Mixed finite element methods for intrinsic elasticity with defects

Kaibo Hu, Espen Sande and Ragnar Winther

When defects or other incompatibilities are present in elastic and plastic materials, it could be advantageous to use an intrinsic model of elasticity/plasticity where the strain tensor is a major variable. This new paradigm has a close relation with the elasticity complex. In this talk, we design and analyze a class of mixed finite element methods for such a model that involves the linearized curvature operator $\mathrm{inc} := \mathrm{curl}^{\mathrm{T}} \mathrm{curl}$. Specifically, we will look at the fourth order problem

$$\begin{aligned} \mathrm{inc}\,\mathrm{inc}\,\underline{\underline{E}} &= \underline{\underline{K}}, \\ \mathrm{div}\,\underline{\underline{E}} &= 0, \end{aligned}$$

where $\underline{\underline{E}}$ is a symmetric matrix field. The Bernstein-Gelfand-Gelfand (BGG) construction plays a vital role in the study of the proposed finite element methods.