Regular Decompositions:
Discrete, Boundary-Aware, and p-Stable

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The term Helmholtz-type decomposition of $\mathbf{H}(\mathbf{curl},\Omega)$, $\Omega \subset \mathbb{R}^3$, refers to stable splittings of the form

$$\mathbf{H}(\mathbf{curl},\Omega) = (H^1(\Omega))^3 + \mathbf{grad}\, H^1(\Omega)\ .$$

These splittings have become a key tool in both the theoretical and numerical analysis of spaces of **curl**-conforming vectorfields, related variational problems, and the corresponding finite element spaces (edge elements).

Their discrete counterparts provide stable splittings of piecewise polynomial finite element subspaces of $\mathbf{H}(\mathbf{curl},\Omega)$ into (i) piecewise polynomial continuous vector fields on Ω, (ii) gradients of piecewise polynomial continuous scalar finite element functions, and (iii) a "small" remainder.

We show the existence of such decompositions for Nédélec's tetrahedral edge element spaces of any polynomial degree with stability depending only on the geometry of the domain and the shape regularity of the mesh. Our decompositions also respect homogeneous boundary conditions on a part of the boundary of Ω. Key tools for our construction are continuous regular decompositions, projection-based interpolation, and quasi-interpolation with low regularity requirements.

References

[1] R. Hiptmair and C. Pechstein, *Discrete regular decompositions of tetrahedral discrete 1-forms*, Tech. Rep. 2017-47, Seminar for Applied Mathematics, ETH Zürich, Switzerland, 2017. To appear in Radon Series on Computational and Applied Mathematics, De Gruyter, U. Langer and D. Pauly, eds.