Master project: Combinatorial Matrices

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- **What is this about?** As a lazy (but not bad) answer I suggest that you take a look at parts of the following video talks:
  - **Gilbert Strang (MIT, Boston)** gives a lecture on some basic combinatorial matrices *Click here*
  - My friend and long-time collaborator **Richard Brualdi (Univ. of Wisconsin, Madison)** gives a talk on combinatorial matrices *Click here*
  - Here **I (UiO)** give a talk at a conference in Atlanta on some topics I like *Click here*

  Your background: Like mathematics, taken our basic courses ..., MAT1120. The courses MAT3100 and MAT2250 are very useful, and MAT2200 and/or MAT2400 or MAT3000-courses give a good basis.

- **Example: a famous theorem.** A real matrix $A = [a_{ij}]$ is called *doubly stochastic* if $a_{ij} \geq 0$ for every i, j and the sum in every row, and in every column, is 1. In particular, if all entries are integers we obtain a *permutation matrix*; it has exactly one 1 in every row and column. A famous theorem due to G. Birkhoff and J. von Neumann says the following: *A real $n \times n$ matrix A is doubly stochastic if and only if it can be written as a convex combination*

  $$A = \lambda_1 P_1 + \lambda_2 P_2 + \cdots + \lambda_k P_k$$

  for some $\lambda_i \geq 0$ $(i \leq k)$ with $\sum_i \lambda_i = 1$ and permutation matrices $P_1, P_2, \ldots, P_k$ for some k.

  A question: Look at this for $n = 2$. What are then the doubly stochastic matrices? Try to prove the theorem in this special case.

- **If you consider a master in this area: what to do?** Then you send an email to me (see above) and we arrange a meeting. You tell about your background, and something (in the videos) you liked, and your preference concerning theory/algorithms/applications. Then, after some thought, I suggest a topic. If you say "Yes, this is for me!!", we fill in some form, and make plans.

- **Oh, I forgot....** Other projects are in **optimization** or **Laplacian matrices and spectral partitioning**.

WELCOME!