On the role of the Helmholtz-Leray projector for a novel pressure-robust discretization theory for the incompressible Navier-Stokes equations

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The talk discusses several physical regimes of the incompressible Navier–Stokes equations with respect to the role of the pressure and the role of the Helmholtz– Leray projector in the Navier–Stokes momentum balance. It is emphasized that not the forces in the momentum balance themselves matter for the dynamics of the flow field, but their Helmholtz–Leray projector. Since the Helmholtz–Leray projector vanishes for arbitrary gradient fields, a semi norm and a corresponding equivalence class of forces play naturally a major role for the evolution of incompressible flows.

Novel pressure-robust mixed finite element methods are designed for an appropriate discrete treatment of this equivalence class of forces. On the contrary, classical, (only) inf-sup stable mixed methods do not care about the existence of such an equivalence class. In order to deliver accurate simulation results for the discrete velocities, they have to resort to expensive high order ansatz spaces, in order to reduce a corresponding consistency error of an appropriately defined discrete Helmholtz–Leray projector.

Finally, the talk will indicate suitable applications for efficient and accurate low-order pressure-robust mixed methods like flows with stagnation points.