The Regge finite element family and its applications

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Regge calculus, proposed by Tulio Regge in 1961, is an elegant discretization of general relativity, akin to a finite difference method. It may also be viewed as a finite element method for the Einstein equations, using piecewise constant elements. Despite its low order, the Regge calculus is consistent, as shown by Cheeger et al in 1984. Viewed as a finite element method, it is nearly a conforming method, as shown by Christiansen in 2011. We will describe a recent new family of finite elements, the generalized Regge elements, extending the Regge elements to all polynomial orders and valid in any number of dimensions. These are structure-preserving finite elements for second-order covariant tensors, such as metrics. We will discuss their implementation and performance and applications in mechanics, where they connect to the Hellan-Herman-Johnson elements for plates and the TDNNS elements for elasticity. Returning to the original application of general relativity, we will give evidence that the Regge calculus approach, though consistent, is not stable. This work comes from the thesis of Lizao Li.