Winter school on

Connes' embedding problem and quantum information theory

January 7–11, 2019

room 108, 1th floor of NH Abel's building, University of Oslo

The school is a part of the project *Pure Mathematics in Norway, 2018-2022*, supported by the **Bergen Research Foundation** and the **Tromsø Research Foundation**.

Programme

Monday, January 7

10.30 – 11.00 Welcome – Coffee/tea

11.00 – 12.00 **Narutaka Ozawa**: *Connes's Embedding Problem and its equivalents, I*

12.00 – 14.00 Lunch (12th floor, NH Abel's building)

14.00 – 15.00 **Magdalena Musat**:
Von Neumann algebras meet Quantum Information Theory, I

15.00 – 15.30 Coffee/tea

15.30 – 16.30 **Vern Paulsen**: *C^*-algebras and non-local games, I*

16.45 – 17.15 Extra lecture – **Alexander Müller-Hermes**:
Decomposability of linear maps under tensor powers

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Tuesday, January 8

09.30 – 10.30 **Narutaka Ozawa**: *Connes's Embedding Problem and its equivalents, II*

10.30 – 11.00 Coffee/tea

11.00 – 12.00 Special guest lecture – **Mikael Rørdam**:
Factorizable maps, traces on free product C-algebras and CEP*

12.00 – 14.00 Lunch (12th floor, NH Abel's building)

14.00 – 15.00 **Vern Paulsen**: *C^*-algebras and non-local games, II.*

15.00 – 15.30 Coffee/tea

15.30 – 16.30 **Benoît Collins**: *A non-commutative probability point of view on the Connes embedding problem, I*

Wednesday, January 9

09.30 – 10.30 **Magdalena Musat**:
Von Neumann algebras meet Quantum Information Theory, II

10.30 – 11.00 Coffee/tea

11.00 – 12.00 **Vern Paulsen**: *C^*-algebras and non-local games, III*

12.00 – 14.00 Lunch (12th floor, NH Abel's building)

14.00 – 15.00 **Benoît Collins**: *A non-commutative probability point of view on the Connes embedding problem, II*

15.00 – 15.30 Coffee/tea

15.30 – 16.30 **Narutaka Ozawa**: *Connes's Embedding Problem and its equivalents, III*

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Thursday, January 10

09.30 – 10.30 **Magdalena Musat**:
Von Neumann algebras meet Quantum Information Theory, III

10.30 – 11.00 Coffee/tea

11.00 – 12.00 **Benoît Collins**: *A non-commutative probability point of view on the Connes embedding problem, III*

12.00 – 14.00 Lunch (12th floor, NH Abel's building)

14.00 – 15.00 **Vern Paulsen**: *C^*-algebras and non-local games, IV*

15.00 – 15.30 Coffee/tea

15.30 – 16.30 **Narutaka Ozawa**: *Connes's Embedding Problem and its equivalents, IV*

16.45 – 17.15 Extra lecture – **Jitendra Prakash**:
Non-closure of the set of quantum correlations

18.30 **Social dinner** at *Der Peppern Gror*, Fritjof Nansens plass 7, Oslo.

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Friday, January 11

09.30 – 10.30 **Benoît Collins**: *A non-commutative probability point of view on the Connes embedding problem, IV*

10.30 – 11.00 Coffee/tea

11.00 – 12.00 **Magdalena Musat**:
Von Neumann algebras meet Quantum Information Theory, IV

12.00 – 14.00 Lunch (12th floor, NH Abel's building)

End

Winter school, UiO, Jan. 7–11, 2019 – Participants

Erik Bédos (Univ. of Oslo)
Eirik Berge (NTNU)
Benoît Collins (Kyoto Univ.)
Ingrid Dæhlen (Univ. of Oslo)
Emilie Elkiær (Univ. of Copenhagen)
Luca Gazdag (Univ. of Oslo)
Erik Habbestad (Univ. of Oslo)
Mads S. Jakobsen (NTNU)
Magnus Landstad (NTNU)
Nadia S. Larsen (University of Oslo)
Franz Luef (NTNU)
Rubén Martos Prieto (Univ. of Copenhagen)
Alexander Müller-Hermes (Univ. of Copenhagen)
Magdalena Musat (University of Copenhagen)
Sergey Neshveyev (Univ. of Oslo)
Tobias K. Netskar (Univ. of Oslo)
Petter Nyland (NTNU)
Sveinung K. Nøding (Univ. of Oslo)
Tron Omland (Oslo)
Narutaka Ozawa (RIMS Kyoto)
Vern Paulsen (Univ. of Waterloo)
Jitendra Prakash (Univ. of Copenhagen)
Mikael Rørdam (Univ. of Copenhagen)
Gaute Schwartz (Univ. of Oslo)
Christian Skau (NTNU)
Eirik Skrettingland (NTNU)
Erling Størmer (Univ. of Oslo)
Lars Tuset (Oslo Metropolitean University)
Makoto Yamashita (Univ. of Oslo)

Winter school, UiO, Jan. 7–11, 2019 – Abstracts

• **Benoît Collins** (Kyoto University, Japan):

A non-commutative probability point of view on the Connes embedding problem

Abstract: We will review the micro state approach to the Connes problem developed in free probability, as well as some relations between this problem and free entropy/free dimension questions. We will also consider some moment problems in matrix algebras and finite von Neumann algebras, and mention some reformulations of the Connes problem, some positive results, and some no-go results. This second part will be inspired from works from/with Dykema and Brannan (among others). Time allowing, we will discuss non commutative real algebraic reformulations of the Connes problem, after Klep and Schweighofer and related topics.

• **Magdalena E. Musat** (University of Copenhagen, Denmark):

Von Neumann algebras meet Quantum Information Theory

Abstract: The study of quantum correlations arising under two different assumptions of commutativity of observables, initiated by Tsirelson in the 80's, has proven over the last decade to have deep interconnections with important problems in operator algebras theory, including various reformulations of the Connes Embedding Problem. In very recent work with M. Rørdam, we show that in every dimension $n \geq 11$, the set of $n \times n$ correlation matrices arising from unitaries in finite dimensional von Neumann algebras is not closed. As a consequence, in each such dimension there are quantum channels that admit type II_1-von Neumann algebras as ancillas, but not finite dimensional ones.

I will also discuss in (more) detail the class of quantum channels that posess a certain factorizability property (introduced by Anantharaman-Delaroche). The study of these channels has lead to counterexamples to the Asymptotic Quantum Birkhoff Conjecture, as well as to further reformulations of the Connes Embedding Problem.

• **Narutaka Ozawa** (RIMS Kyoto, Japan):

Connes's Embedding Problem and its equivalents

Abstract: I will survey the operator algebraic aspects of Connes' Embedding Conjecture and Tsirelson's problem. I will also cover some of the recent works of W. Slofstra.

• **Vern Paulsen** (University of Waterloo, Canada):

C^*-algebras and non-local games

Abstract: There are currently several different mathematical models that attempt to describe the conditional probabiity densities that can occur when two labs in an entangled state conduct a finite set of quantum experiments. The Tsirelson conjectures are concerned with whether or not these various models give rise to the same sets of conditional probability densities. Thanks to the work of a number of researchers we now know that one of these conjectures is equivalent to Connes' embedding conjecture.

Many of the best results on these conjectures have come from the study of certain families of games, called non-local games. In these talks we will introduce these ideas and show that for each synchronous non-local game, there is an affiliated C^*-algebra whose representation theory tells us if the game has a perfect strategy in each of the possible models. In the case of the graph isomorphism game, this C^*-algebra is related to the quantum permutation group and the game theory perspective gives new information about this C^*-algebra.

Special guest lecture

• **Mikael Rørdam** (University of Copenhagen, Denmark):

Factorizable maps, traces on free product C-algebras and CEP*

Abstract: We give a reformulation of the notion of factorizable maps on matrix algebras in terms of traces of the universal unital free product of the given matrix algebra by itself, whereby the factorizable map that admits a factorization through a finite-dimensional C*-algebra corresponds to a trace that factors through a finite-dimensional C*-algebra. The universal free product of a matrix algebra by itself is known to be residually finite dimensional and weakly semiprojective. Using these facts one can show that the weak* closure of the set of traces on this C*-algebra that factor through a finite dimensional C*-algebra is equal to the set of hyperlinear traces on the free product. The latter is identical with all traces precisely if Connes Embedding Problem has an affirmative answer. We also characterize the weak* closure of traces factoring through a finite dimensional C*-algebra on an arbitrary residually finite-dimensional C*-algebra. This is joint work with Magdalena Musat.

Extra lectures

• **Alexander Müller-Hermes** (University of Copenhagen, Denmark):

Decomposability of linear maps under tensor powers

Abstract: Both completely positive and completely copositive maps stay decomposable under tensor powers, i.e., under tensoring the linear map with itself. But are there other examples of maps with this property? We show that this is not the case: Any decomposable map, that is neither completely positive nor completely copositive, will lose decomposability eventually after taking enough tensor powers. Moreover, we establish explicit bounds to quantify when this happens. To prove these results, we use a symmetrization technique from the theory of entanglement distillation and analyze when certain symmetric maps become non-decomposable after taking tensor powers. Finally, we apply our results to construct new examples of non-decomposable positive maps and establish a connection to the positive partial transpose squared conjecture.

• **Jitendra Prakash** (University of Copenhagen, Denmark):

Non-closure of the set of quantum correlations

Abstract: Consider a bipartite system with two observers, Alice and Bob, who are performing measurements in their labs. There are two models of quantum mechanics which describe the joint lab of Alice and Bob — the quantum model and the commuting quantum model. Tsirelson's original question asked whether these two models were essentially the same. We shall show that these two models are different for bipartite systems with five quantum experiments and binary outcomes for each experiment, by using the notion of correlation functions of graphs. (This is a joint work with Ken Dykema and Vern Paulsen.)