Quantum differentials on cross product Hopf algebras

Ryan Aziz
Joint work with Shahn Majid
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Prelims

Quantum Riemannian geometry by quantum groups approach :

- Differentials on an algebra A is $A - A$-bimodule Ω^1 (space of 1-forms) :
  - $\mathrm{d} : A \to \Omega^1$ (differential map) s.t. $\mathrm{d}(ab) = (\mathrm{d}a)b + a\mathrm{d}b$ (Leibniz rule)
  - $\Omega^1 = \mathrm{span}\{a\mathrm{d}b\}$ (surjectivity)
  - $\ker \mathrm{d} = k.1$ (connectedness, conditional).
- Exterior algebra means a DGA $\Omega = \oplus_{n\geq 0}\Omega^n$ on A generated by $\Omega^0 = A, \mathrm{d}A$ with
  - $\mathrm{d} : \Omega^n \to \Omega^{n+1}$ s.t. $\mathrm{d}(\omega\tau) = (\mathrm{d}\omega)\tau + (-1)^{|\omega|}\omega\mathrm{d}\tau$ (graded-Leibniz rule)
  - $\mathrm{d}^2 = 0$.

Prelims

- Ω^1 is left(resp.right) covariant if it is a left(resp.right) A-comodule algebra with $\Delta_L : \Omega \to A \otimes \Omega^1$, $\Delta_L \mathrm{d} = (\mathrm{id} \otimes \mathrm{d})\Delta$ (resp.$\Delta_R : \Omega^1 \otimes \Omega^1 \otimes A$, $\Delta_R \mathrm{d} = (\mathrm{d} \otimes \mathrm{id})\Delta$).
- Ω^1 is bicovariant if it is both left and right covariant.
- Can be extended to have Ω left/right/bicovariant.
- [Brzeziǹski '93] Ω^1 bicovariant $\Rightarrow$ Ω super-Hopf algebra ($\mathbb{Z}_2$-graded)

$$\Delta_*|_{\Omega_0} = \Delta, \quad \Delta_*|_{\Omega^1} = \Delta_L + \Delta_R$$

$$\Delta_*(\mathrm{d}a\mathrm{d}b) = \Delta_*(\mathrm{d}a)\Delta_*(\mathrm{d}b)$$

Motivation and Problem

- Knowing only Ω^1 and Ω^2, we can build elements of noncommutative geometry (metric, connection, torsion, curvature) algebraically on the DGA.
- In nice cases, we can recover the Dirac operators as in Connes' approach but does not require it as axiom.
- Fundamental problem : there will be many Ω^1 and Ω^2 on a given Hopf algebra A.
- Woronowicz construction of bicovariant Ω^1 :

$$\Omega^1 \cong A \otimes A^+/I; \quad A^+ = \ker\epsilon; \quad I : \text{ad-stable right ideal}$$

- No general result known, but for some cases Ω^1 are classified:
  - coquasitriangular Hopf algebra A (Bauman, Schmidt '98)
  - the Sweedler-Taft algebra $U_q(b_+)$ (Oeckl '99).

Overview

We introduce a method (different from Woronowicz) to construct DGAs on all main type of cross (co)product Hopf algebras :

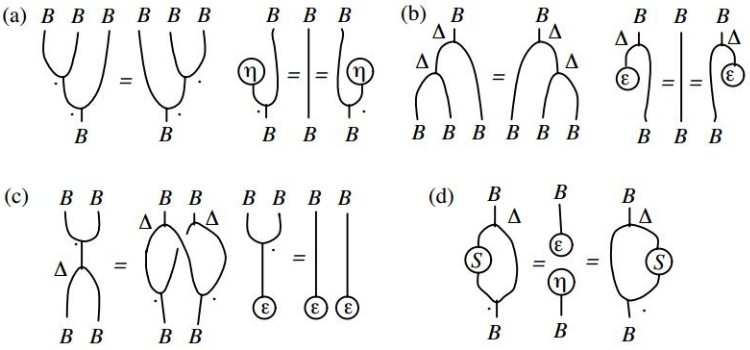
- On double cross product $A \hookrightarrow A \bowtie H \hookleftarrow H$.
- On double cross coproduct $A \twoheadleftarrow A \blacktriangleright\!\!\blacktriangleleft H \twoheadrightarrow H$.
- On bicrossproduct $A \hookrightarrow A \blacktriangleright\!\!\triangleleft H \twoheadrightarrow H$.
- On biproduct $A \overset{\hookrightarrow}{\twoheadleftarrow} A {\triangleright\!\!\!<} B$ (Here B is a braided Hopf algebra)

Overview

- Assumption : $\Omega(A), \Omega(H), \Omega(B)$ are **strongly bicovariant exterior algebras**.
- Their differentials are built by using their super version, e.g. $\Omega(A{\blacktriangleright\!\!\triangleleft}H) := \Omega(A){\blacktriangleright\!\!\triangleleft}\Omega(H)$ gives a strongly bicovariant exterior algebra on $A{\blacktriangleright\!\!\triangleleft}H$, etc.
- We do not classify all Ω^1 but the resulting exterior algebra is natural in the sense it (co)acts on its factor **differentiably**.
- In this talk, we will focus on differentials on biproduct $A{\cdot\!\!\!>\!\!\!\lhd}B$.

Braided Hopf algebras

Def (Majid '90s) : Let $\mathcal{C}$ be braided monoidal category. $B \in \mathcal{C}$ is a braided Hopf algebra if it is algebra + coalgebra + antipode $S : B \to B$ s.t.



e.g $\Delta(bc) = b_{\underline{(1)}}\Psi(b_{\underline{(2)}} \otimes c_{\underline{(1)}})c_{\underline{(2)}}$.

Biproduct Hopf algebras

- If A is ordinary Hopf algebra and B is braided Hopf algebra in $\mathcal{M}_A^A$ crossed module (or Drinfeld-Radford-Yetter module) category, then there is a biproduct $A \rtimes\!\!\cdot B$ (or the *Radford-Majid bosonisation* of B) built in $A \otimes B$ with

$$(a \otimes b)(c \otimes d) = ac_{(1)} \otimes (b \triangleleft c_{(2)})d$$

$$\Delta(a \otimes b) = a_{(1)} \otimes b_{\underline{(1)}}{}^{\overline{(0)}} \otimes a_{(2)} b_{\underline{(1)}}{}^{\overline{(1)}} \otimes b_{\underline{(2)}}$$

  for all $a, c \in A$, $b, d \in B$.
- Example : $\mathbb{C}_q[P] = \mathbb{C}_q[GL_2] \rtimes\!\!\cdot \mathbb{C}_q^2 \cong \mathbb{C}_q[SL_3]/(t^i{}_j | i > j)$ a deformation of maximal parabolic $P \subset SL_3$

Super Crossed Modules

- Let A be a super Hopf algebra, i.e. $A = A_0 \oplus A_1$.
- Let $V = V_0 \oplus V_1$ be a super right A-crossed module over a super-Hopf algebra A if
  1. V is a super right A-module by $\triangleleft : V \otimes A \to V$
  2. V is a super right A-comodule by $\Delta_R : V \to V \otimes A$ denoted $\Delta_R v = v^{\overline{(0)}} \otimes v^{\overline{(1)}}$, such that

$$\Delta_R(v \triangleleft a) = (-1)^{|v^{\overline{(1)}}||a_{(1)}|+|v^{\overline{(1)}}||a_{(2)}|+|a_{(1)}||a_{(2)}|} v^{\overline{(0)}} \triangleleft a_{(2)} \otimes (Sa_{(1)}) v^{\overline{(1)}} a_{(3)}$$

for all $v \in V$ and $a \in A$.
- The category $\mathcal{M}_A^A$ of super right A-crossed modules is a prebraided category with the braiding $\Psi : V \otimes W \to W \otimes V$,

$$\Psi(v \otimes w) = (-1)^{|v||w^{\overline{(0)}}|} w^{\overline{(0)}} \otimes (v \triangleleft w^{\overline{(1)}})$$

and braided if A has invertible antipode

Strongly bicovariant exterior algebras

(Majid - Tao '15) Ω is **strongly bicovariant** if it is :

- a super-Hopf algebra with super-degree given by the grade mod 2
- super-coproduct Δ_* grade preserving and restricting to the coproduct of A
- d is a **super coderivation** in the sense

$$\Delta_* \mathrm{d}\omega = (\mathrm{d} \otimes \mathrm{id} + (-1)^{|\ |} \mathrm{id} \otimes \mathrm{d}) \Delta_* \omega$$

Lemma (Majid - Tao '15)

Ω *Strongly bicovariant* $\Rightarrow$ Ω *bicovariant*

Lemma

$\Omega(A), \Omega(H)$ *strongly bicovariants* $\Rightarrow \Omega(A \otimes H) := \Omega(A)\underline{\otimes}\Omega(H)$ *is strongly bicovariant on* $A \otimes H$ *with* $\mathrm{d} = \mathrm{d}_A \otimes id + (-1)^{|\ |} \mathrm{id} \otimes \mathrm{d}_H$.

Differentiable Coaction

- Let A be Hopf algebra, $\Omega(A)$ be its exterior algebra.
- Let $B \in \mathcal{M}^A$ be comodule algebra, $\Omega(B)$ is A-covariant, i.e. the coaction $\Delta_R : \Omega(B) \to \Omega(B) \otimes A$ (denoted by $\Delta_R \eta = \eta^{\overline{(0)}} \otimes \eta^{\overline{(1)}}$) is a comodule map.
- Δ_R **is differentiable** if it extends to a degree-preserving map $\Delta_{R*} : \Omega(B) \to \Omega(B) \underline{\otimes} \Omega(A)$ of exterior algebras such that

$$\mathrm{d}_B \Delta_{R*} = \mathrm{d} \Delta_{R*}$$

or explicitly

$$\Delta_{R*} \mathrm{d}_B \eta = \mathrm{d}_B \eta^{\overline{(0)}*} \otimes \eta^{\overline{(1)}*} + (-1)^{|\eta|} \eta^{\overline{(0)}*} \otimes \mathrm{d}_A \eta^{\overline{(1)}*},$$

where $\Delta_{R*} \eta = \eta^{\overline{(0)}*} \otimes \eta^{\overline{(1)}*} \in \Omega(B) \underline{\otimes} \Omega(A)$.

Differentiable action

- Let A be Hopf algebra, $\Omega(A)$ be its exterior algebra.
- Let $B \in \mathcal{M}_A$ be a module algebra, $\Omega(B)$ is A-covariant, i.e. the action $\triangleleft : \Omega(B) \otimes A \to \Omega(B)$ is a module map.
- The action $\triangleleft$ **is differentiable** if it extends to a degree preserving map $\triangleleft : \Omega(B) \otimes \Omega(A) \to \Omega(A)$ such that

$$\mathrm{d}_B \triangleleft = \triangleleft \mathrm{d}$$

or explicitly

$$\mathrm{d}_B(\eta \triangleleft \omega) = (\mathrm{d}_B \eta) \triangleleft \omega + (-1)^{|\eta|} \eta \triangleleft (\mathrm{d}_A \omega)$$

for all $\eta \in \Omega(B), \omega \in \Omega(A)$.

Super Biproducts

Assumption :

1. B is a braided Hopf algebra in $\mathcal{M}_A^A$ s.t. they form $A \mathrel{\triangleright\!\!<} B$
2. $\Omega(B) \in \mathcal{M}_A^A$ with differentiable action and coaction
3. $\Omega(B)$ is a super braided Hopf algebra in super crossed module category $\mathcal{M}_{\Omega(A)}^{\Omega(A)}$ with d_B a super coderivation

Then we have super biproduct $\Omega(A) \mathrel{\triangleright\!\!<} \Omega(B)$

$$(\omega \otimes \eta)(\tau \otimes \xi) = (-1)^{|\eta||\tau_{(1)}|} \omega \tau_{(1)} \otimes (\eta \triangleleft \tau_{(2)}) \xi$$

$$\Delta_*(\omega \otimes \eta) = (-1)^{|\omega_{(2)}||\underline{\eta_{(1)}}^{\overline{(0)}*}|} \omega_{(1)} \otimes \underline{\eta_{(1)}}^{\overline{(0)}*} \otimes \omega_{(2)} \underline{\eta_{(1)}}^{\overline{(1)}*} \otimes \underline{\eta_{(2)}}$$

for all $\omega, \tau \in \Omega(A)$ and $\eta, \xi \in \Omega(B)$.

Differentials by Super Biproducts

Theorem

1. *Under the assumptions above,* $\Omega(A{\triangleright\!\!\!<}B) := \Omega(A){\triangleright\!\!\!<}\Omega(B)$ *is a strongly bicovariant exterior algebra on* $A{\triangleright\!\!\!<}B$ *with differential map*

$$\mathrm{d}(\omega \otimes \eta) = \mathrm{d}_A\omega \otimes \eta + (-1)^{|\omega|}\omega \otimes \mathrm{d}_B\eta$$

for all $\omega \in \Omega(A)$, $\eta \in \Omega(B)$.

2. *The canonical* $\Delta_R : B \to B \otimes A{\triangleright\!\!\!<}B$ *given by* $\Delta_R b = b_{\underline{(1)}}{}^{\overline{(0)}} \otimes b_{\underline{(1)}}{}^{\overline{(1)}} \otimes b_{\underline{(2)}}$ *is differentiable, i.e it extends to* $\Delta_{R*} : \Omega(B) \to \Omega(B)\underline{\otimes}\Omega(A{\triangleright\!\!\!<}B)$ *by*

$$\Delta_{R*}\eta = \eta_{\underline{(1)}}{}^{\overline{(0)}*} \otimes \eta_{\underline{(1)}}{}^{\overline{(1)}*} \otimes \eta_{\underline{(2)}}$$

Differential on $A \rtimes V(R)$

- Let $R \in M_n(\mathbb{C}) \otimes M_n(\mathbb{C})$ be q-Hecke (PR has two eigen-values).
- Let $A(R)$ be an FRT algebra generated by $\mathbf{t} = ({t^i}_j)$ with

$$R\mathbf{t}_1\mathbf{t}_2 = \mathbf{t}_2\mathbf{t}_1 R, \quad \Delta\mathbf{t} = \mathbf{t} \otimes \mathbf{t}$$

- $A = A(R)[D^{-1}]$, $D \in A(R)$ central, grouplike.
- $\Omega(A(R))$ has

$$(\mathrm{d}\mathbf{t}_1)\mathbf{t}_2 = R_{21}\mathbf{t}_2\mathrm{d}\mathbf{t}_1 R, \quad \mathrm{d}\mathbf{t}_1\mathrm{d}\mathbf{t}_2 = -R_{21}\mathrm{d}\mathbf{t}_2\mathrm{d}\mathbf{t}_1 R$$

$$\mathrm{d}D^{-1} = -D^{-1}(\mathrm{d}D)D^{-1}, \quad \Delta_*\mathrm{d}\mathbf{t} = \mathrm{d}\mathbf{t} \otimes \mathbf{t} + \mathbf{t} \otimes \mathrm{d}\mathbf{t}$$

- Let $V(R) \in \mathcal{M}^A$ a braided covector algebra generated by $\mathbf{x} = (x_i)$ with $q\mathbf{x}_1\mathbf{x}_2 = \mathbf{x}_2\mathbf{x}_1 R, \quad \Delta_R\mathbf{x} = \mathbf{x} \otimes \mathbf{t}$
- $\Omega(V(R)) \in \mathcal{M}^{\Omega(A)}$ has

$$(\mathrm{d}\mathbf{x}_1)\mathbf{x}_2 = \mathbf{x}_2\mathrm{d}\mathbf{x}_1 qR, \quad -\mathrm{d}\mathbf{x}_1\mathrm{d}\mathbf{x}_2 = \mathrm{d}\mathbf{x}_2\mathrm{d}\mathbf{x}_1 qR, \quad \Delta_{R*}\mathrm{d}\mathbf{x} = \mathrm{d}\mathbf{x}\otimes\mathbf{t}+\mathbf{x}\otimes\mathrm{d}\mathbf{t}$$

Differential on $A{\triangleright\!\!\!<}V(R)$

Theorem

Let $A = A(R)[D^{-1}]$ *with* R *q-Hecke and* $V(R)$ *the right-covariant braided covector algebra. Then* $\Omega(V(R))$ *is a super-braided-Hopf algebra with* $x_i, \mathrm{d}x_i$ *primitive in* $\mathcal{M}^{\Omega(A)}_{\Omega(A)}$ *with* $\Delta_{R*}\mathrm{d}\mathbf{x} = \mathrm{d}\mathbf{x} \otimes \mathbf{t} + \mathbf{x} \otimes \mathrm{d}\mathbf{t}$ *and*

$$\mathbf{x}_1 \triangleleft \mathbf{t}_2 = \mathbf{x}_1 q^{-1} R_{21}^{-1}, \quad \mathrm{d}\mathbf{x}_1 \triangleleft \mathbf{t}_2 = \mathrm{d}\mathbf{x}_1 q^{-1} R$$

$$\mathbf{x}_1 \triangleleft \mathrm{d}\mathbf{t}_2 = (q^{-2} - 1)\mathrm{d}\mathbf{x}_1 P, \quad \mathrm{d}\mathbf{x}_1 \triangleleft \mathrm{d}\mathbf{t}_2 = 0,$$

and $\Omega(A{\triangleright\!\!\!<}V(R)) := \Omega(A){\triangleright\!\!\!<}\Omega(V(R))$ *with*

$$\mathbf{x}_1 \mathbf{t}_2 = \mathbf{t}_2 \mathbf{x}_1 q^{-1} R_{21}^{-1}, \quad \mathrm{d}\mathbf{x}_1 . \mathbf{t}_2 = \mathbf{t}_2 \mathrm{d}\mathbf{x}_1 q^{-1} R,$$

$$\mathbf{x}_1 \mathrm{d}\mathbf{t}_2 = \mathrm{d}\mathbf{t}_2 . \mathbf{x}_1 q^{-1} R_{21}^{-1} + (q^{-2} - 1)\mathbf{t}_2 \mathrm{d}\mathbf{x}_1 P, \quad \mathrm{d}\mathbf{x}_1 \mathrm{d}\mathbf{t}_2 = -\mathrm{d}\mathbf{t}_2 \mathrm{d}\mathbf{x}_1 q^{-1} R$$

$$\Delta \mathbf{x} = 1 \otimes \mathbf{x} + \mathbf{x} \otimes \mathbf{t}, \quad \Delta_* \mathrm{d}\mathbf{x} = 1 \otimes \mathrm{d}\mathbf{x} + \mathrm{d}\mathbf{x} \otimes \mathbf{t} + \mathbf{x} \otimes \mathrm{d}\mathbf{t}.$$

Differential on Quantum Parabolic Group

- For $R = R_{gl_2}$, then $A = \mathbb{C}_q[GL_2]$ generated by $t^1{}_1 = a, t^1{}_2 = b, t^2_1 = c, t^2{}_2 = d$ with

$$ba = qab, \quad ca = qac, \quad db = qbd, \quad dc = qcd$$

$$da - ad = (q - q^{-1})bc, \quad ad - q^{-1}bc = da - qcb = D$$

$$\Delta t^i{}_j = t^i{}_k \otimes t^k{}_j$$

- Let $V(R) = \mathbb{C}_q^2 \in \mathcal{M}^{\mathbb{C}_q[GL_2]}$ a two-dimensional quantum plane with $x_2x_1 = q$, $\underline{\Delta} x_i = 1 \otimes x_i + x_i \otimes 1$ and $\Delta_R x_i = x_j \otimes t^j_i$

Differential on Quantum Parabolic Group

- $\Omega(\mathbb{C}_q^2)$ has

$$(\mathrm{d}x_i)x_i = q^2 x_i \mathrm{d}x_i, \quad (\mathrm{d}x_1)x_2 = qx_2\mathrm{d}x_1$$

$$(\mathrm{d}x_2)x_1 = qx_1\mathrm{d}x_2 + (q^2-1)x_2\mathrm{d}x_1$$

$$(\mathrm{d}x_i)^2 = 0, \quad \mathrm{d}x_2\mathrm{d}x_1 = -q^{-1}\mathrm{d}x_1\mathrm{d}x_2$$

- By requiring differentiability on $\Delta_R : \mathbb{C}_q^2 \to \mathbb{C}_q^2 \otimes \mathbb{C}_q[GL_2]$, it enforces us to use the following $\Omega(\mathbb{C}_q[GL_2])$

$$\mathrm{d}a.a = q^2 a\mathrm{d}a, \quad \mathrm{d}a.b = qb\mathrm{d}a, \quad \mathrm{d}b.a = qa\mathrm{d}b + (q^2-1)b\mathrm{d}a$$

$$\mathrm{d}d.a = a\mathrm{d}d, \quad \mathrm{d}b.c = c\mathrm{d}b + (q-q^{-1})d\mathrm{d}d, \quad \text{etc.}$$

$$\Delta_*\mathrm{d}t^i{}_j = \mathrm{d}t^i{}_k \otimes t^k{}_j + t^i{}_k \otimes \mathrm{d}t^k{}_j$$

Differential on Quantum Parabolic Group

$\Omega(\mathbb{C}_q^2)$ is a super braided Hopf algebra in $\mathcal{M}_{\Omega(\mathbb{C}_q[GL_2])}^{\Omega(\mathbb{C}_q[GL_2])}$ by

$$x_1 \triangleleft \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} q^{-2}x_1 & (q^{-2}-1)x_2 \\ 0 & q^{-1}x_1 \end{pmatrix}, \quad x_2 \triangleleft \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \cdots$$

$$x_1 \triangleleft \begin{pmatrix} \mathrm{d}a & \mathrm{d}b \\ \mathrm{d}c & \mathrm{d}d \end{pmatrix} = \begin{pmatrix} (q^{-2}-1)\mathrm{d}x_1 & (q^{-2}-1)\mathrm{d}x_2 \\ 0 & 0 \end{pmatrix}$$

$$\mathrm{d}x_1 \triangleleft \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \mathrm{d}x_1 & 0 \\ 0 & q^{-1}\mathrm{d}x_1 \end{pmatrix}$$

$$x_2 \triangleleft \begin{pmatrix} \mathrm{d}a & \mathrm{d}b \\ \mathrm{d}c & \mathrm{d}d \end{pmatrix} = \cdots, \quad \mathrm{d}x_2 \triangleleft \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \cdots$$

$$\mathrm{d}x_i \triangleleft \mathrm{d}t^k{}_l = 0, \quad \Delta_R x_i = x_j \otimes t^j{}_i, \quad \Delta_{R*}\mathrm{d}x_i = \mathrm{d}x_j \otimes t^j{}_i + x_j \otimes \mathrm{d}t^j{}_i$$

$$\underline{\Delta} x_i = x_i \otimes 1 + 1 \otimes x_i, \quad \underline{\Delta}_* \mathrm{d}x_i = \mathrm{d}x_i \otimes 1 + 1 \otimes \mathrm{d}x_i.$$

Differential on Quantum Parabolic Group

Then (i) $\Omega(\mathbb{C}_q[P]) = \Omega(\mathbb{C}_q[GL_2]\ltimes\mathbb{C}^2) := \Omega(\mathbb{C}_q[GL_2])\ltimes\Omega(\mathbb{C}_q^2)$ with sub-exterior algebras $\Omega(\mathbb{C}_q[GL_2])$, $\Omega(\mathbb{C}_q^2)$ and cross relations and super coproduct

$$x_1\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} q^{-2}ax_1 & q^{-1}bx_1 + (q^{-2}-1)ax_2 \\ q^{-2}cx_1 & q^{-1}dx_1 + (q^{-2}-1)cx_2 \end{pmatrix}, \quad x_2\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \cdots$$

$$\mathrm{d}x_1.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a\mathrm{d}x_1 & q^{-1}b\mathrm{d}x_1 \\ c\mathrm{d}x_1 & q^{-1}d\mathrm{d}x_1 \end{pmatrix}, \quad \mathrm{d}x_2.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \cdots$$

$$x_1\begin{pmatrix} \mathrm{d}a & \mathrm{d}b \\ \mathrm{d}c & \mathrm{d}d \end{pmatrix} = \cdots, \quad x_2\begin{pmatrix} \mathrm{d}a & \mathrm{d}b \\ \mathrm{d}c & \mathrm{d}d \end{pmatrix} = \cdots$$

$$\Delta x_i = 1\otimes x_i + \Delta_R(x_i), \quad \Delta_*(\mathrm{d}x_i) = 1\otimes \mathrm{d}x_i + \Delta_{R*}(\mathrm{d}x_i).$$

(ii) $\Delta_R : \mathbb{C}_q[GL_2] \to \mathbb{C}_q[GL_2]\otimes\mathbb{C}_q[P]$ is differentiable

$$\Delta_R x_i = 1\otimes x_i + x_j\otimes t^j{}_i, \quad \Delta_{R*}\mathrm{d}x_i = 1\otimes\mathrm{d}x_i + \mathrm{d}x_j\otimes t^j{}_i + x_j\otimes \mathrm{d}t^j{}_i$$

Overview

- The canonical coactions $\Delta_R : A \to A \otimes H \bowtie A$ and $\Delta_L : H \to H \bowtie A \otimes H$ are differentiable, i.e. they extend to

$$\Delta_{R*} : \Omega(A) \to \Omega(A) \underline{\otimes} \Omega(H) \bowtie \Omega(A)$$

$$\Delta_{L*} : \Omega(H) \to \Omega(H) \bowtie \Omega(A) \underline{\otimes} \Omega(H)$$

  making $\Omega(H)$ and $\Omega(A)$ super $\Omega(H \bowtie A)$-comodule algebras
- The canonical coaction $\Delta_R : H \to H \otimes A \blacktriangleright\!\!\triangleleft H$ is differentiable, i.e. it extends to

$$\Delta_{R*} : \Omega(H) \to \Omega(H) \underline{\otimes} \Omega(A) \blacktriangleright\!\!\triangleleft \Omega(H)$$

  making $\Omega(H)$ a super $\Omega(A \blacktriangleright\!\!\triangleleft H)$-comodule algebra.

Overview

- $A{\bowtie}H$ acts on f.d. A^* as module algebra by

$$(\phi{\triangleleft}h)(a)=\phi(h{\triangleright}a),\quad \phi{\triangleleft}a=\langle\phi_{(1)},a\rangle\phi_{(2)},$$

Similarly for a left action on H^*. However, for differentiability, we would need $\Omega(A^*)$ or $\Omega(H^*)$ to be specified.

Thank you for your attention