# Risky risk assessments.

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- Actuarial assessments often based on an underlying model:
  Our basic assumption.

- If $X$ is a risk variable, then
  Reality: $M \longrightarrow X$.
  Our model: $\widehat{M} \longrightarrow \widehat{X}$.
  Draw conclusions from $\widehat{M}$.

- Why is $\widehat{M} \neq M$?
  (1) The future is unknown!
  (2) The past isn't perfectly known.

**Disability modelling:**

(1) Disability intensites decades ahead needed.
They may change!

(2) What we have to go on is limited:
Disability data scarce.

**Large claims insurance:**

(1) The future is next year.
Not that different from the present one (?).

(2) Limited number of events for model building.
Data often very scarce.

# Outline of the talk.

- Discuss two case studies:
  - (1) Disability evaluation.
  - (2) A supreme court decision on captive risk.
- General comments on
  - (1) Approach to error.
  - (2) What the academic world should do (perhaps).

- Disability intentities depend on

  **age**,

  **covariates** such as

  gender, salary, company type,

  company size, rural/urban.

- Use of models:

  To quantify portfolio risk.

# Disability modelling: approach.

- Two main techniques:

  Regression (log-linear, GLM)

  Intensity modelling with covariates.

- Intensity modelling natural:

  The **unifying** viewpoint.

  Software available.

- Mathematical formulation.

  **Proportional hazard**:

  intensity $\lambda(t;\mathbf{x})$, covariates $\mathbf{x} = (x_1, \ldots, x_p)$:

  $\lambda(t;\mathbf{x}) = \lambda_0(t)e^{\beta_1 x_1 + \cdots + \beta_p x_p}$.

  $\lambda_0(t)$: Intensity when $\mathbf{x} = 0$.

# Data from a Norwegian insurance company[a].

- Description of data:
  340000 individuals over seven years.
  Average age 39 and average time observed 3 years.
  Around 2000 disabilities (0.6%).

- **Cox regression**[b,c] handles irregular data patterns.

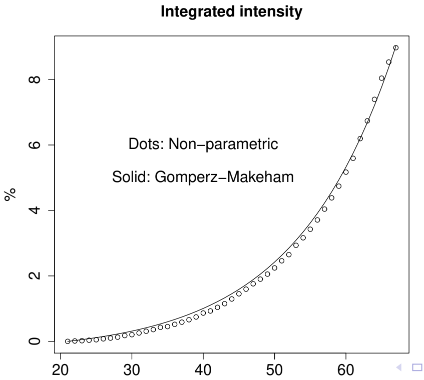
[a]Myking, A.M. (2020). Modellering og risiko i uføreportføljer. Master thesis, Department of Mathematics, University of Oslo.
[b] Cox, D. (1972). Regression models and life-tables. *Journal Royal Statistical Society*.
[c]Efron, B. (1977). The efficiency of Cox's likelihood function for censored data. *Journal American Statistical Association*.

# Fitted baseline **integrated** intensity

- The plot:

  The integrated intensity: $\int_{t_0}^{t} \lambda_0(s)ds, \quad t_0 = 18$
  for a woman earning 500000 NOK.

- Curvature: Higher intensity at higher age.



# Regression coeffcients

- Estimated coefficients (standard error in parenthesis).

| | |
|---|---|
| Male versus female | -0.34 (0.05) |
| Income (in 100000 NOK) | -0.14 (0.01) |
| Urban versus rural | -0.35 (0.07) |
| Sector: Production versus primary | 0.51 (0.18) |
| Sector: Services versus primary | 0.38 (0.18) |
| Company size: Middle versus small | 0.17 (0.05) |
| Company size: Large versus small | 0.19 (0.06) |

- Everything statistically significant.

- Portfolio calculations:
  Standard actuarial methods,
  adding all payments $\times$ probabilities.

- Test case:
  Portfolio with 50000 **active** individuals.
  Average age 41 years.
  Disability pension 250000 NOK anually up to 67.
  A lot of other conditions.

- Expected, discounted, net cost: 807.5 million NOK.

- What might the error in this assessment be?

- Important:
  How disabilities develope in the future.
  Judgment more than quantitative analysis.

- Error we can quantify:
  Uncertainty in what we have seen,
  i.e in the disability model.

- Method: Bootstrapping[a]
  Simulating historical data, refitting model,
  recalculating cost. Repeating 100 times or more.

[a] Efron, B. again

# Estimation error in example.

- Based on 1000 simulations.

- Historical data:

  $n = 346000$ or $n = 86500$ individuals

  Same percentage of disabled, same age distribution.

- Results caused by estimation error:

| | $n = 346000$ | $n = 86500$ |
|---|---|---|
| Bias | 1.1% | −8.8% |
| SD | 22.9% | 44.8% |

- SD $\approx a/\sqrt{n}$ where $a$ depends on situation[a].

  **How sensitively does a vary?**

[a] Bølviken, E and Myking, A.M. (2020). How much data does disability modelling require? (under preparation).

# Case 2: Large claims insurance.

- Solvency Capital for **Captive**.
- **Captive**: Insurance company owned by a mother company.
- My example: Captive **CX** insuring **oil** company **CO**,
  both owned by mother **MO**.
- Big tax incentives for CX to charge CO high premia.
  In Norway:Oil tax 55% **above** the normal 23%!

# A legal dispute almost 20 years ago.

- Participants:

  CX and MO against the Norwegian government.

- Issue: Had CX and MO been too greedy?

  Had premia been too high compared to the risk?

- Taxman position:

  The solvency capital was too low.

- Two legal issues:

  1 Was the solvency capital below 99%?

  2 If it was, the activty of CX might no be insurance (?).

- CX had 8 clients with maximal responsibility (US$):

| Company | Year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|
| D1 | 47 | 86 | 120 | 147 |
| D2 | 45 | 68 | 72 | 144 |
| D3 | 34 | 34 | 34 | 144 |
| D4 | 34 | 34 | 49 | 49 |
| D5 | 17 | 72 | 107 | 127 |
| D6 | 29 | 64 | 154 | 114 |
| D7 | 17 | 72 | 167 | 127 |
| D8 | 28 | 68 | 88 | 119 |

- Summary for CX (US$):

| | Year 1 | year 2 | year 3 | year 4 |
|---|---|---|---|---|
| Total premia | 45 | 64 | 161 | 55 |
| Total reinsurance premia | 10 | 19 | 16 | 8 |
| Solvency Capital | **25** | **40** | **80** | **131** |

- Information on expected number of claims $\lambda$ unavailable.

- Claim statistics (22 events) available,
  Pareto distribution fitted,
  $f(z) = (\alpha/\beta)(1 + z/\beta)^{-(1+\alpha)}$ with $\alpha \approx 6$.
  $E(Z) = \beta/(\alpha - 1)$.

- But how to deal with claim frequency:
  Use the formula $P = (1 + \gamma)\lambda E(Z)$
  where $P$ is premium and $\gamma$ the loading
  and solve for $\lambda$.

- CX under-capitalized in year 1-3, more doubtfull in year 4.
- But what about uncertainty?
- Outcome court battle:
  Court of appeals: The government won completely.
  Supreme court: Goverment lost 3-2 on Issue 2.

# Uncertainty in risk conclusions.

- The risk evaluation in court depended on
  claim distribution (uncertain),
  the loading which was varied.

- Was the court interested in uncertainty at all?

- Should we be?
  (1) When evaluating solvency capital?
  (2) When evaluating expected cost?

# Attidue towards “risk in risk”: Personal view

- We should not quantifying all kinds of uncertainty.
  Bayesian analyses lose transparency.
- Bootstrap techniques as in Example 1:
  Yield **some** of the uncertainty.
- Personal judgment on top of everything.
  Good **communication** from the analyst essential.

# What should professors be doing? Personal view

- Is too much time spent on mathematical issues
  and on complex, unjustifiable models?

- More emphasis on software to deal with "risk in risk".

- More emphasis on promoting communication skill.