Conference: Mapping problems and Complex manifolds in projective spaces

Department of Mathematics, University of Oslo, Norway
3–7 December 2018
Venue: Clarion Collection Hotel Gabelshus, Gabels gate 16, 0272 Oslo
Organisers: John Erik Fornæss (NTNU, Norway), Tuyen Trung Truong (UiO, Norway) and Erlend Fornæss Wold (UiO, Norway)

Titles and Abstracts of invited talks

(in alphabet order of the speakers' names)

- Antonio Alarcón (Universidad de Granada, Spain)
  Title: Complete complex submanifolds in the ball
  Abstract: We shall prove that, for any pair of integers n and q with $1 \leq q < n$, any smooth complete closed complex submanifold V of pure codimension q in the open unit ball $\mathbb{B}_n$ of $\mathbb{C}^n$ is, under necessary assumptions, a fibre of a holomorphic submersion from $\mathbb{B}_n$ to $\mathbb{C}^q$ all of whose fibres are complete. Moreover, we shall show that every complete closed complex hypersurface in $\mathbb{B}_n$ is defined by a holomorphic function on $\mathbb{B}_n$ all of whose level sets are complete.

- Eric Bedford (Stony Brook University, USA)
  Title: Reinhard domains in Complex Dynamics
  Abstract: We discuss the dynamics of complex Henon Maps and the rotation domains they produce.

- Judith Brinkschulte (University of Leipzig, Germany)
  Title: On CR manifolds with mixed Levi signature
  Abstract: So-called q-pseudoconcave CR manifolds (of arbitrary codimension) arise naturally in mathematics. Their Levi-forms have eigenvalues of different signs. I will discuss some global properties of such CR manifolds, in particular global CR embeddability results.

- Barbara Drinovec Drnovšek (University of Ljubljana, Slovenia)
  Title: Proper holomorphic curves attached to domains
  Abstract: The image of a proper holomorphic map from an open Riemann surface is an analytic subvariety of the ambient space which is called a proper holomorphic curve. We present sufficient conditions for the existence of proper holomorphic curves which intersect a compact set L in a complex manifold X exactly at a given point. This is joint work with Marko Slapar.

- Peter Ebenfelt (UC San Diego, USA)
  Title: Local classification of real hypersurfaces in $\mathbb{C}^2$
  Abstract: The local classification of real hypersurfaces of finite type in $\mathbb{C}^2$ is by now well understood by work of E. Cartan, Tanaka, Chern–Moser, Kolar. In this talk, we shall discuss the classification of the remaining class, namely those of infinite type. This is more

subtle than the finite type case, and the classification in some sense parallels the Poincaré-Dulac classification of singular vector fields. We find the appearance of resonances and Stokes phenomena. This is joint work with Kossovskiy and Lamel.

- Charles Favre (École Polytechnique, Paris, France)
  Title: Entropy of endomorphisms over arbitrary fields
  Abstract: The entropy of endomorphisms of the complex projective space was computed by Gromov. I will discuss the problem of estimating the entropy of such maps over arbitrary non-Archimedean fields.

- Xiaojun Huang (Rutgers University, USA)
  Title: Regular finite type conditions for pseudoconvex real hypersurfaces in $\mathbb{C}^n$ and the Bloom conjecture
  Abstract: This is a joint work with Wanke Yin. We discuss the connection between the type defined by the Lie-Bracket of vector fields and the finite type condition in terms of the trace of Levi form; and provide a partial solution to an old conjecture of Bloom asked about 40 years ago.

- Jakob Hultgren (Universitetet i Oslo, Norway)
  Title: Coupled Kähler-Einstein metrics
  Abstract: An important theme in complex algebraic geometry is to study various types of canonical metrics, for example Kähler-Einstein metrics and cscK metrics. This seminar will be about a new type of canonical metrics called coupled Kähler-Einstein metrics, introduced by David Witt Nyström and myself. I will present general existence and uniqueness results and connections to algebraic geometry and, if time permits, zoom in on the toric case.

- Lucas Kaufmann (National University of Singapore)
  Title: Dynamics of correspondences on Riemann Surfaces
  Abstract: Let X be a compact Riemann surface. A (holomorphic) correspondence f on X is a holomorphic multi-valued map from X to itself. Each point of X has d images and d' pre-images counting multiplicity.

  As in the case of maps we can iterate f and study its dynamics. When d and d' are different the global dynamics of f is well understood and f admits a canonical invariant measure with many good properties.

  In this talk I intend to present some results concerning the case d = d'. Surprisingly, under a mild and necessary condition, f admits two canonical measures that share the role of the global description of the system. As an application, we can consider the action of subgroups of PSL(2,C) on $\mathbb{P}^1$ and recover some classical results about random products of matrices.

  This is a joint work with Tien Cuong Dinh and Hao Wu.

- Frank Kutzschebauch (Universitaat Bern, Switzerland)
  Title: Embedding Riemann surfaces with isolated punctures into $\mathbb{C}^2$
  (Joint work with Pierre-Marie Poloni)
  Abstract: We enlarge the class of open Riemann surfaces known to be holomorphically embeddable into the plane by allowing them to have additional isolated punctures compared to the known embedding results.

Theorem 0.1. The following open Riemann surfaces admit a proper holomorphic embedding into $\mathbb{C}^2$:

– the Riemann sphere with a (nonempty) countable closed subset with at most 2 accumulation points removed,

– any compact Riemann surface of genus 1 (torus) with a (nonempty) closed discrete set with at most one accumulation point removed,

– any hyperelliptic Riemann surface with a discrete closed set C removed with the properties that C contains a fibre $F = R^{-1}(p)$ (consisting either of two points or a single Weierstrass point) of the Riemann map R and all accumulation points of C are contained in that fibre F.

The same holds if X is as above with additionally a finite number of smoothly bounded regions removed.

The second and the third case with no accumulation points in the closed discrete set correspond to the Theorem of Sathaye.

- Finnur Lárusson (University of Adelaide, Australia)
  Title: Chaotic holomorphic automorphisms of Stein manifolds with the volume density property
  Abstract: I will report on joint work with Leandro Arosio. Let X be a Stein manifold of dimension $n \geq 2$ satisfying the volume density property with respect to an exact holomorphic volume form. For example, X could be $\mathbb{C}^n$, any connected linear algebraic group that is not reductive, the Koras-Russell cubic, or a product $Y \times \mathbb{C}$, where Y is any Stein manifold with the volume density property. We prove that chaotic automorphisms are generic among volume-preserving holomorphic automorphisms of X. In particular, X has a chaotic holomorphic automorphism. Fornæss and Sibony proved (but did not explicitly state) this for $X = \mathbb{C}^n$ in 1997. We follow their approach closely. Peters, Vivas, and Wold showed that a generic volume-preserving automorphism of $\mathbb{C}^n$, $n \geq 2$, has a hyperbolic fixed point whose stable manifold is dense in $\mathbb{C}^n$. This property can be interpreted as a kind of chaos. We generalise their theorem to a Stein manifold as above.

- Viet-Anh Nguyen (The University of Lille, France)
  Title: Upper bound for the number of periodic points for holomorphic maps
  Abstract: Using the super-potentials and the densities of positive closed currents, we obtain a sharp upper bound for the number of isolated periodic points of holomorphic maps on compact Kahler manifolds whose actions on cohomology are simple. This is a joint-work with Tien-Cuong Dinh (National University of Singapore) and Duc-Viet Vu (University of Cologne).

- John Christian Ottem (Universitetet i Oslo, Norway)
  Title: Positive vector bundles and submanifolds
  Abstract: The talk will survey various positivity properties of vector bundles and subschemes on projective varieties. I will also explain how this theory can be used to construct a counterexample to a problem of Andreotti-Grauert.

- Jasmin Raissy (Universite Paul Sabatier, France)
  Title: Stable manifolds for germs of biholomorphisms
  Abstract: Given a germ of a holomorphic diffeomorphism $F \in \mathrm{Diff}(\mathbb{C}^2, 0)$ with an invariant formal curve Γ, we are interested in understanding under which hypotheses one can

ensure the existence of orbits converging to the origin tangentially to the formal curve Γ. I will show that it is enough to assume that the restricted diffeomorphism $F|_\Gamma$ is either hyperbolic attracting or rationally neutral non-periodic (which are the conditions that the diffeomorphism it should satisfy, if Γ were convergent, in order to have orbits converging to the origin). Moreover it turns out that F has finitely many stable manifolds, either open domains or parabolic curves, consisting of and containing all converging orbits asymptotic to Γ. These results generalize to the case when Γ is a formal periodic curve of F. (Joint work with L. Lopez-Hernanz, J. Ribon and F. Sanz-Sanchez.)

- Tyson Ritter (Universitetet i Stavanger, Norway)
  Title: Carleman approximation by holomorphic automorphisms of Danielewski surfaces
  Abstract: In 2014, Kutzschebauch and Wold proved a remarkable result on the Carleman approximation of smooth isotopies of non-compact totally real submanifolds of $\mathbb{C}^n$, by holomorphic automorphisms of $\mathbb{C}^n$. I will begin by explaining their result and some of the main ingredients in the proof, and then discuss how their argument can be adapted to give Carleman approximation of smooth isotopies of non-compact curves in a complex Danielewski surface, by holomorphic automorphisms of the Danielewski surface. This result has an application in the construction of proper holomorphic embeddings of open Riemann surfaces into complex Danielewski surfaces. This is joint work with Andrist.

- Avinash Sathaye (University of Kentucky, USA)
  Title: Epimorphism Problems
  Abstract: Given a polynomial ring $A_n = k^{[n]}$ over a field k, a hypersurface $F \in A_n$ is said to be **an abstract hyperplane** if its coordinate ring $A_n/(F)$ is isomorphic to $k^{[n-1]}$- the coordinate ring of a hyperplane. Moreover F is said to be a **coordinate hyperplane** if there are polynomials $G_1, \cdots, G_{n-1}$ such that $A = k[F, G_1, \cdots, G_{n-1}]$.

  The conjecture named after Abhyankar-Sathaye proposes that every abstract hyperplane is a coordinate hyperplane when k has characteristic zero.

  While trivial for $n = 1$, the nontrivial proof of the conjecture for $n = 2$ by Abhyankar and Moh inspired several advances in Affine Algebraic Geometry, including extensive theory of affine curves with one place at infinity. The conjecture is intimately connected with the problem of identifying the structure of the automorphism group of A_n for $n \geq 3$.

  We shall discuss known cases of the conjecture for $n = 3$ and ideas for further progress.

- Nikolay Shcherbina (University of Wuppertal, Germany)
  Title: On core and hyperbolicity of some model domains.
  Abstract: We prove that a pseudoconvex domain D in $\mathbb{C}^2$ of the form $D = \{(z, w) : v > G(z, u)\}$, where $w = u + iv$ and G is continuous on $\mathbb{C}_z \times \mathbb{R}_u$, has an empty core (and, therefore, Kobayashi hyperbolic) if and only if the graph $\Gamma(G)$ contains no graphs $\Gamma(g)$ of entire functions g of the form $w = g(z)$. We also discuss similar questions in higher dimensions.

- Nessim Sibony (Universite Paris-Sud (Orsay), France)
  Title: Unique ergodicity for generic foliations of Kähler surfaces
  Abstract: Consider a foliation in the projective plane admitting a unique invariant algebraic curve. Assume that the foliation is generic in the sense that its singular points are hyperbolic. With T.C Dinh, we showed that there is a unique positive dd^c-closed (1,1)-current of mass 1 which is directed by the foliation. This is the current of integration on the invariant

curve. A unique ergodicity theorem for the distribution of leaves follows: for any leaf L, appropriate averages on L converge to the current of integration on the invariant curve. The result uses an extension of our theory of densities for currents. We will describe recent extensions, with T.C Dinh and V.A Nguyen, dealing with foliations on compact Kähler surfaces. Foliations with or without invariant curves are now considered. This improves on previous results, with J. E. Fornæss, for foliations (without invariant algebraic curves) on the projective plane..

- Duc-Viet Vu (University of Cologne, Germany)
  Title: Equidistribution of Fekete points of large order
  Abstract: Let K be a compact with piecewise smooth boundary in n-dimensional Euclidean space and N_k the dimension of the vector space of the restrictions of real polynomials of n variables to K. A Fekete point of order k is a point in K^{N_k} maximising the determinant of a certain matrix of Vandermonde type. These points are important in the interpolation problem of continuous functions on K by polynomials. By using deep tools from complex geometry, pluripotential theory and the Cauchy-Riemann geometry, we prove that Fekete points of large order are equidistributed toward a canonical measure associated to K with an explicit speed of convergence. Such a equidistribution is crucial for approximation of Fekete points in practice.

- Paul Arne Østvær (Universitetet i Oslo, Norway)
  Title: A^1-contractible varieties
  Abstract: Motivic homotopy theory gives a way of viewing algebraic varieties and topological spaces as objects in the same category, where homotopies are parametrised by the affine line. In particular, there is a notion of A^1-contractible varieties. Affine spaces are A^1-contractible by denition. The Koras-Russell threefold KR dened by the equation

$$x + x^2y + z^2 + t^3 = 0$$

  in A^4 is the first nontrivial example of an A^1-contractible smooth affine variety. We will discuss this example in some detail, and speculate on whether one can use motivic homotopy theory to distinguish between KR and A3.
  References:
  A. Dubouloz, J. Fasel: Families of A^1-contractible affine threefolds, Algebr. Geom. 5, 1–14 (2018).
  M. Hoyois, A. Krishna, P. A. Østvær: A^1-contractibility of Koras-Russell threefolds, Algebr. Geom. 3, 407–423 (2016).
  F. Morel, V. Voevodsky: A^1-homotopy theory of schemes, Inst. Hautes Etudes Sci. Publ. Math. 90, 45-143 (2001).
