# SCV-OSLO-CONFERENCE 2022

Marking the retirement of Erik Løw University of Oslo, December 1 - 3, 2022 All talks will be in Niels Henrik Abels Hus, 12th floor. Blindern Campus, Moltke Moe's Vei 35.

# **SPEAKERS:**

- Mats Andersson (Chalmers)
- Rafael Andrist (Ljubljana)
- Leandro Arosio (Tor Vergata)
- Robert Berman (Chalmers)
- Barbara D. Drnovšek (Ljubljana)
  - Jimmy Johansson (Chalmers)
  - Franc Forstnerič (Ljubljana)
  - Frank Kutzschebauch (Bern)
  - Han Peters (Amsterdam)
- Alexander Rashkovski (Stavanger)
  - Jean Ruppenthal (Wuppertal)
    - Josua Schott (Bern)
    - Weixia Zhu (Vienna)
    - Per Åhag (Umeå)

Organisers: John Erik Fornæss, Frank Kutzschebauch, Håkan Samuelsson Kalm, László Lempert, and Erlend F. Wold



TROMSØ RESEARCH FOUNDATION

SCV-OSLO-CONFERENCE 2022			
	Thursday	Friday	Saturday
10:00-10:45	Andersson	Berman	Åhag
10:15-11:15	Coffiee	Coffiee	Coffiee
11:15-12:00	Schott	Forstneric	Johansson
12:00-13:15	Lunch Break	Lunch Break	Andrist
13:15-14:00	Peters	Drnovsek	Kutzschebauch
14:15-1500	Arosio	Rashkovski	
15:15-16:00	Zhu	Ruppenthal	

### MATS ANDERSSON

Title: Poincaré-Lelong type formulas and generalized Segre numbers

Abstract: I will present a Poincaré-Lelong type formula for a holomorphic morphism  $g: S \to E$ , where S and E are vector bundles over a complex manifold X. The construction is global, and has some global applications. It also suggests a definition of pointwise numbers of a coherent sheaf on X, generalizing the well-known Hilbert-Samuel multiplicities and Segre numbers of ideal sheaves.

#### ANDRIST, RAFAEL

Title: Symplectic holomorphic automorphisms of Calogero-Moser spaces

Abstract: A Calogero-Moser space describes the completed phase space of a system of finitely many particles with a certain Hamiltonian in classical physics. Since the past two decades, these spaces are also an object of ongoing study in pure mathematics. In particular, a Calogero-Moser space of n particles is known to be a smooth complex-affine variety, and to be diffeomorphic to the Hilbert scheme of n points in the affine plane.

A result of Ellingsrud-Strømme computes the Borel-Moore homology of Hilbert schemes; and by Poincaré duality we then also know their de Rham cohomology. In particular, we can conclude that every symplectic vector field on a Calogero-Moser space is in fact Hamiltonian, which allows us to work with Hamiltonian functions instead of vector fields.

We establish the symplectic/Hamiltonian holomorphic density property for the Calogero-Moser spaces and describe their group of symplectic holomorphic automorphisms.

#### LEANDRO AROSIO

#### Title: Gromov hyperbolicity methods in holomorphic iteration

Abstract: Karlsson proved in 2001 that the classical Denjoy-Wolff theorem about iteration of holomorphic functions in the unit disc can be generalized to the setting of nonexpanding maps of Gromov hyperbolic metric spaces. In this talk we will consider the same setting, and we will discuss the interplay between the horofunction compactification and the Gromov compactification. Applying our results to the case of strongly pseudoconvex domains and of convex domains of D'Angelo finite type we will obtain the existence of horospheres, a Julia Lemma, and an analog of the Denjoy-Wolff theorem for backward iteration. This is based on joint works with Fiacchi-Gontard-Guerini and Fiacchi-Guerini-Karlsson.

#### ROBERT BERMAN

Title: Sharp bounds on the height of Fano varieties

Abstract: According to a result of Fujita-Liu the n- dimensional complex projective space may be characterized as the Fano variety which has the largest algebraic degree among all K-semistable Fano varieties. In this talk - which is a non-technical introduction to a joint work with Rolf Andreasson - an arithmetic analog of this result is discussed, where the role of degree is played by Falting's height. The height is formulated in terms of arithmetic intersection theory and depends on the choice of a metric on the anti-canonical line bundle of the corresponding Fano variety X. Our main result establishes an arithmetic analog of Fujita-Liu's result for toric Fano varieties of dimension  $n \leq 6$ . The conjectural extension to any dimension nhinges on a conjectural gap-hypothesis for the algebraic degree.

#### BARBARA DRINOVEC DRNOVŠEK

Title: Proper holomorphic maps in Euclidean spaces which avoid unbounded convex sets.

Abstract: We show that under a mild condition on a closed convex set  $E \subset \mathbb{C}^n$ the following holds. Given a Stein manifold X with dim X < n, a compact  $\mathcal{O}(X)$ convex set  $K \subset X$ , and a holomorphic map  $f_0 : U \to \mathbb{C}^n \setminus E$  from an open neighbourhood  $U \subset X$  of K, we can approximate  $f_0$  uniformly on K by proper holomorphic maps  $f : X \to \mathbb{C}^n$  such that  $f(X) \subset \mathbb{C}^n \setminus E$ . This is joint work with Franc Forstnerič.

# FRANC FORSTNERIČ

Title: Proper holomorphic embeddings: from classical to recent.

Abstract: In this talk, I shall survey the methods used in constructions of proper holomorphic mappings and embeddings of Stein manifolds, emphasizing the fertile interplay between peak function techniques, Oka theory, and Andersen-Lempert theory. A recent joint result concerning proper holomorphic maps and embeddings in Euclidean spaces avoiding large closed convex sets will be presented in more detail by Barbara Drinovec Drnovšek.

#### JIMMY JOHANSSON

Title: Twisted resolutions, Ext groups, and residue currents

Abstract: Let F and G be coherent sheaves over a complex manifold X. I will describe a result by Toledo and Tong on how the Ext groups  $\operatorname{Ext}^{k}(F,G)$  can be represented as the cohomology of a certain Čech cochain complex which is formed from a family of locally free resolutions of F over a Stein cover of X. This construction naturally reveals Grothendieck's local-to-global spectral sequence that relates the Ext sheaves with the Ext groups.

Under the assumption that G is locally free, I will describe an explicit isomorphism, using residue currents, between this representation of the Ext groups and a representation that is formed using the Dolbeault complex of G. This is based on joint work with Richard Lärkäng.

#### FRANK KUTZSCHEBAUCH

Title: Holomorphic Factorization of special automorphisms of rank two vector bundles" (joint work with George Ionita)

Abstract: The factorization for matrices of determinant one into unipotent factors of the standard type

$$\left(\begin{array}{cc}1&Z\\0&1\end{array}\right),\quad \left(\begin{array}{cc}1&0\\Z&1\end{array}\right)$$

has been solved by Ivarsson and the speaker some years ago. In case of vector bundle automorphisms no such standard type unipotent automorphisms exist. We create finitely many pairs  $U_i^+, U_i^-$  replacing the standard pair and prove that every null homotopic holomorphic special automorphism is a finite product of such. This involves the use of two different Oka principles, one of Forster-Ramspott type for Oka pairs (where the fibration is not a submersion) and one of Oka-Forstnerič type (where the fibration is a submersion). The topological solution to the second Oka problem makes use of a special form of the Whitehead lemma for continuous functions developed in our work. In contrast to the standard form of Whitehead lemma, which is true for any ring, our special form is wrong for holomorphic functions. Unfortunately the work of Hultgren and Wold, who proved continuous factorization into unipotent factors, cannot be applied for our purposes. This is a work in progress, the higher rank bundles will be considered next.

## HAN PETERS

Title: History repeating?

Abstract: Some eight years ago, together with Erik Løw and two others, I worked on a problem motivated by accelerator physics. The main physical problem was be translated to a linear algebra problem, asking whether some explicit but potentially very large matrix with polynomial entries is invertible for generic values of the coefficients. The problem was never solved in full generality. Now, a different physical motivation leads to another question regarding matrices with polynomial entries. Will history repeat itself?

#### Alexander Rashkovski

Title: Asymptotic multiplicities and Monge-Ampère masses

Abstract: It was shown recently by Chi Li that there exist psh functions whose residual Monge-Ampère masses are not controlled by Demailly's (and any other analytic) approximations of the functions. We approach the control problem by studying graded systems of ideals and their asymptotic multiplier ideals. This gives us a sufficient condition for the analytic control over the residual masses of psh functions. The talk is based on a joint paper with Dano Kim (Seoul National University).

#### JEAN RUPPENTHAL

Title: About "canonical" sheaves at isolated singularities: coherence and explicit realisation

Abstract: The canonical line bundle and the corresponding canonical sheaf belong to the most important geometric/analytic objects associated to a complex manifold. They play a crucial role e.g. in classification theory, Serre duality or vanishing theorems. If we consider singular varieties instead of smooth manifolds, then there exist various possibilities to generalize the canonical sheaf to that setting. One can consider for example the Grothendieck(-Barlet-Henkin-Passare) dualizing sheaf or the Grauert-Riemenschneider  $L^2$ -sheaf. In this talk, we will discuss another possible generalization, i.e., the sheaf of  $L^2$  holomorphic n-forms with a certain boundary condition at the singular set. This sheaf is essential for  $L^2 - \overline{\partial}$ theory on singular spaces, but difficult to understand. I will present a new and surprisingly simple proof of its coherence at isolated singularities and describe it (more or less) explicitly for isolated canonical Gorenstein singularities, particularly in dimension two (DuVal singularities).

## Josua Schott

Title: Factorization of holomorphic mappings into the symplecic group.

Abstract: A symplectic matrix M whose entries are holomorphic functions on a reduced Stein space X can be decomposed into a finite product of elementary symplectic matrices if and only if M is null-homotopic. The proof is very geometric and I want to sketch how an application of the wonderful Oka principle makes this result possible

#### Weixia Zhu

# Title: Spectral stability of the complex Laplacians

Abstract: In physical sciences, exact values are oftentimes difficult—in some cases, impossible—to obtain, and approximate values are observed and utilized instead. It is thus important to know whether a quantified under-study remains stable when other parameters are slightly perturbed. In this talk, we will discuss the stability of complex Laplacians' spectrum when the underlying analytic or geometric structures are slightly perturbed. In particular, we study spectral stability of the Kohn Laplacian and related it to stability of embeddability of CR manifolds as further questions. This talk is based on joint work with Siqi Fu and Howard Jacobowitz.

# Per Åhag

Title: On the regularity of the complex Hessian equation

Abstract: We shall investigate the regularity of a solution to the Dirichlet problem for the complex Hessian equation, which has a density of the *m*-Hessian measure that belongs to  $L^q$ , for  $q \leq \frac{n}{m}$ . This is joint work with Rafal Czyz.