher smoothness enables a higher accuracy per degree of freedom. This superior performance has been observed numerically since long time, and recently is supported theoretically by error estimates with constants that are explicit, not only in the mesh size, but also in the polynomial degree and the smoothness. Moreover, the isogeometric approach based on maximally smooth spline spaces over uniform grids turns out to be an excellent choice for addressing eigenvalue problems: it gives a very good approximation of the full spectrum, except for a very small portion of spurious outliers.

In this talk we review some recent results on explicit error estimates for approximation with highly smooth splines. These estimates are sharp or close to sharp in several interesting cases and are actually good enough to cover convergence to eigenfunctions of classical differential operators under so-called k-refinement. We also discuss how subspaces of maximally smooth splines, which are optimal in the sense of Kolmogorov n-widths, can be selected to make outlier-free isogeometric discretizations. The talk is based on joint work with Carla Manni and Espen Sande..

Approximation properties of subdivision based isogeometric discretizations

T. Takacs
RICAM, Austrian Academy of Sciences

Thursday 11.20, Auditorium 1

In this study we consider domains that are composed of an infinite sequence of self-similar rings and corresponding finite element discretization spaces (or isogeometric spaces) over those domains. The rings are parameterized using piecewise polynomial or tensor-product B-spline mappings of degree q over quadrilateral meshes. Subdivision schemes create, in the limit, such domains. We then consider finite element discretizations which, over each ring, are mapped, piecewise polynomial functions of degree p. We study approximation properties over such recursively parameterized domains. The main finding is that, for generic isoparametric discretizations (i.e., where $p = q$), the approximation properties always depend only on the degree of polynomials that can be reproduced exactly in the physical domain and not on the degree p of the mapped elements. Especially, in general, L^∞-errors converge at most with the rate h^2, where h is the mesh size, independent of the degree $p = q$. This has implications for subdivision based isogeometric discretizations, as we will see in this talk.

Hierarchical B-splines for Isogeometric Analysis: Local projector and mixed isogeometric methods

D. Toshniwal, K. Dijkstra and K. Shepherd
Delft University of Technology, The Netherlands

Thursday 10.40, Auditorium 1

Isogeometric Analysis (IGA) advocates for the direct utilization of spline- based geometric representations for performing spline-based numerical sim- ulations. Over the last decade and a half, this philosophy has been applied to diverse applications with great success, and has yielded theoretical de- velopments that cement the status of smooth splines as excellent tools for approximation. In this talk I will focus on the use of Truncated Hierarchical B-spline (THB-spline) spaces and discuss the formulation of a local projector for them. The projector is an extension of the Bezier projection proposed by Thomas et al. (2015) and relies on a characterization of linear independence of THB-splines over certain macro-elements. I will also discuss applications of THB-splines to the development of mixed isogeometric methods.

Recent developments in genuine multi-sided surface representation and editing

M. Vaitkus, P. Salvi and T. Várady
Department of Control Engineering and Information Technology
Budapest University of Technology and Economics

Wednesday 15.00, Auditorium 1

The representation of multi-sided surface patches has been a major area of research within CAGD for the last several decades. While tensor product Bézier and NURBS patches have been the standard choice for representing free-form geometry, they have well-known limitations when surface regions of general shape and topology need to be modeled. For a variety of practical applications (e.g. vertex blending and curve network based design) genuine multi-sided surface representations offer certain advantages over traditional trimming or macro-patch approaches. In recent years, ribbon-based representations have been the subject of intense study, effectively generalizing Bézier or Hermite splines from regular grids to the multi-sided setting.

After giving a brief overview of the state-of-the-art in genuine multi-sided surface patches, we describe recent developments regarding Generalized Bézier (GB) and Generalized B-spline (GBS) patches over curved domains. In particular we discuss some recent results regarding the constrained editing and interior control of GB(S) patches. Supported by the Hungarian Scientific Research Fund (OTKA, No. 145970).

NURBS-based geometry processing for Additive Manufacturing

J. Vallejo, J. Sanchez-Reyes, J.M. Chacon andP.J. Nunez
IMACI - ETSII Ciudad Real, Univ. de Castilla-La Mancha (Spain)

Friday 11.00, Auditorium 1

We propose a streamlined workflow for Additive Manufacturing with a seam- less transfer from the initial CAD description to the final G-code. Adhering to the NURBS standard for geometry processing at all steps avoids multiple representations, polygonal approximations, and associated errors. The traditional workflow consists of an initial polygonization of the object (e.g., via STL), slicing this approximation, offsetting the polygonal sections, and finally generating G-code made up of polyline trajectories (G1-commands). Our proposal bypasses the polygonal approximation and then proceeds to: (1) Direct slicing of the CAD model in the NURBS environment provided by the CAD system Rhino3D; (2) Path planning, including offset trajectories, in this NURBS environment; (3) Accurate G-code generation (circular arcs, Bézier cubics, and polylines). We stick to the NURBS standard for G-code generation, a possibility overlooked in commercial applications, by exploiting the possibilities of exiting firmware incorporating G2/3 (circular arcs) and G5 (cubic Béezier curves) commands. Since trajectories resulting from offsetting in Rhino3D usually restrict to circles and polynomial (cubic or quadratic) splines, the exact conversion into G2/3 and G5 code is readily performed via standard NURBS geometry processing, such as knot-insertion and degree-elevation.

Planar quintic Pythagorean-hodograph curves: new algebraic and geometric characterizations

K. Hormann, L. Romani and **A. Viscardi**
University of Torino

Thursday 16.00, Auditorium 2

It is well known that planar cubic PH curves are characterized algebraically by a single equation involving the edges of the control polygon as complex numbers, which has a very elegant geometric translation as the similarity of two triangles. For the quintic case, instead, while an algebraic and a geometric characterization are available, the former needs different sets of two equations in order to address all possible cases, the latter requires various artificial auxiliary points and there is no direct connection between the two.

Here we present, on the one hand, a compact algebraic characterization of planar quintic PH curves that is composed of only two equations working in all cases and, on the other hand, a geometric characterization based on the similarity of two quadrilaterals. These two characterizations are the proper counterparts of the cubic case and are obtained from a slightly different analytic representation of the hodograph of the curve with respect to the one commonly used in the literature.

A C^s-smooth mixed degree isogeometric spline space over planar multi-patch domains

M. Kapl, A. Kosmav c and **V. Vitrih**
University of Primorska, Koper, Slovenia

Thursday 12.20, Auditorium 1

In the talk we present a method for the construction of a C^s-smooth mixed degree isogeometric spline space over planar bilinearly parameterized multi-patch domains, which possesses the degree $p = 2s + 1$ in a small neighborhood around the edges and vertices of the multi-patch domain, and the smallest possible degree $p = s+1$ in all other parts of the domain. The C^s-smooth mixed degree spline space is generated as the span of locally supported basis functions corresponding to the single patches, edges and vertices of the multi-patch domain. The degree reduction of the functions in the interior of the patches is achieved by introducing an appropriate mixed degree underlying spline space over the unit square $[0, 1]^2$ to define the functions on the individual patches, and allows the solving of high order PDEs with functions of mostly low degree.

The construction will be further extended on the basis of examples to the class of bilinear-like G^s multi-patch parameterizations, which enables the modeling of multi-patch domains with curved patch interfaces and boundaries. Finally, the potential of the C^s-smooth mixed degree isogeometric spline space for isogeometric analysis will be demonstrated by several numerical examples of solving high-order PDEs via the isogeometric Galerkin method or via the isogeometric collocation technique.

Approximation and Envelope Computation using Polygon Rolling Motions

J. Vráblíková, B. Jüttler and Z. Šír
Johannes Kepler University, Linz, Austria

Wednesday 15.20, Auditorium 1

We analyze the approximation of planar rigid body motions by polygon rolling motions, i.e., by piecewise rotational and translational motions. These motions can be realized by two polygons (which are called the central polygons) rolling on each other. It is shown that the approximation error, which is measured by the Hausdorff distance between curves in a suitable kinematic image space, converges quadratically to zero. This implies the same rate of convergence for the individual point trajectories under the two motions. Furthermore, the trajectory of any point under the approximated motion is an arc spline, i.e., a curve composed of circular arcs and line segments. Thus, given a planar object with an arc spline boundary, its envelope under a polygon rolling motion is also an arc spline and can be computed efficiently. Finally we analyze the error obtained by combining the arc spline approximation of planar domains with the approximation by polygon rolling motions in the context of envelope computation.

Spectra alignment of kernel matrices and applications

T. Wenzel and A. Iske
Universität Hamburg

Wednesday 15.40, Auditorium 2

Kernel methods provide versatile tools for scattered data approximation, numerical analysis and machine learned. For this, the kernel matrix is a key quantity, allowing to analyze e.g. stability and accuracy of kernel based approximation methods. In this contribution we analyze an alignment within the spectral decomposition of these matrices for kernels of different smoothness and present implications for error estimates of kernel based approximation. We discuss applications for the stability of kernel based approximation as well as for inverse statements, which allow to conclude the smoothness of a function based on approximation rates.

Isotropic Geometry and Applications in Geometric Computing

K. Yorov, M. Skopenkov, and H. Pottmann
King Abdullah University of Science and Technology

Wednesday 17.20, Auditorium 2

We study surfaces with a constant ratio of principal curvatures in Euclidean and isotropic geometries and characterize rotational, channel, ruled, helical, and translational surfaces of this kind under some technical restrictions (the latter two cases only in isotropic geometry). We use the interlacing of various meth- ods of differential geometry, including line geometry and Lie sphere geometry, ordinary differential equations, and elementary algebraic geometry. This study investigates the geometric properties of surfaces with a constant ratio of princi- pal curvatures (CRPC) across Euclidean and isotropic geometries. We explore a range of surfaces, including rotational, channel, ruled, helical, and translational, particularly under the constraints of isotropic geometry.

Optimal uniform approximation by planar parametric polynomials

A. Vavpetic and **E. Zagar**
Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

Thursday 15.20, Auditorium 2

The Remez algorithm is an iterative procedure to find the best uniform polynomial approximation of a given continuous function. In this talk, we will generalize it for finding the best uniform approximation of a planar parametric curve by a parametric polynomial of a fixed degree. The problem is highly nonlinear and appears much more difficult as the well known functional case.

Haar Framelets on Spheres and Graphs: Construction and Applications

X. Zhuang
City University of Hong Kong

Wednesday 17.00, Auditorium 1

Deep learning has revolutionized modern society and people's daily lives during the past decade, ranging from automated driving, online shopping, and AI assistants to intelligent surveillance systems, medical diagnosis, drug discovery, and so on. Spherical neural networks (SNNs) and Graph neural networks (GNNs) are powerful deep learning methods for machine learning tasks on non-Euclidian data, e.g., CMB data, social networks, and citation networks. Framelet systems, like the traditional wavelet systems for Euclidean data (signals, images, videos, etc.), provide a powerful tool for multiresolution analysis of such data. Based on a general framework for the construction of Haar-type framelets on compact sets, we establish Haar framelet systems on various domains and demonstrate how to build Haar framelet systems that can be efficiently and effectively utilized for the design of deep neural network architecture. We demonstrate state-of-the-art performances of the framelet neural networks in some deep learning tasks.

Affine Lofting: Advancements in Lofting Techniques by Achieving Enhanced Surface Continuity and User Interactivity

J.J. Ågotnes and T.E. Henriksen
The Arctic University of Tromsø

Thursday 12.00, Auditorium 2

Lofting is a frequently used tool in computer-aided design (CAD) and computer- aided geometric design (CAGD). The basis of operation of such a tool is to draw sectional curves that are then interpolated. Many methods are in use today, however, there still are some limitations to this tool. In this presentation, we present a novel approach to a method of a lofting tool where the continuity of the surface generated by the tensor product is limited by the order of the blending function applied, i.e. a surface with C^k-continuity, where k is the order of the blending function. The method is based on linear and quadratic Lagrange basis functions to generate sub-surfaces which are then blended using the appropriate blending function. In addition, each curve in the set has an added affine matrix which results in a surface in which the user can update the parameters in its corresponding matrix to live edit their lofted surface.