Best space of a fixed dimension

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Consider the space $\mathbb{S}_{p,k,n}$ of piecewise polynomials $[0,1] \to \mathbb{R}$ of degree p on the uniform partitions in n segments and having k continuous derivatives. Let $C_{p,k,n}$ be the best constant such that: $\forall f \in H^{p+1}(0,1)$

$$||f - \Pi^{\perp} f||_{L^2} \le C_{p,k,n} ||\partial^{p+1} f||_{L^2}.$$

Which pair of parameters (k, n) minimizes $C_{p,k,n}$ for a fixed degree and space dimension? We can not answer the question completely. However, we can compare the interesting cases k = -1 used in DG, k = 0 used in FEM and k = p - 1 used in IGA. The answer extends to broken Sobolev spaces and to tensor product spaces.