

# $C^*$ -algebras and geometry of groups and semigroups

University of Oslo 2-6 May 2022

## Abstracts and schedule

**Isabelle Baraquin (University Burgundy Franche-Comté)**

*Analysis and probability on compact quantum groups*

In this talk, we will first present a result from Diaconis, Shahshahani and Evans. Let  $M$  be a random matrix chosen from the unitary group  $U(n)$  and distributed according to the Haar measure. Then, for  $j \in \mathbb{N}$ , the  $\text{Tr}(M^j)$ 's are independent and distributed as some complex Gaussian random variables when  $n$  tends to  $+\infty$ .

We will then introduce some tools of noncommutative mathematics and look at this type of results in the context of free quantum groups and finite quantum groups.

**Adrien Le Boudec (ENS Lyon)**

*On the geometry of graphs of actions of solvable groups*

Let  $G$  be a finitely generated group. Classically, geometric group theory studies the geometry of the Cayley graph of  $G$ . Now to every action of  $G$  on a set, one can associate a graph, called the Schreier graph of the action. Contrary to the case of the Cayley graph, the geometry of a single Schreier graph does not reflect the geometry of the group  $G$ , for instance because such a graph can be very small, although the group  $G$  is not. Hence we are led to consider all Schreier graphs of  $G$  as a whole, and we are interested in geometric properties common to all of them. In the talk we will focus on the case where  $G$  is a solvable group, and we will be interested in lower bounds for the growth of all Schreier graphs of faithful actions of  $G$ . For large classes of solvable groups, we are able to give explicit lower bounds, that are sharp for many motivating examples. This includes solvable groups that are linear over the field of rational numbers, and metabelian groups. This is joint work with Nicolas Matte Bon.

**Matthijs Borst (TU Delft)**

*Bimodule coefficients, Riesz transforms on Coxeter groups and strong solidity*

In deformation-rigidity theory it is often important to know whether certain bimodules are weakly contained in the coarse bimodule. Consider a bimodule  $H$  over the group algebra  $\mathbb{C}[\Gamma]$ , with  $\Gamma$  a discrete group. The starting point of the talk is that if a dense set of the so-called coefficients of  $H$  is contained in the Schatten  $\mathcal{S}_p$  class  $p \in [2, \infty)$  then the  $n$ -fold tensor power  $H^{\otimes_{\Gamma} n}$  for  $n \geq p/2$  is quasi-contained in the coarse bimodule. We apply this to gradient bimodules associated with the carré du champ of a symmetric quantum Markov semi-group.

For Coxeter groups we give a number of characterizations of having coefficients in  $\mathcal{S}_p$  for the gradient bimodule constructed from the word length function. We get equivalence of: (1) the gradient- $\mathcal{S}_p$  property introduced by the second named author, (2) smallness at infinity of a natural compactification of the Coxeter group, and for a large class of Coxeter groups: (3) walks in the Coxeter diagram called parity paths.

We derive three strong solidity results: two are known, one is new. The first result is a concise proof of a result by T. Sinclair for discrete groups admitting a proper cocycle into a  $p$ -integrable representation. The second result is strong solidity for hyperbolic right-angled Coxeter groups. The final – and new – result extends current strong solidity results for right-angled Hecke von Neumann algebras beyond right-angled Coxeter groups that are small at infinity.

**Chris Bruce (Queen Mary University / University of Glasgow)**

*Algebraic semigroup actions:  $C^*$ -algebras and groupoids*

Each algebraic semigroup action naturally gives rise to a concrete  $C^*$ -algebra. I will explain a method for finding groupoid models for such  $C^*$ -algebras, and then present results characterizing certain properties of these groupoids in terms of dynamical properties of the initial algebraic semigroup action.

As a consequence, we obtain structural results for the  $C^*$ -algebras arising from several large classes of algebraic semigroup actions. In special cases, we recover results on simplicity and pure infiniteness by Cuntz–Vershik and Brownlowe–Larsen–Stammeier. Time permitting, I will also discuss rigidity phenomenon in this setting.

This is joint work with Xin Li (University of Glasgow).

**Antje Dabeler (University of Münster)**

*Exotic group  $C^*$ -algebras of higher rank Lie groups*

An exotic group  $C^*$ -algebra of a non-amenable group  $G$  is a  $C^*$ -completion of  $C_c(G)$  that lies naturally in between the reduced and the universal group  $C^*$ -algebra of  $G$ . Using asymptotic properties of matrix coefficients of representations of  $G$ , one can construct potentially exotic group  $C^*$ -algebras. For simple Lie groups with real rank one, this construction was shown to define exotic group  $C^*$ -algebras, by Samei and Wiersma and by de Laat and Siebenand. I will explain how similar methods can be used to show the existence of exotic group  $C^*$ -algebras of higher rank simple Lie groups, e.g.  $SL(n, \mathbb{C})$  with  $n \geq 3$ .

**Ulrik Enstad (Stockholm University)**

*Sufficient density conditions for coherent systems arising from discrete series representations*

Many important function systems in harmonic analysis are coherent systems, which means that they arise from the action of a discrete series representation on a single vector. A central question is to determine the spanning properties (e.g. completeness or frame) of coherent systems when discretized to a lattice. In this talk I will discuss the problem of characterizing when a given discrete series and lattice admits a vector that gives rise to a coherent frame, and when the vector can be chosen with additional regularity. The two problems require von Neumann algebraic and  $C^*$ -algebraic techniques respectively. The talk is based on joint works with Erik Bédos and Jordy Timo van Velthoven.

**Mario Klisse (TU Delft)**

*On the isomorphism class of  $q$ -Gaussian  $C^*$ -algebras.*

In 1991 Bożejko and Speicher introduced a non-commutative version of Brownian motion by defining a family of algebras depending on a parameter  $-1 \leq q \leq 1$  that are nowadays commonly known as the  $q$ -Gaussian algebras. These algebras interpolate between the extreme Bosonic case  $q = 1$  and the Fermionic case  $q = -1$ . For  $q = 0$  they coincide with Voiculescu's free Gaussians. The  $q$ -Gaussians can be studied on the level of  $*$ -algebras, on the level of  $C^*$ -algebras, and on the level of von Neumann algebras. Whereas it is easily seen that in the  $*$ -algebraic setting the  $q$ -Gaussians all coincide, as soon as one passes to the operator algebraic level the question for the dependence on the parameter  $q$  becomes notoriously difficult.

After introducing the necessary background on  $q$ -Gaussians, by considering the so-called Akemann-Ostrand property of the canonical inclusion we will discuss the dependence of the isomorphism class of  $q$ -Gaussian  $C^*$ -algebras on the parameter  $q$ . This partially answers a question by Nelson and Zeng.

The talk is based on joint work with Matthijs Borst, Martijn Caspers and Mateusz Wasilewski.

**Tim de Laat (University of Münster)**

*Origami expanders*

I will explain the construction of a new class of expanders, from measure-preserving affine actions with spectral gap on origami surfaces, in each genus  $g > 0$ . These actions are the first examples of actions with spectral gap on surfaces of genus  $g > 1$ . We prove that the new expanders, which we call origami expanders, are coarsely distinct from the classical expanders obtained via the Laplacian as Cayley graphs of finite quotients of a group. In genus  $g = 1$ , this implies that the Margulis expander, and hence the Gabber–Galil expander, is coarsely distinct from the Selberg expander. This is joint work with Goulnara Arzhantseva, Dawid Kielak and Damian Sawicki.

**Waltraud Lederle (University of Louvain)**

*Compact uniformly recurrent subgroups*

Given a locally compact, Hausdorff group, the set of closed subgroups carries a compact, Hausdorff topology and is commonly referred to as the "Chabauty space" of the group. The group acts on it by conjugation. A minimal, invariant subset is then called uniformly recurrent subgroup, in short URS. This terminology was introduced by Glasner and Weiss. URS are thought of as dynamical counterpart to the more well-known IRS (invariant random subgroup). We prove that the union of a URS consisting of compact subgroups generates a compact normal subgroup. This is joint work with Pierre-Emmanuel Caprace and Gil Goffer.

**Kang Li (University of Erlangen-Nuremberg)**

*Kirillov's orbit method to the Baum-Connes conjecture for algebraic groups*

The orbit method for the Baum-Connes conjecture was first developed by Chabert and Echterhoff in the study of permanence properties for the Baum-Connes conjecture. Together with Nest they were able to apply the orbit method to verify the conjecture for almost connected groups and p-adic groups.

In this talk, we will discuss how to prove the Baum-Connes conjecture for linear algebraic groups over local fields of positive characteristic along the same idea. It turns out that the unitary representation theory of unipotent groups plays an essential role in the proof. As an example, we will concentrate on the Jacobi group, which is the semi-direct product of the symplectic group with the Heisenberg group. It is well-known that the Jacobi group has Kazhdan's property (T), which is an obstacle to prove the Baum-Connes conjecture. If time permits, we will also discuss my recent joint work with Maarten Solleveld about quasi-reductive groups.

**Diego Martinez (University of Münster)**

*Locally finite semigroups and quasidiagonality of Roe algebras*

In this talk we will study some geometric aspects of inverse semigroups. For this, we shall equip them with a suitably right invariant and proper metric, and then prove that the semigroups with asymptotic dimension 0 are, precisely, those whose finitely generated sub-semigroups are finite. In turn, this condition ensures that the uniform Roe algebra of the semigroup is strongly quasi-diagonal, and we shall also prove that this is a characterization in terms of the Roe algebra of the semigroup. Likewise, time permitting, we shall study the quasi-diagonality of the uniform Roe algebra, and give an algebraic characterization of it.

**Sam Mutter (Newcastle University)**

*The domino complex: K-theory and geometry*

Kumjian and Pask, motivated by Robertson and Steger, first introduced the notion of a  $k$ -rank graph as a higher-dimensional analogue of a directed graph. The graph  $C^*$ -algebras associated to higher-rank graphs are more flexible than those of conventional graphs, in that there are fewer restrictions on their  $K$ -theory.

We describe " $k$ -cube groups", which act freely and transitively on a product of  $k$ -many trees, and inducing a  $k$ -rank graph. Simultaneously, these groups can be visualised as a collection of higher-dimensional cubes, and we can glue together these cubes whenever they share a face, producing a cube complex called the domino complex.

In this talk, we draw parallels between the topology of the domino complex, and the structure of the associated higher-rank graph algebra by means of its  $K$ -theory, demonstrating a new link between the fields of geometry and functional analysis.

**Sergey Neshveyev (University of Oslo)**

*Crystallization of  $C^*$ -algebras*

Given a  $C^*$ -algebra  $A$  with an almost periodic time evolution we define a new  $C^*$ -algebra  $A_c$  whose states parameterize the ground states of  $A$ . We then show, partly by examples, partly by general results, that  $A_c$  contains nontrivial information about the equilibrium states on  $A$  for low temperatures and its  $K$ -theory. (Joint work with Marcelo Laca and Makoto Yamashita.)

**Piotr Nowak (IM PAN Warszawa)**

*Sums of squares and higher cohomology*

Ozawa proved a characterization of property (T) for discrete groups by positivity of a certain element of the real group ring, where positivity is understood as being equal to a sum of hermitian squares.

I will discuss a generalization of this idea obtained jointly with Uri Bader to higher group cohomology with coefficients in unitary representations, as well as present a recent result with Marek Kaluba and Piotr Mizerka on the cohomology of  $SL_3(\mathbb{Z})$ .

**Eduard Ortega (NTNU Trondheim)**

*Left cancellative small categories and their associated algebras*

In this talk I will explain how to associate an étale groupoid to a left cancellative small category. We will show that certain categories with a length function can be written as a Zappa-Zsép product of a free subcategory and the groupoid of invertible elements. This talk is based in a common project with Enrique Pardo.

**Sanaz Pooya (Stockholm University)**

*Higher Kazhdan projections: K-theory and  $\ell^2$ -Betti numbers*

The Baum-Connes conjecture provides topological tools to compute the K-theory of reduced group  $C^*$ -algebras. It has been confirmed for large classes of groups, such as amenable groups, but also for some groups with Kazhdan's property (T). Property (T) and its strengthening are driving forces in the search for potential counterexamples to the conjecture. Having property (T) for a group is characterised by the existence of a certain projection in the universal group  $C^*$ -algebra of the group, known as the Kazhdan projection. It is this projection and its analogues in other completions of the group ring, which obstruct certain methods of proof for the Baum-Connes conjecture. In this talk, I will introduce a generalisation of Kazhdan projections. Employing these projections we establish a link between surjectivity of the coarse Baum-Connes assembly map and calculations of ( $\ell^2$ -)Betti numbers of the group. Further, I will introduce delocalised  $\ell^2$ -Betti numbers and exhibit an approximation theorem involving these invariants and higher Kazhdan projections. If time permits, I will explain explicit calculations of the K-theory classes of higher Kazhdan projections. This is based on joint work with K. Li and P. Nowak and work in progress with H. Wang.

**Mikael de la Salle (ENS Lyon)**

*Spectral gap and stability for groups and non-local games*

The word stable is used to describe a situation when mathematical objects that almost satisfy an equation are close to objects satisfying it exactly. I will study operator-algebraic forms of stability for unitary representations of groups and quantum synchronous strategies for non-local games. In particular, I will present how simple spectral gap estimates can lead to strong quantitative forms of stability. For example, the direct product of two (flexibly) Hilbert-Schmidt stable groups is again (flexibly) Hilbert-Schmidt stable, provided that one of them has Kazhdan's property (T). I will also provide a simple form and simple analysis of a non-local game with few questions, with the property that synchronous strategies with large value are close to perfect strategies involving large Pauli matrices. This simplifies one of the steps (the question reduction) in the recent announced resolution of Connes' embedding problem by Ji, Natarajan, Vidick, Wright and Yuen.

**Eduardo Scarparo (Federal University of Santa Catarina)**

*Almost finiteness and homology of odometers and Cantor minimal dihedral systems*

Almost finiteness is a regularity property for étale groupoids introduced by Matui which resembles the characterization of group amenability via Følner sequences. This property has found applications in the classification program of  $C^*$ -algebras and in homology questions. We will present a complete characterization of almost finiteness for odometers (also known as profinite actions) and provide the first examples of non-free almost finite actions. We will also discuss Cantor minimal dihedral systems and their homology and observe that these systems satisfy Matui's conjecture on homology and  $K$ -theory if and only if the action is free. This is joint work with Eduard Ortega.

**Gaute Schwartz (University of Oslo)**

*TBA*

**Adam Skalski (IMPAN Warszawa)**

*UCT, groupoids, and Haagerup trick and all that*

Motivated by the work of Selçuk Barlak and Xin Li on Cartan subalgebras of nuclear  $C^*$ -algebras on one hand, and the results of Jean-Louis Tu regarding étale groupoids with the Haagerup property on the other hand, we undertake a systematic study of the relationship between the Haagerup property for inclusions of  $C^*$ -algebras and (twisted) groupoid dynamical systems. The key role in this investigation is played by a version of what we call the Haagerup trick, which allows us to average arbitrary bi-module maps on a groupoid crossed product into ‘scalar multipliers’. As a consequence we show in particular that if  $A$  is a separable  $C^*$ -algebra with a Cartan subalgebra  $B$  such that the inclusion has the Haagerup property, then  $A$  satisfies the UCT. Based on the joint work with Bartosz Kwaśniewski and Kang Li.

**Alain Valette (University of Neuchâtel)**

*Explicit Baum-Connes for semi-direct products of  $\mathbb{Z}^2$  by some non-amenable subgroups of  $GL_2(\mathbb{Z})$ .*

Semi-direct products  $\mathbb{Z}^2 \rtimes G$  (with  $G$  non-amenable) are interesting because they satisfy the Baum-Connes conjecture without belonging to one of the large classes (a-T-menable groups and hyperbolic groups) for which the conjecture is known to hold. Thanks to a 3-dimensional model for the classifying space of proper actions of  $\mathbb{Z}^2 \rtimes G$ , the geometric side of the conjecture can be explicitly computed, and so gives intuition for the analytical side. In good cases a proof by hand of the Baum-Connes conjecture can be obtained: we will give two examples. This is part of a joint project with R. Flores, S. Pooya and A. Zumbrunnen.

**Alina Vdovina (Newcastle University)**

*Higher structures in mathematics: buildings,  $k$ -graphs and  $C^*$ -algebras.*

We present buildings as universal covers of certain infinite families of CW-complexes of arbitrary dimension. We will show several different constructions of new families of  $k$ -rank graphs and  $C^*$ -algebras based on these complexes, for arbitrary  $k$ .

The underlying building structure allows explicit computation of the  $K$ -theory as well as the explicit spectra computation for the  $k$ -graphs.

The talk is based on joint papers with Nadia Larsen, Sam Mutter, Christiana Radu.



## Schedule

2<sup>nd</sup> - 6<sup>th</sup> May 2022

## C\*-algebras and geometry of groups and semigroups

Observe the changing lecture rooms!

After each talk we plan 10 minutes for questions and a short break.

	Monday	Tuesday	Wednesday	Thursday	Friday
10:00 – 11:10	Coffee	Martinez	de la Salle	Nowak	Scarpato
11:10 – 11:50	de Laat	Coffee	Coffee	Coffee	Coffee
12:00 – 14:00	Lunch (provided by the workshop)	Li	Ortega	Pooya	Lederle
14:00 – 14:40	Skalski	Lunch (provided by the workshop)	Lunch (provided by the workshop)	Lunch (provided by the workshop)	Neshveyev
14:50 – 15:20	Dabeler	Le Boudec	Valette	Borst	
15:30 – 16:00	Coffee	Klisse	Coffee	Coffee	
16:00 – 16:40	Bruce	Enstad	Vdovina	Mutter	
16:50 – 17:20	Schwartz	Baraquin	Mutter		
17:30 – 20:00	Reception in room 'Abel's lookout'	Dinner at Olivia Hegdehaugsv eien			
		18:00 – 21:00			

All lectures take place on the Blindern campus

<https://www.uio.no/english/about/getting-around/areas/blindern/>

Monday Aud 3 Chemistry builing

<https://www.uio.no/english/about/getting-around/areas/blindern/bl23/>

Monday recptic Abel's lookout Niels Henrik Abel Building

<https://www.uio.no/english/about/getting-around/areas/blindern/bl14/>

Tuesday Aud 5 Vilhelm Bjerknæs Building

<https://www.uio.no/english/about/getting-around/areas/blindern/bl13/>

Wednesday Aud 1 Vilhelm Bjerknæs Building

<https://www.uio.no/english/about/getting-around/areas/blindern/bl27/>

Thursday Aud 2 Georg Sverdrup Building

<https://www.uio.no/english/about/getting-around/areas/blindern/bl27/>

Friday Aud 1 Vilhelm Bjerknæs Building

## Restaurants and Wednesday afternoon activities

Oslo offers a variety of different restaurants.

- Indian food: "Der peppern gror" on Bogstadveien.  
<https://bogstadveien.derpepperngror.no>
- Burgers and pizza's, large selection of beers: Cafe "Laundromat" located about halfway between the department and Thon Europa.  
<http://www.laundromat.no>
- Norwegian food at reasonable prices: "Sofies Mat & Vinhus", Sofies gate 15, located about halfway between the department and Thon Europa.  
<https://www.sofiesmat.no>
- French food: Bistro "L'Ardoise", possibly a bit pricey.  
<https://www.lardoise.no>
- Asian: "Tasty Thai", located about halfway between the department and Thon Europa.  
<https://www.tastythai.no>

If you like to discuss maths on Wednesday afternoon, there is a sitting area on the 7th floor of the maths department. Also, there are two seminar rooms with blackboards that can be used for discussions. Other activities you might like to

- go for a stroll around Akershus Fortress if you are historically inclined,
- go for a stroll in Frognerparken if you care to see a large sculptural display (by the Norwegian artist Vigeland),
- go for a stroll in downtown Oslo by the harbourfront, either near the Opera House and the Munch museum quarter, or in the Tjuvholmen quarter, if you are interested in fashionable, new modern buildings, or
- take metro line no 1 to Frognersteren to enjoy a rather spectacular view of Oslo and its surroundings.

# Map

