Chaos expansions for stochastic evolution equations

Tijana Levajković
University of Innsbruck, Austria

Abstract

In this talk we study parabolic stochastic partial differential equations in the framework of white noise analysis. Particularly, we consider a stochastic Cauchy problem of the form

$$\begin{aligned} \frac{\partial}{\partial t} U(t,x,\omega) &= \mathbf{A}\, U(t,x,\omega) + \mathbf{B} \lozenge U(t,x,\omega) + F(t,x,\omega) \\ U(0,x,\omega) &= U^0(x,\omega)\,, \end{aligned} \tag{1}$$

where $t \in (0,T]$, $\omega \in \Omega$ and $u(t,\cdot,\omega)$ belongs to some Banach space X. The operator $\mathbf{A}$ is densely defined, generating a C_0-semigroup and $\mathbf{B}$ is a linear bounded operator which combined with the Wick product $\lozenge$ introduces convolution-type perturbations into the equations. By applying the method of Wiener-Itô chaos expansions, also known as the propagator method, we reduce the SPDE to an infinite triangular system of deterministic PDEs, which can be solved by induction. Summing up all coefficients of the expansion and proving convergence in an appropriate weight space of stochastic processes, one obtains the solution of the initial SPDE.

In addition, we consider an optimal control problem related to a class of stochastic evolution equations (1) with quadratic cost functional. The optimal solution is thus obtained by combining techniques from white noise analysis with classical theory of optimal control.