Stabilizing a discontinuous Galerkin scheme for incompressible flow by means of divergence and jump penalties

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My talk will present our work on robust discontinuous Galerkin (DG) methods for the incompressible Navier-Stokes equations. The basic DG discretization uses standard techniques, such as the local Lax-Friedrichs flux for the convective term, the symmetric interior penalty method for the viscous term, and central fluxes for the velocity-pressure coupling terms. For flows that are only marginally resolved, this scheme becomes unstable because of a violation of the divergence-free condition and excessive jumps over element boundaries. To address this issue, we have developed suitable penalty terms that enforce the incompressibility constraint as well as inter-element continuity of the velocity field in a weak sense. I will discuss how these penalty terms imitate the mechanisms of the Hdiv-conforming Raviart-Thomas finite elements and link our method to the overall theme of the workshop.

The main motivation for retaining a fully discontinuous approximation space is the computational efficiency. Applying the penalty terms in a postprocessing step, for example in projection type solution methods, is compatible with fast matrix-free implementations and very attractive at high approximation orders. Since our approach is based on consistent penalty terms, it is also by definition generic and provides optimal rates of convergence when applied to laminar flow problems. We have verified the scheme on a series of underresolved test cases, such as the Orr-Sommerfeld stability problem or the viscous and inviscid Taylor-Green vortex problem.