The computational abilities of biological neurons and the relation to their neural network counterparts

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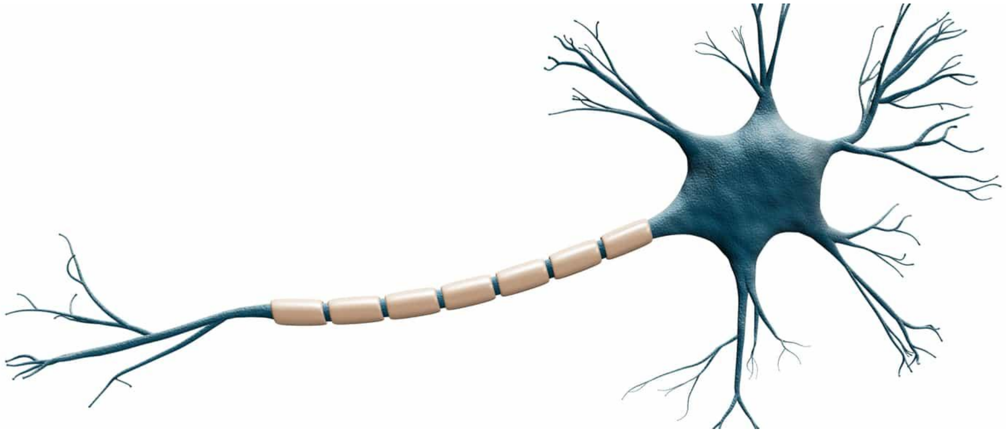
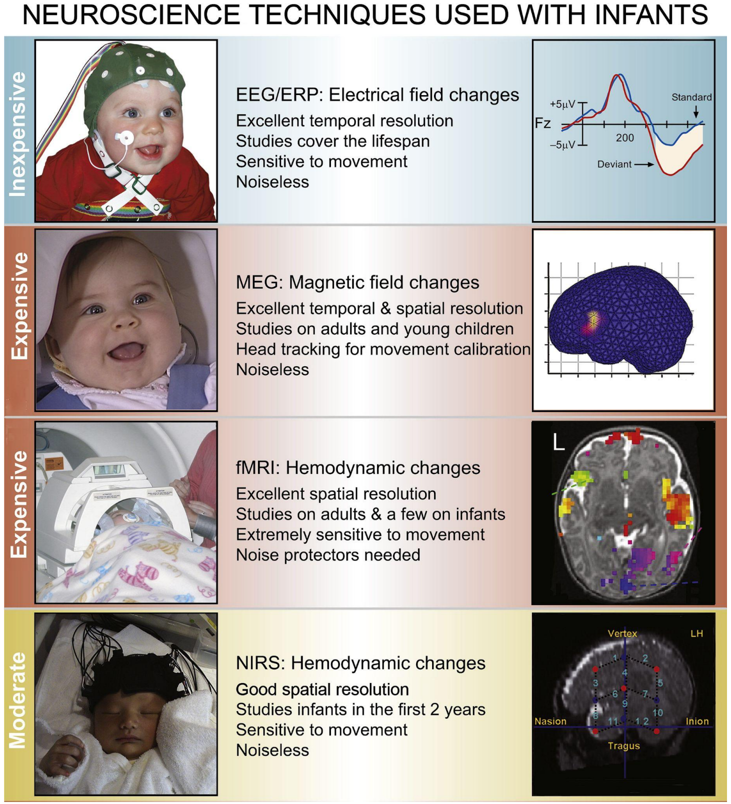
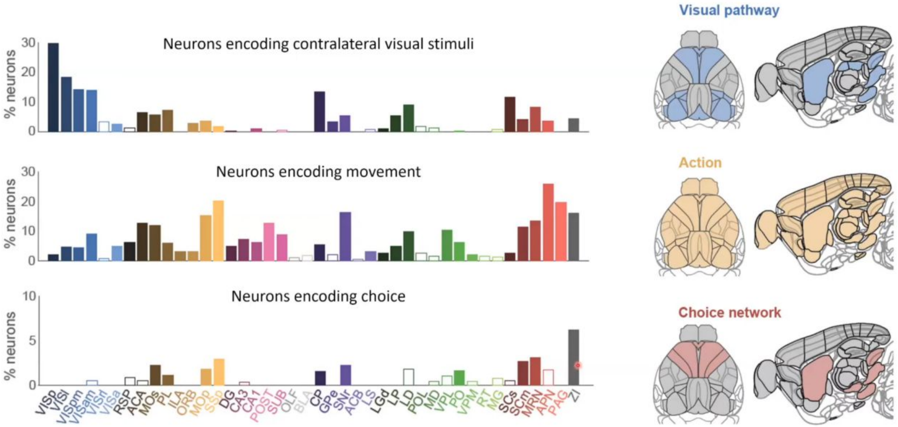
20/07/23



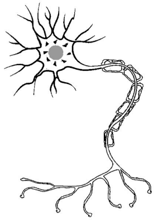
UiO : University of Oslo

This was derived from a collaboration between Tuyen and I where we looked at how children learn

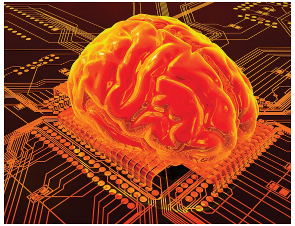




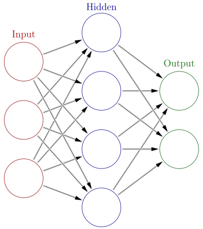
This talk explores the computational ability of neurons



Biological Neurons



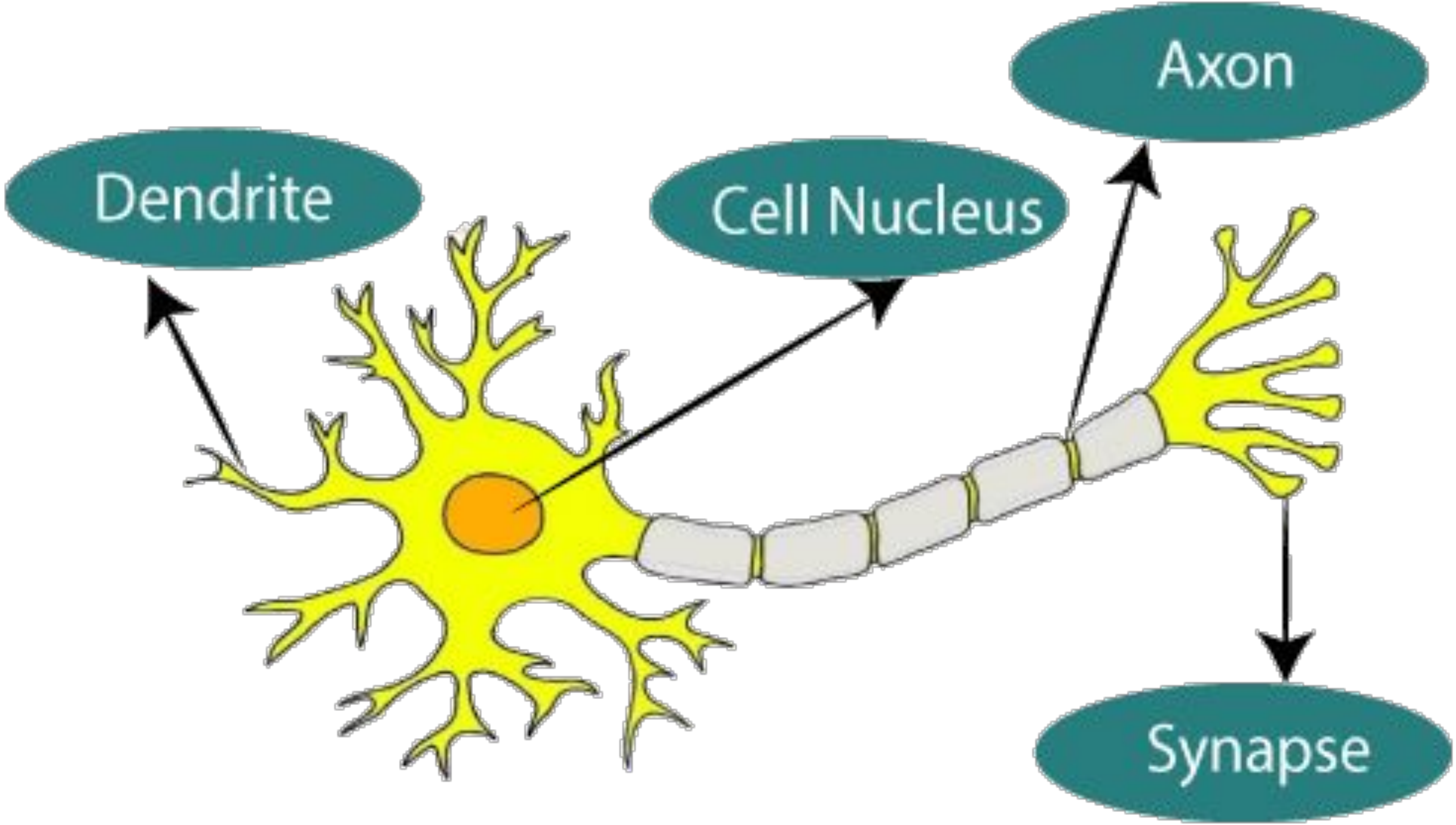
Computational Abilities



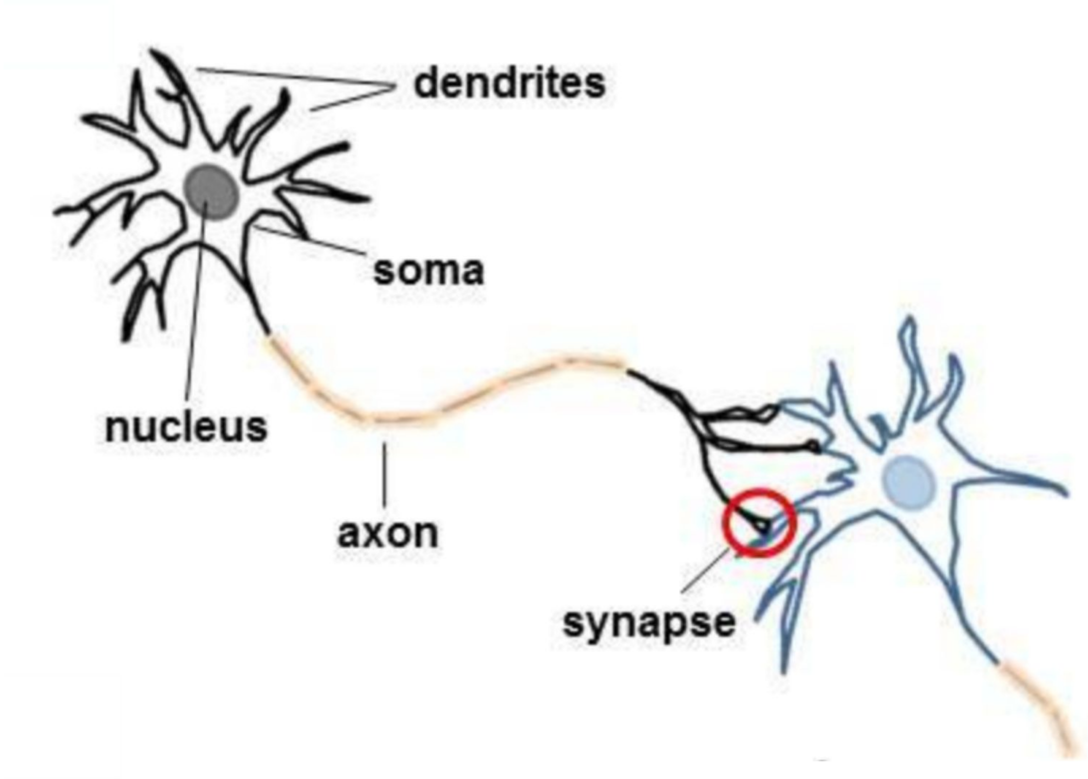
The Deep Learning Context

The neuron are the fundamental unit of the nervous system

The neuron have 4 main components



The synapse is a structure that allows the neuron to transmit a signal to a neighboring cell



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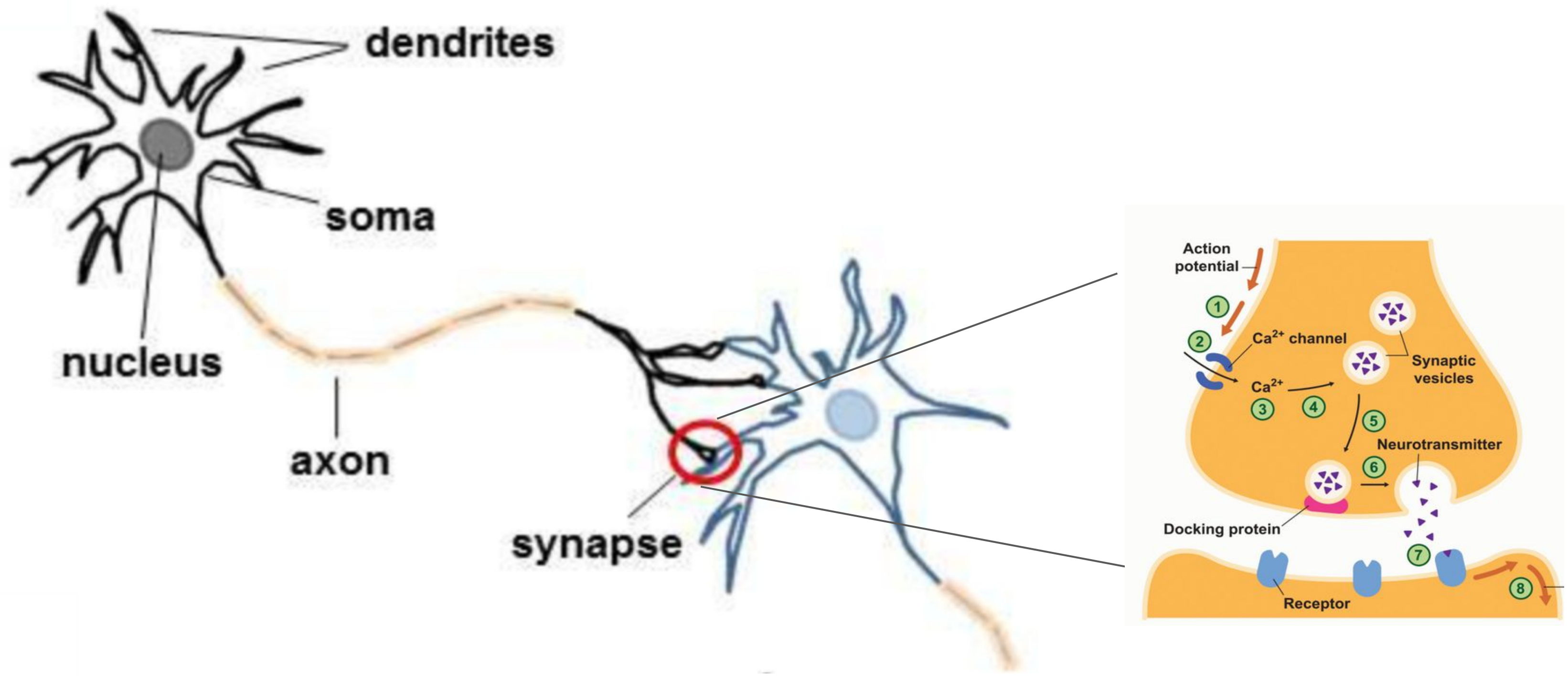
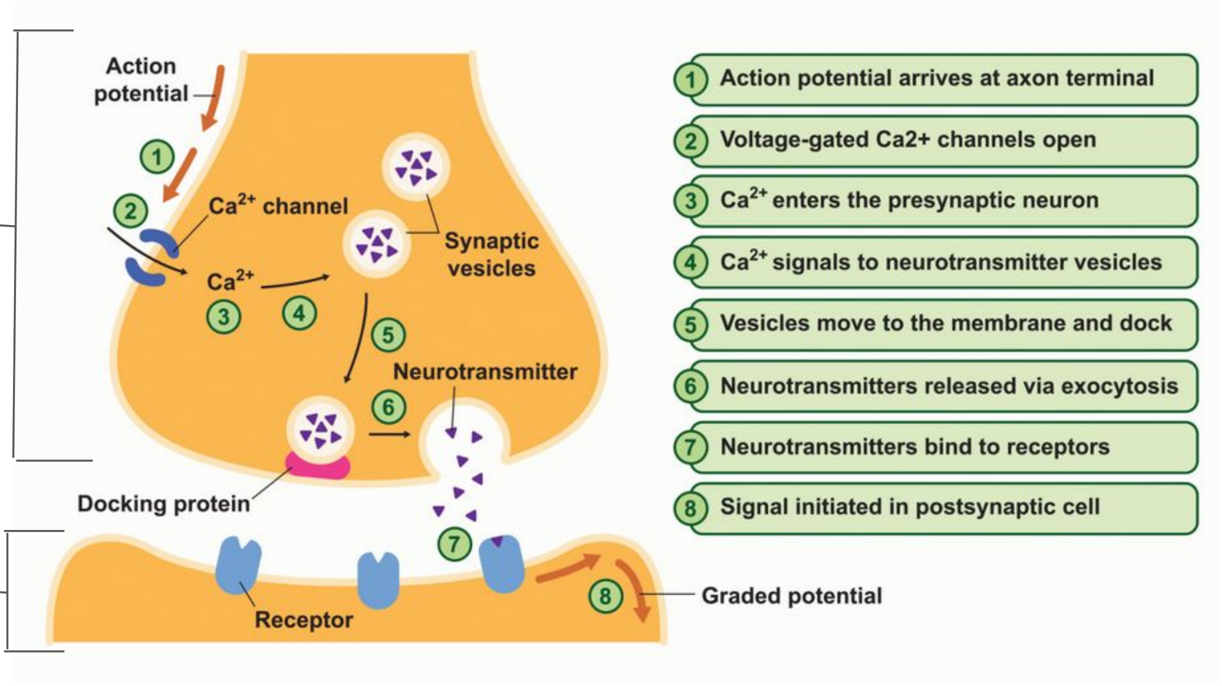


Chemical synapses transfers signals through a biochemical process

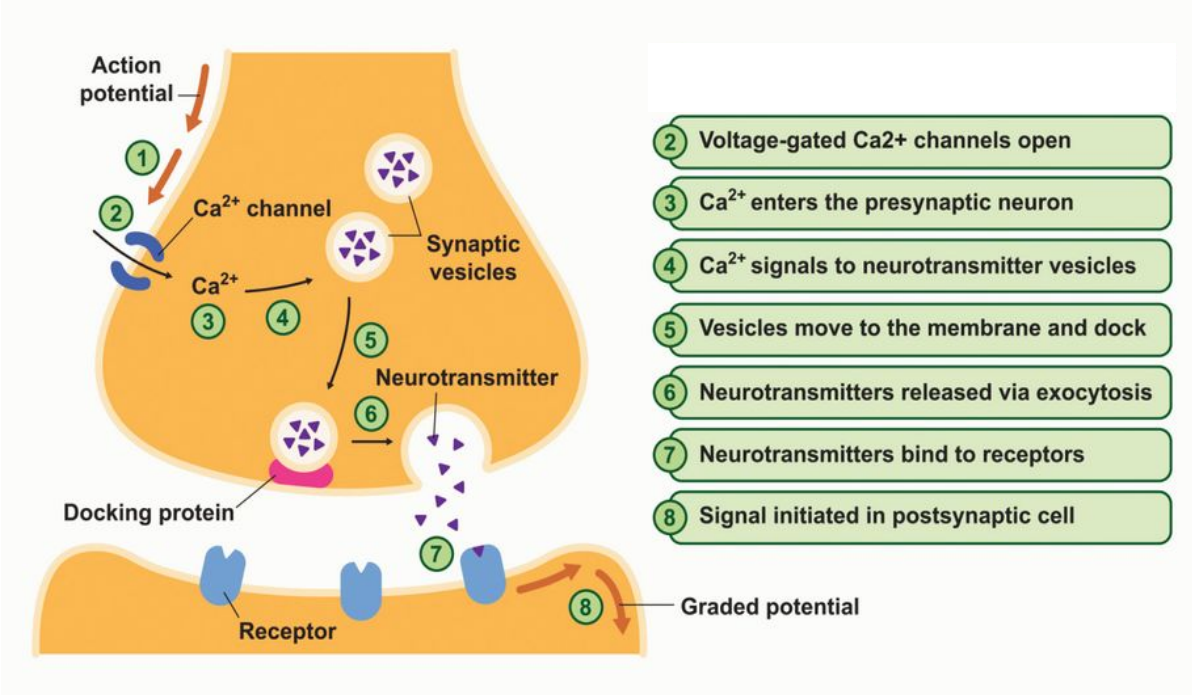
pre-synapse

post-synapse



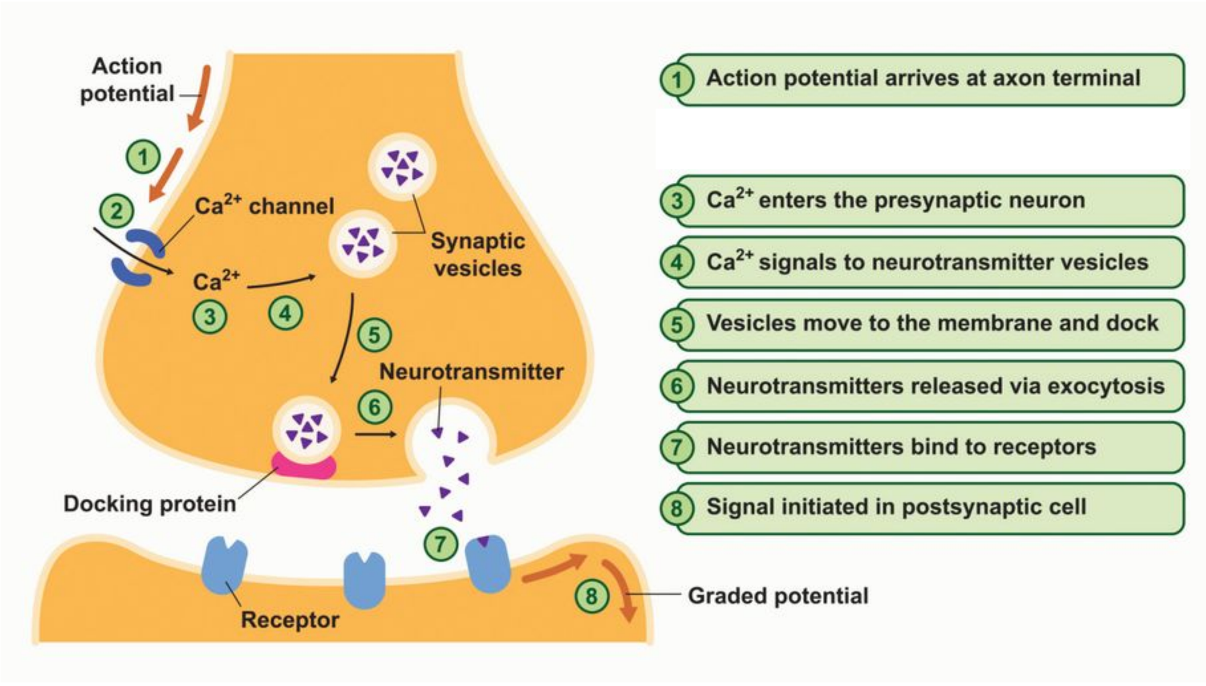
Chemical synapses transfers signals through a biochemical process

1 Action potential arrives at axon terminal



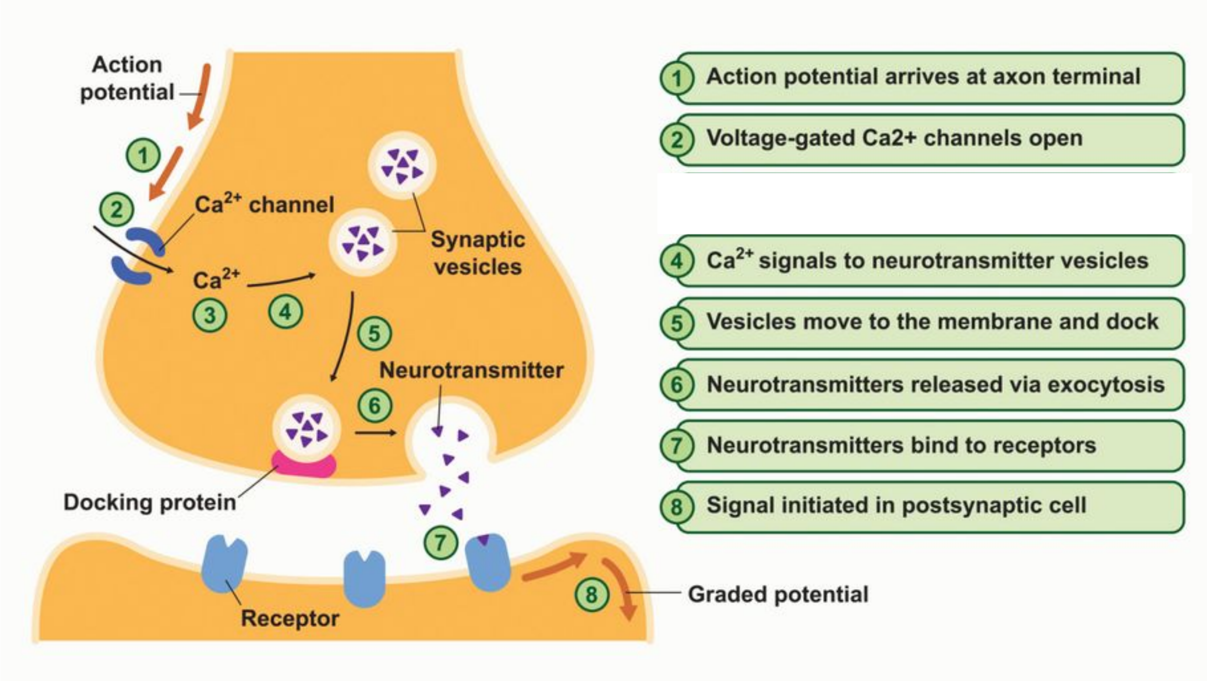
Chemical synapses transfers signals through a biochemical process

2 Voltage-gated Ca2+ channels open



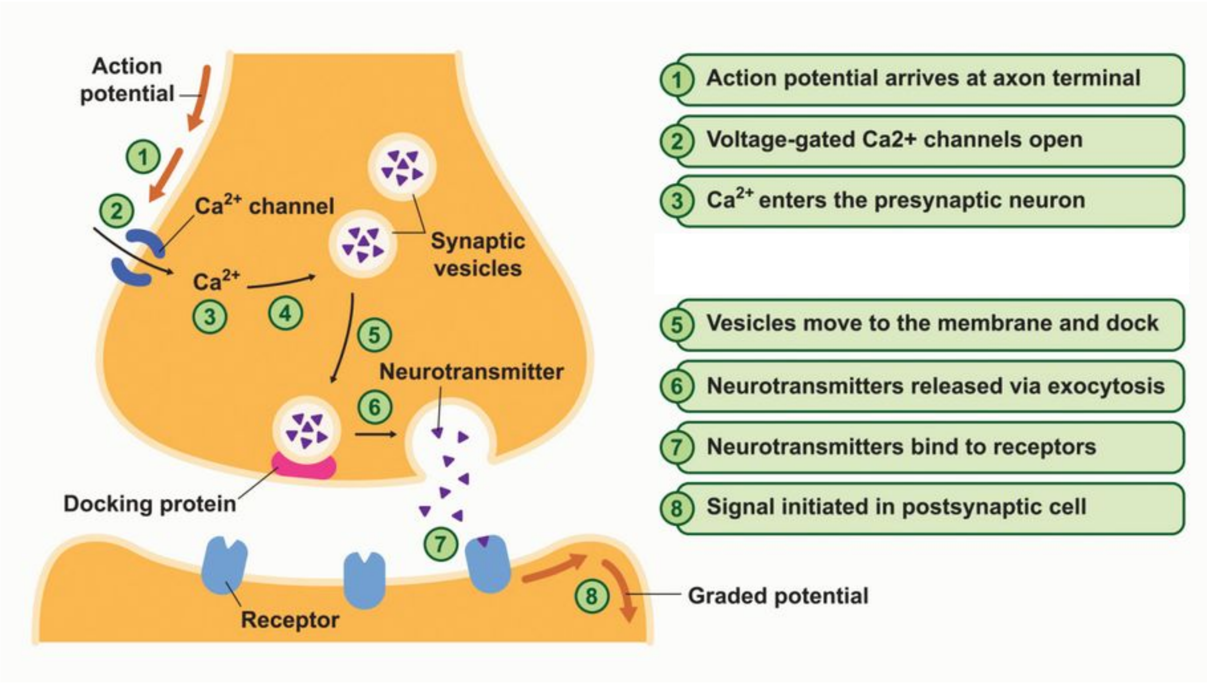
Chemical synapses transfers signals through a biochemical process

3 Ca^{2+} enters the presynaptic neuron



Chemical synapses transfers signals through a biochemical process

4 Ca^{2+} signals to neurotransmitter vesicles



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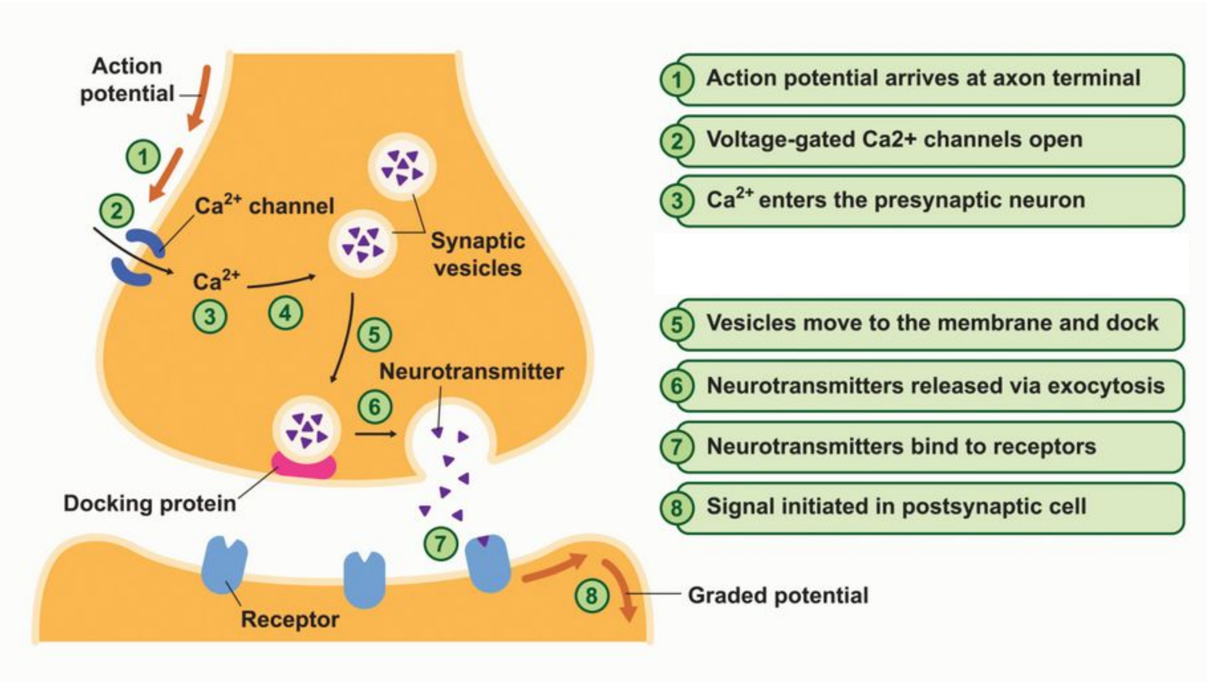
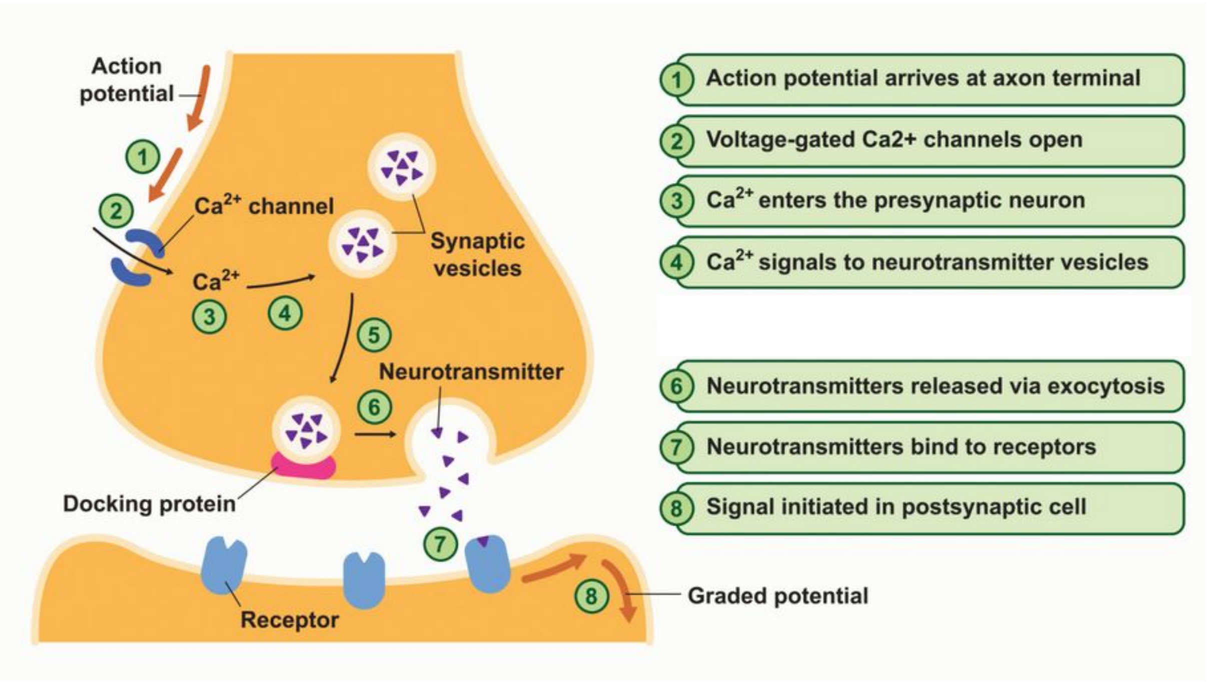


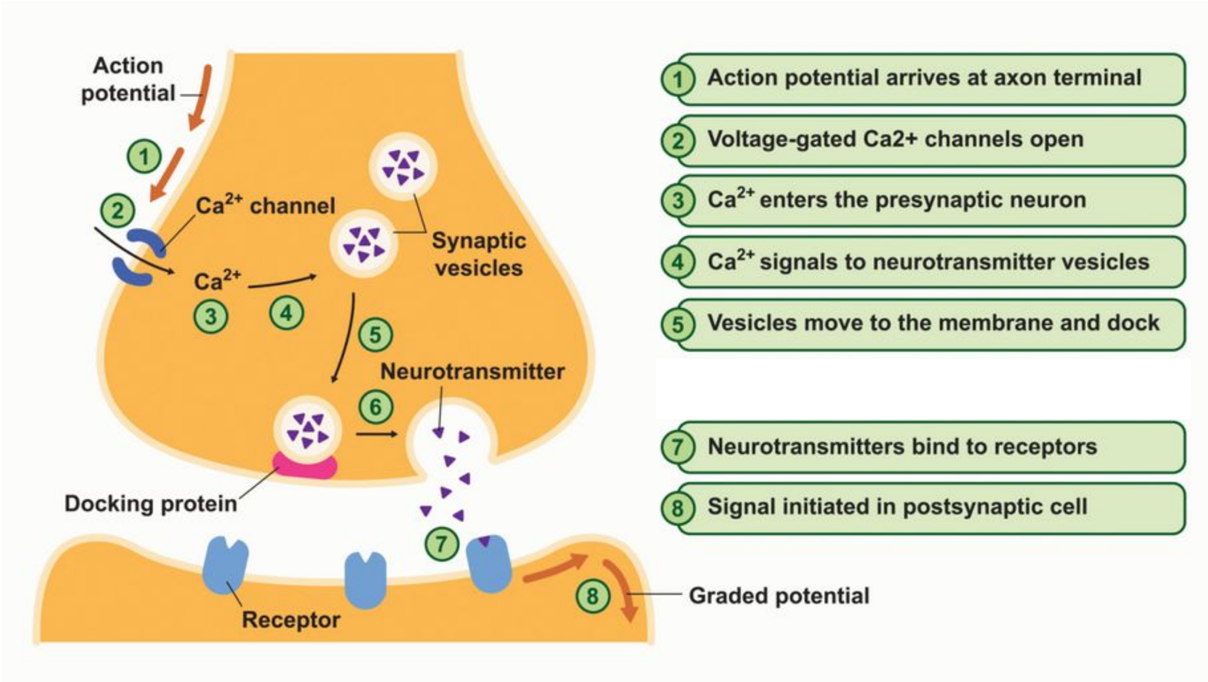
Chemical synapses transfers signals through a biochemical process

5 Vesicles move to the membrane and dock



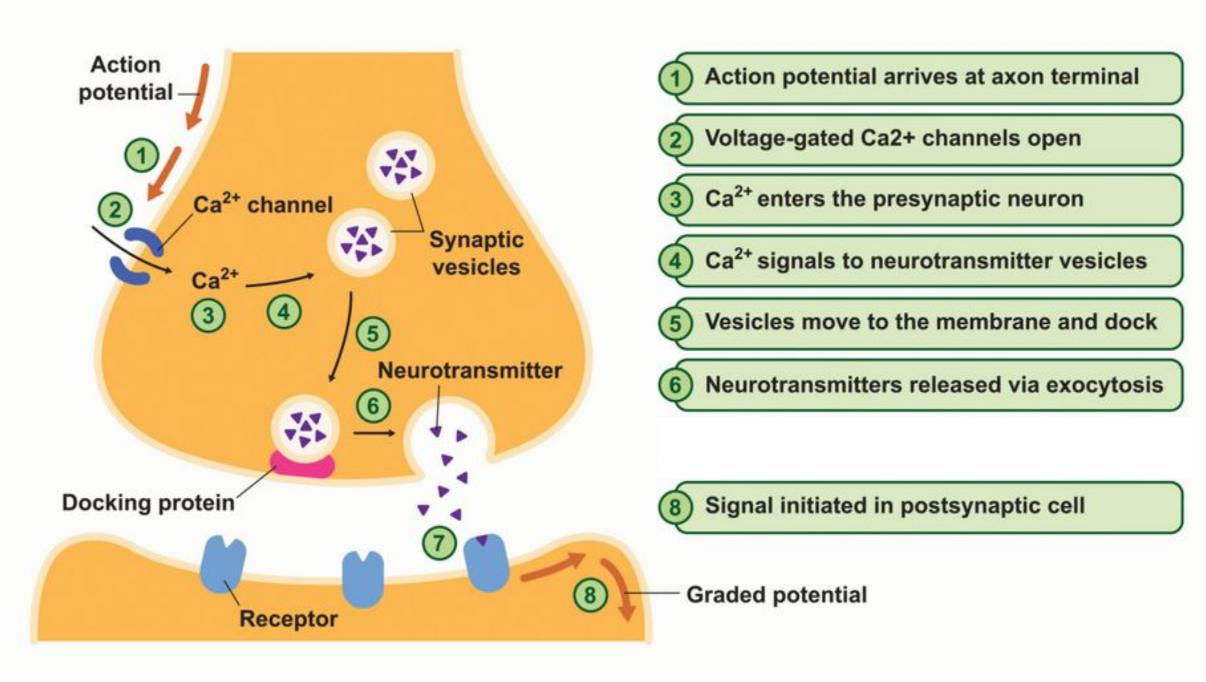
Chemical synapses transfers signals through a biochemical process

6 Neurotransmitters released via exocytosis



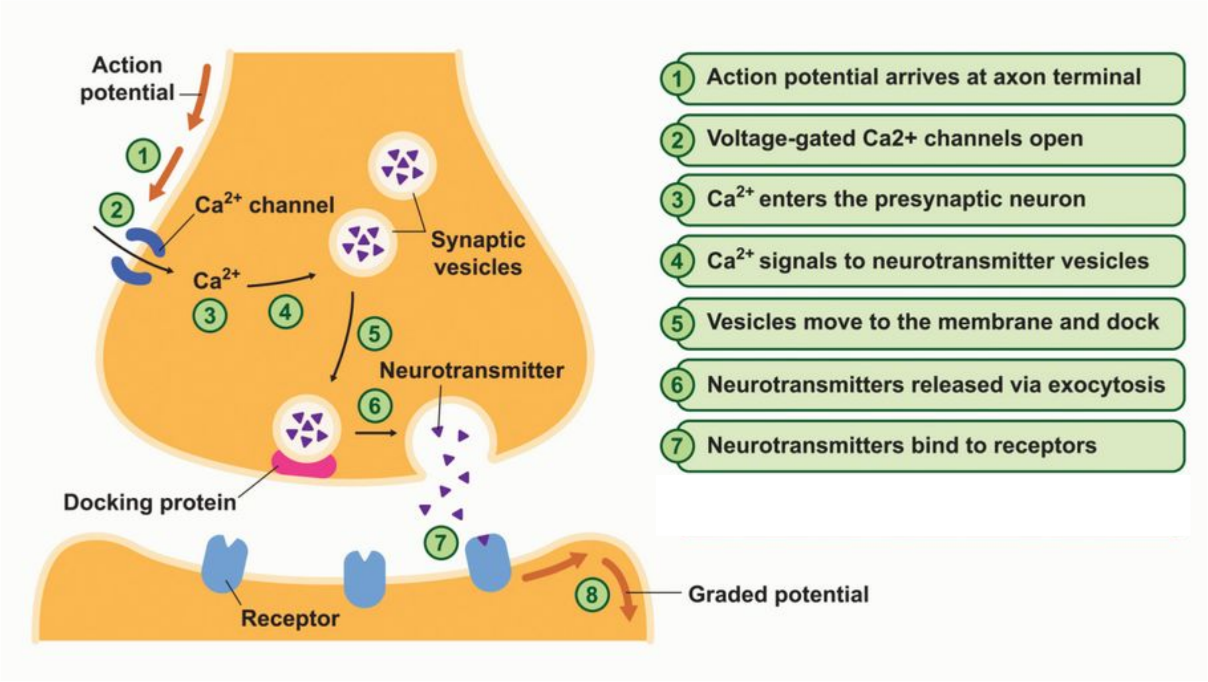
Chemical synapses transfers signals through a biochemical process

(7) Neurotransmitters bind to receptors

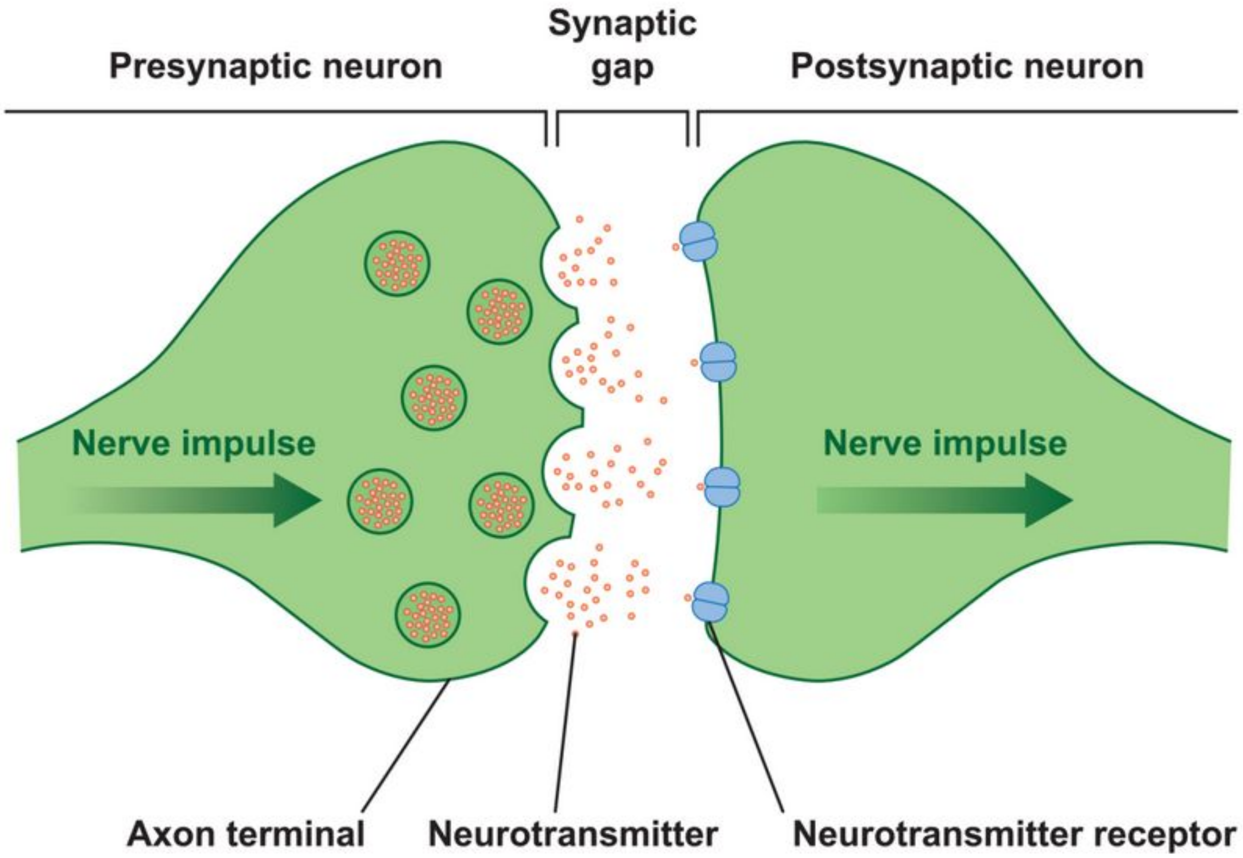


Chemical synapses transfers signals through a biochemical process

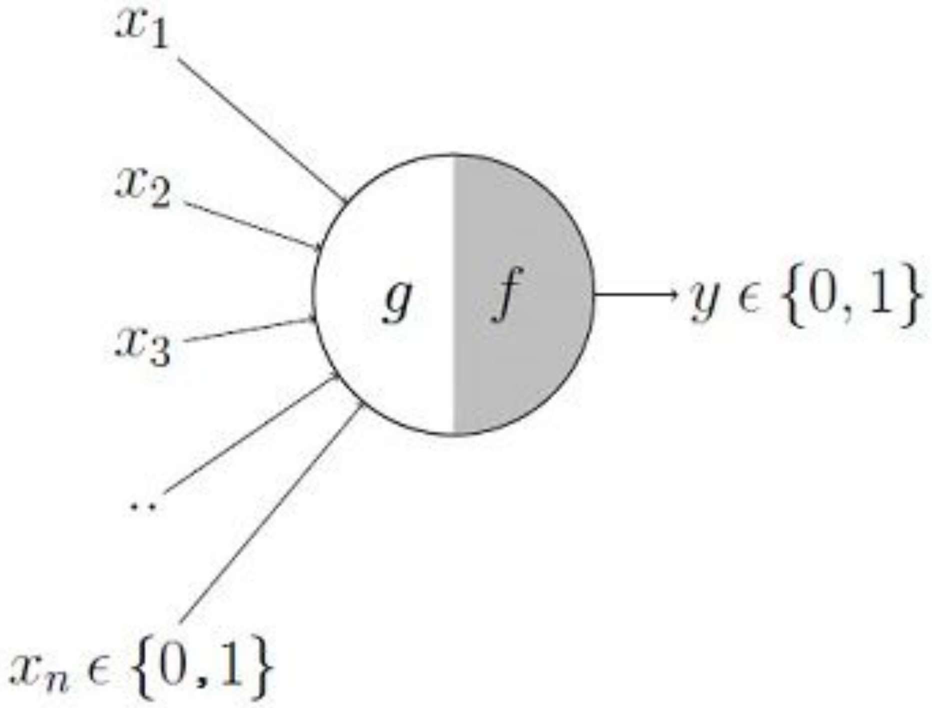
8 Signal initiated in postsynaptic cell



Synapses can also transmit signals electrically



The McCulloch-Pitts Neuron was the original mathemiccal neuron model



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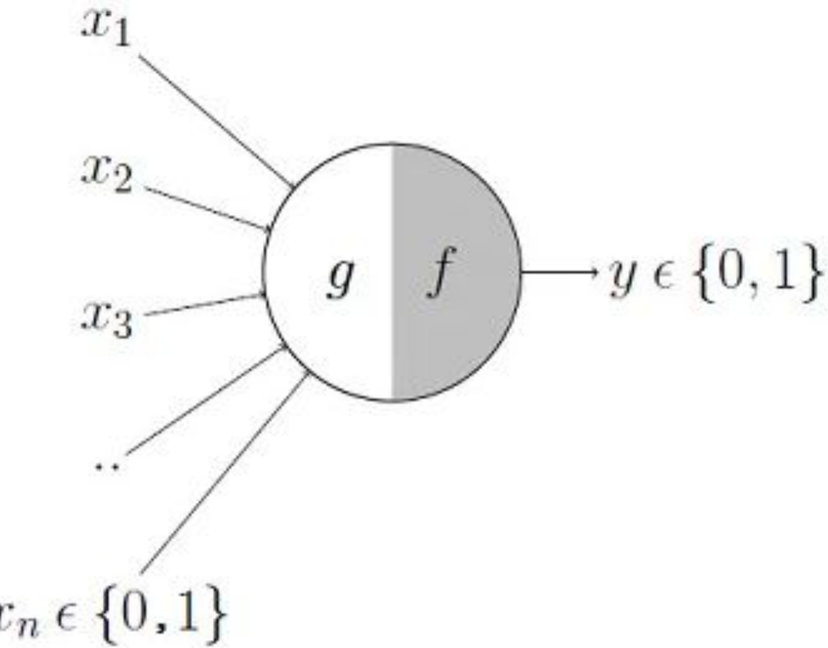
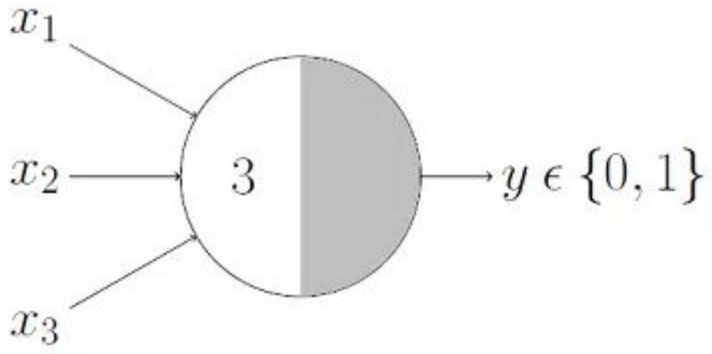


$$g(x_1, x_2, x_3, ..., x_n) = g(\mathbf{x}) = \sum_{i=1}^{n} x_i$$

$$\begin{aligned} y = f(g(\mathbf{x})) &= 1 \quad if \quad g(\mathbf{x}) \geq \theta \\ &= 0 \quad if \quad g(\mathbf{x}) < \theta \end{aligned}$$

You can easily represent AND and OR functions with this model



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$$\theta = 3$$

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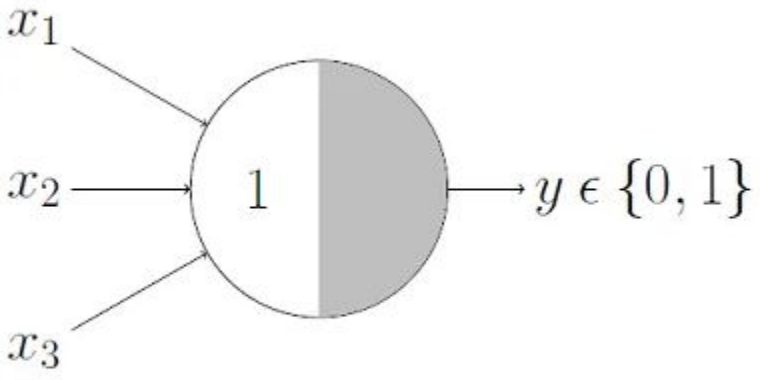
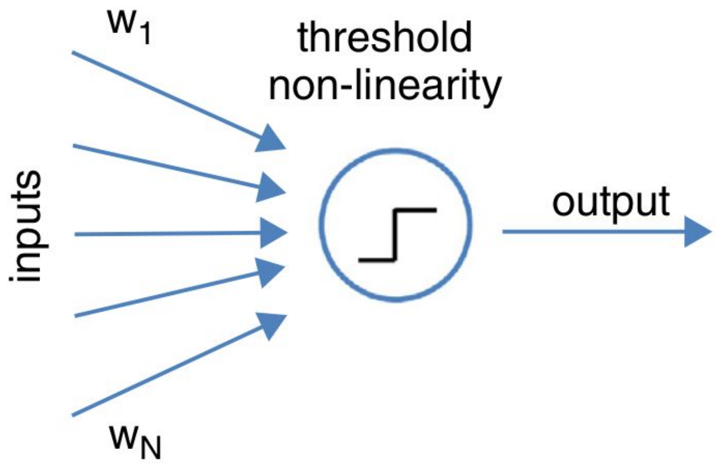
$$\theta = 1$$

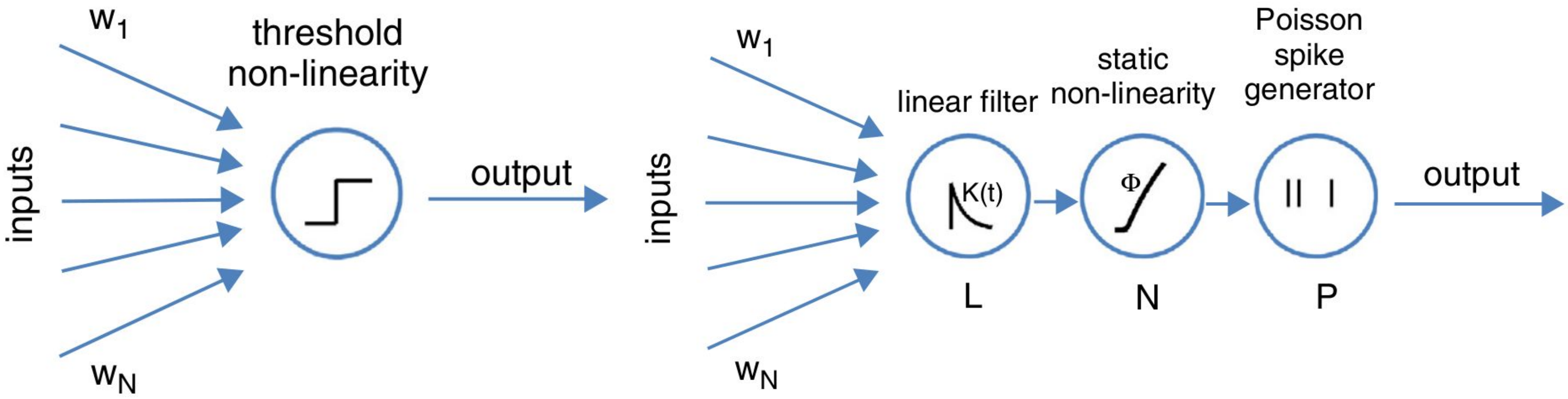


The single neuron model can be summarized a sum of inputs filtered by a threshold function



Brunel et.al, 2021

In modern neuron models the threshold function is replaced by a LNP model



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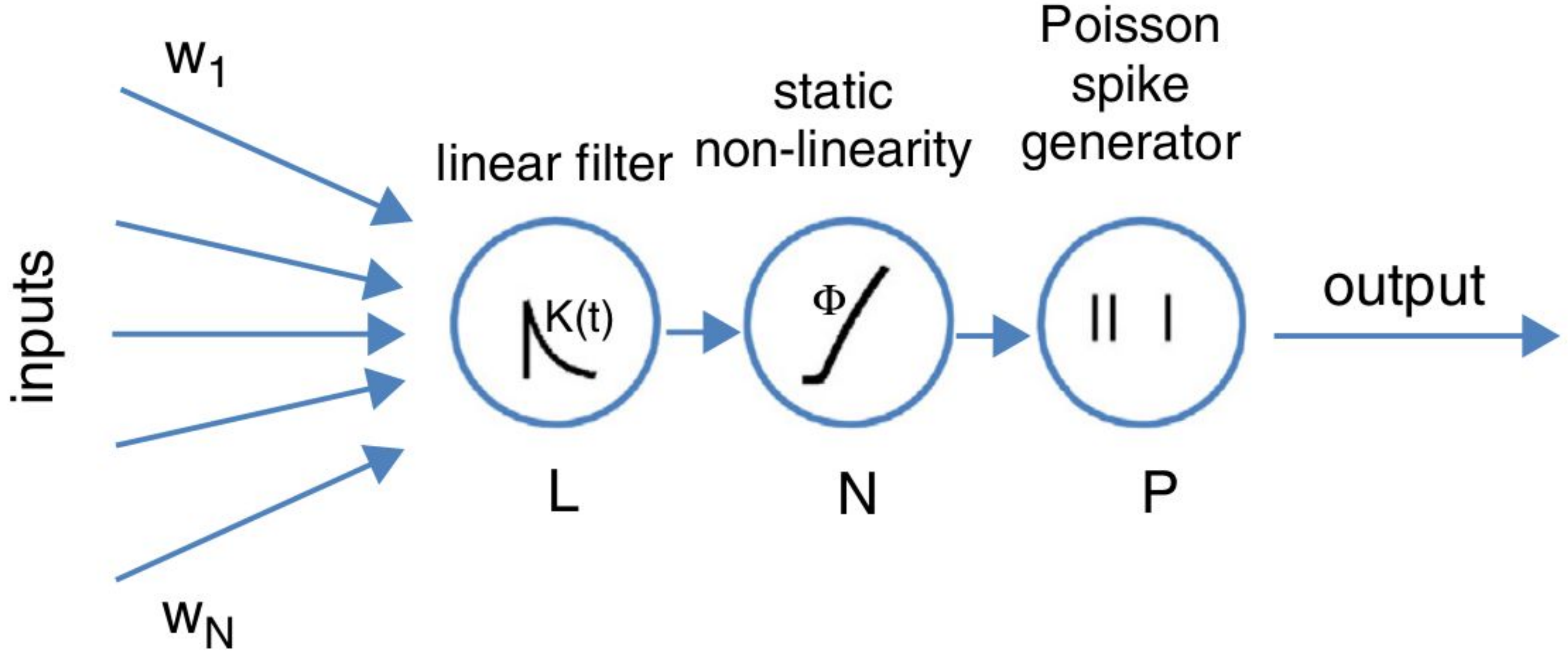
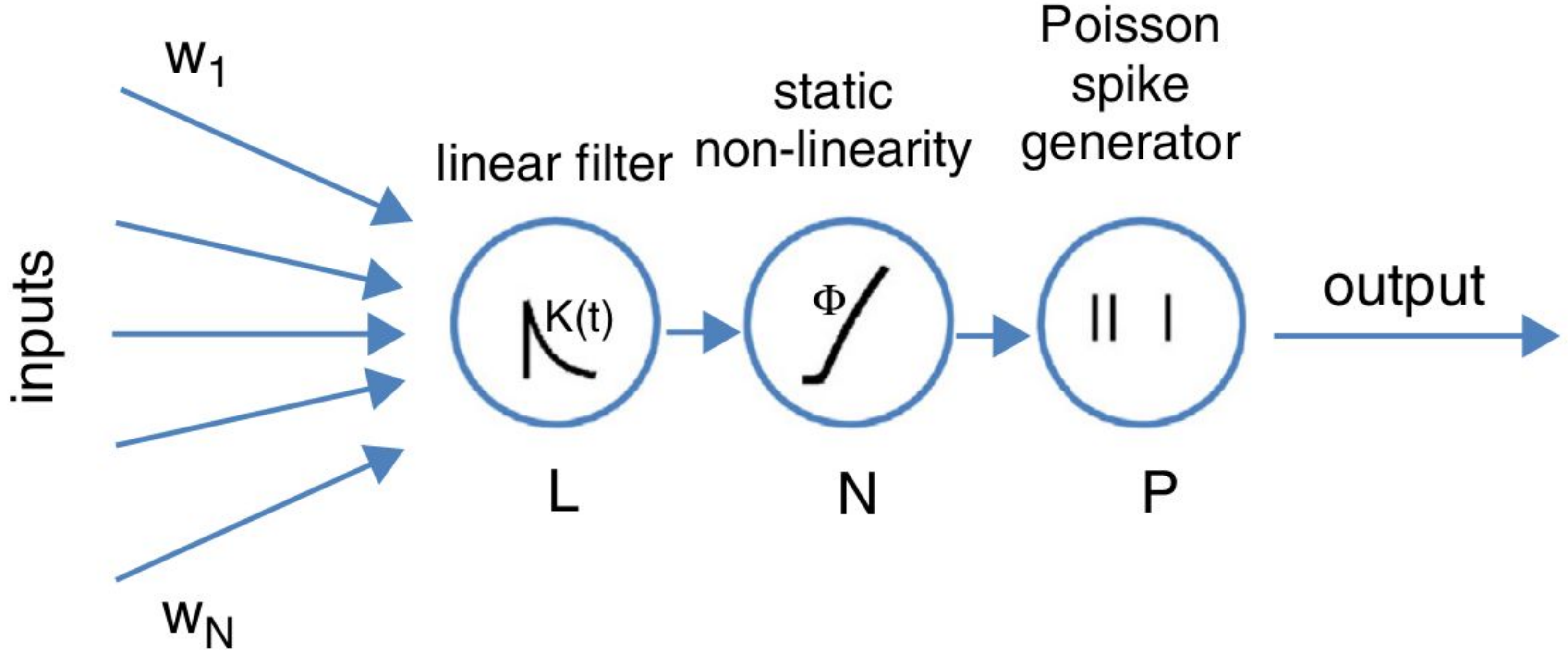


Brunel et.al, 2021

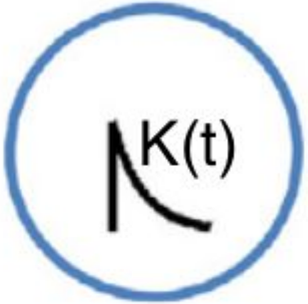
In the following we will be focusing on the neurons related to visual stimuli

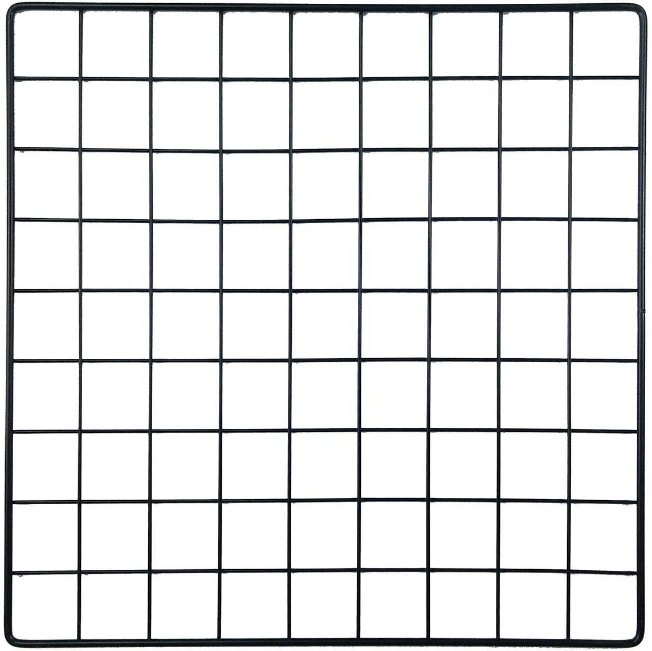
In modern neuron models the threshold function is replaced by a LNP model



Brunel et.al, 2021

The linear filter operates on a pixel matrix input





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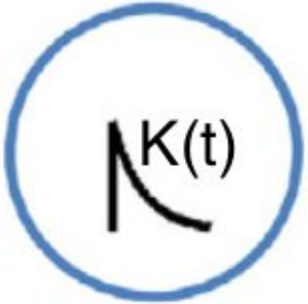

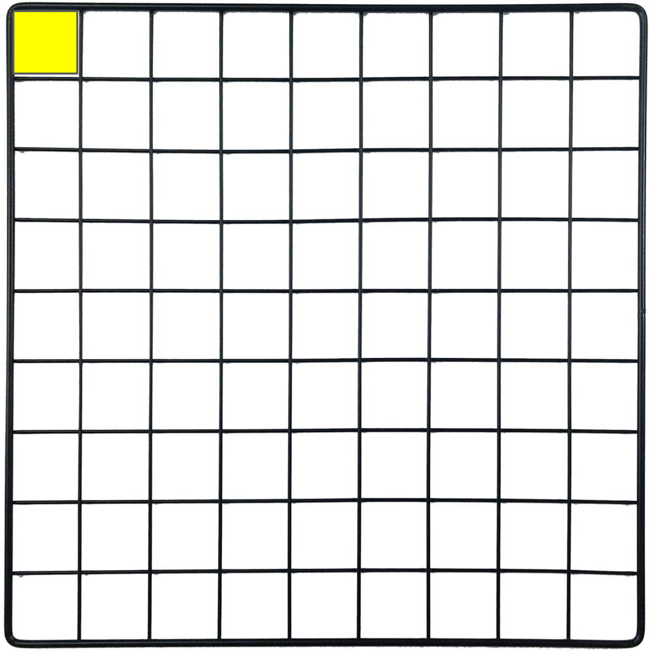

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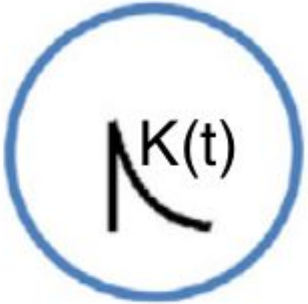

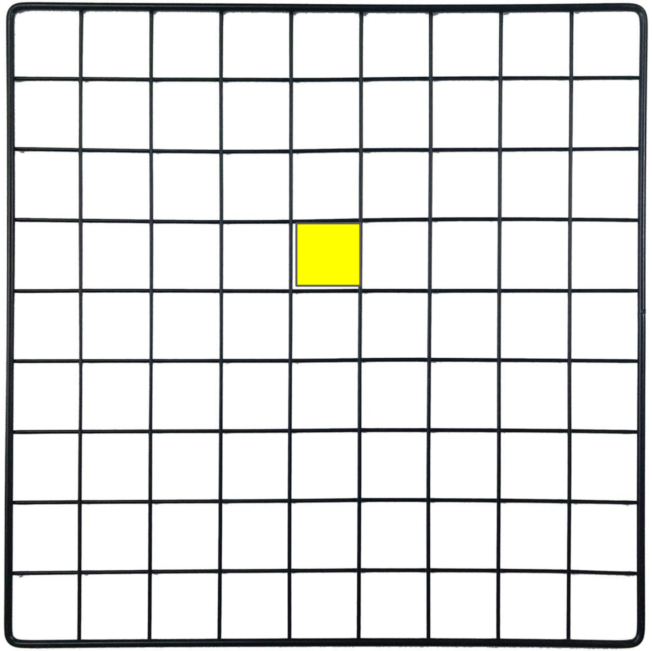

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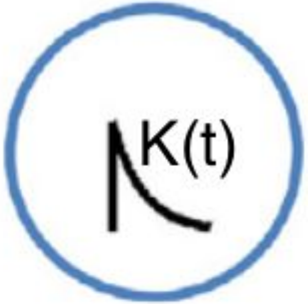
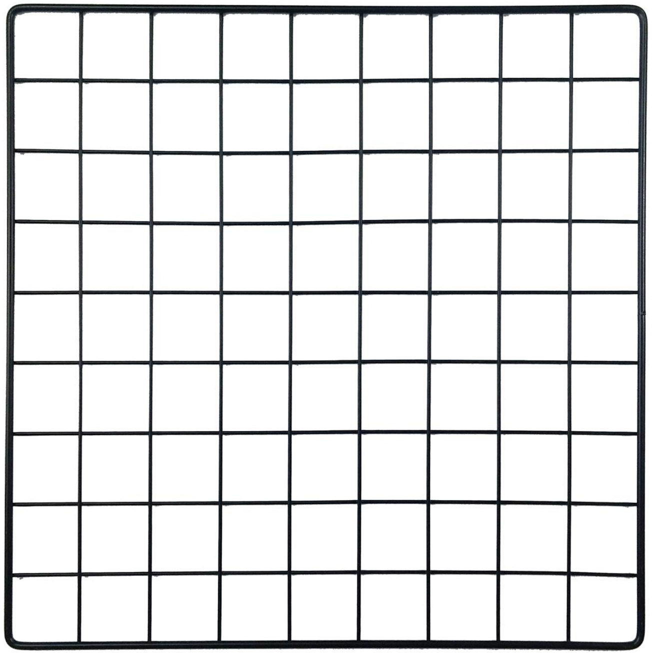
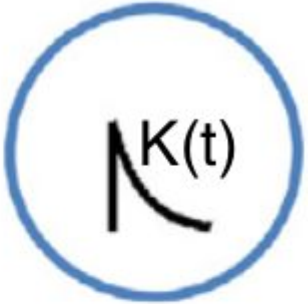
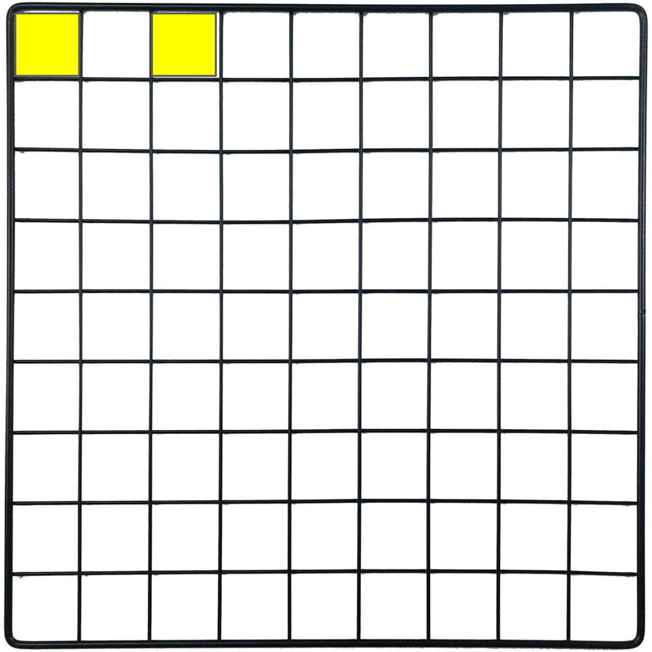




$$\mathbf{x} = [x_1, x_2, \ldots, x_{81}]$$

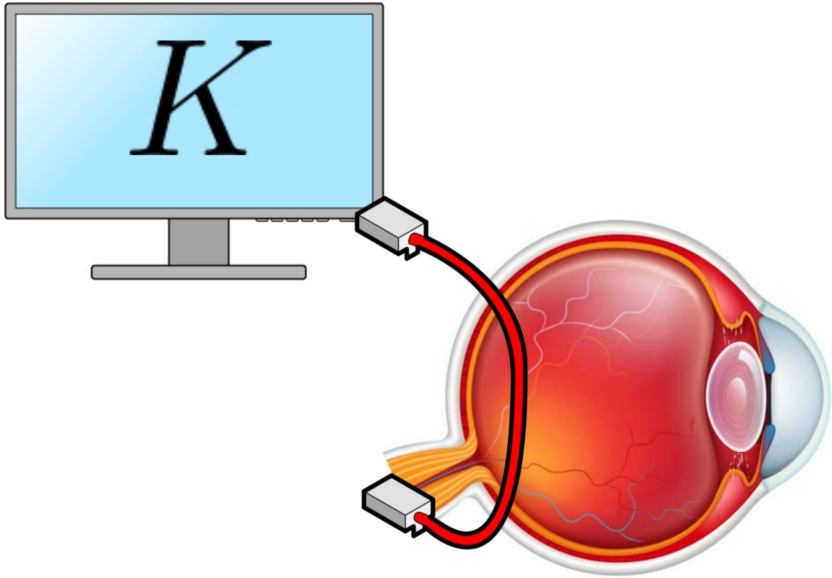
The linear filter operates on a pixel matrix input

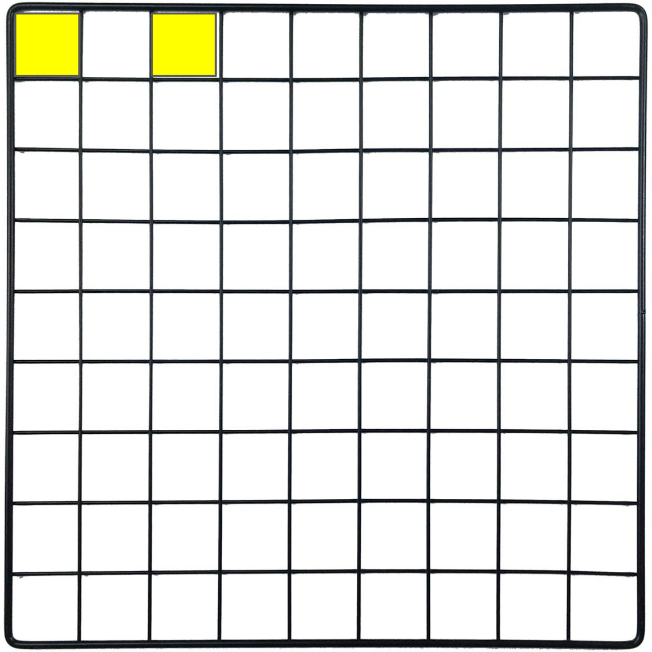




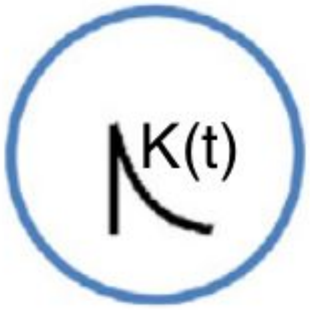
$$\mathbf{x} = [1, 0, 1, \ldots, 0]$$

Detect the flies filter by checking response to all the pixels





The linear filter



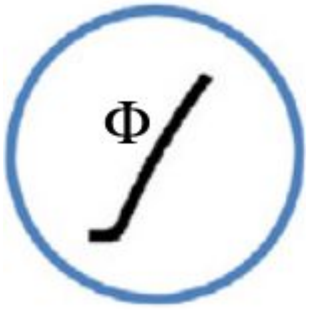
The linear filter is a matrix **k** with same dimensionality as **x**

$$\mathbf{k} = [k_1, k_2, \ldots, k_{81}]$$

The filter is multiplied by the input

$$\mathbf{k} \cdot \mathbf{x}$$

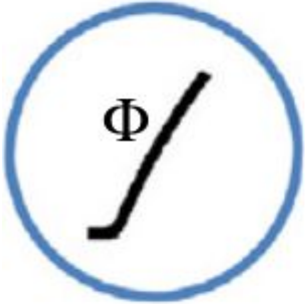
The non-linear function

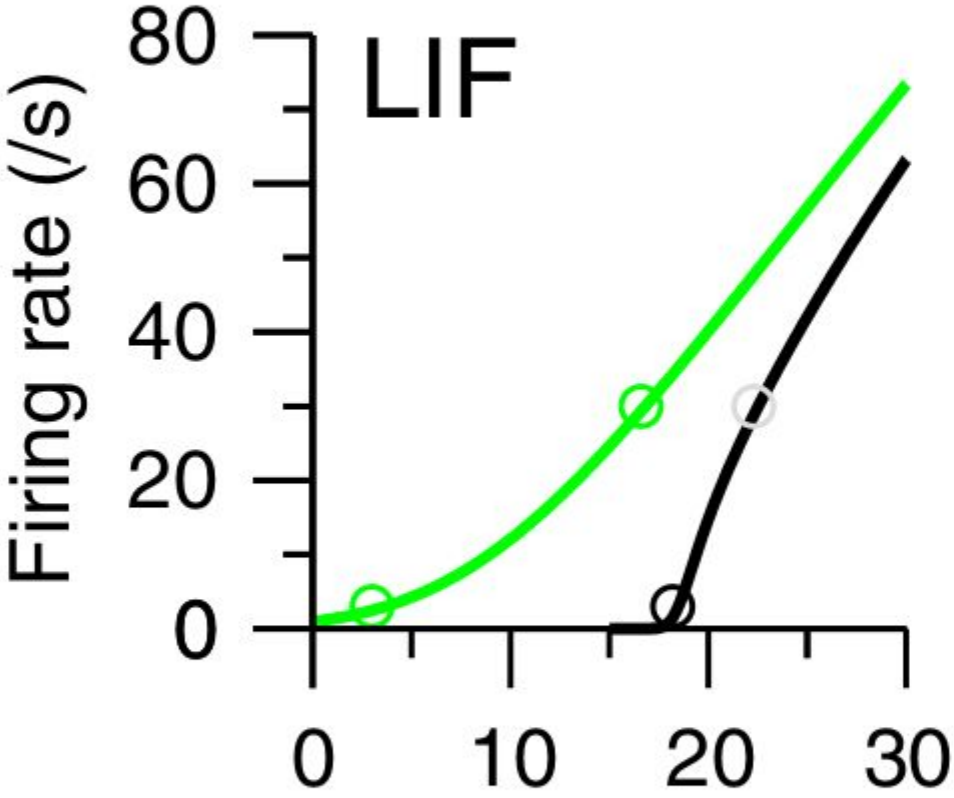


The filter output the input of a non-linear function

$$\lambda = \Phi(\mathbf{k} \cdot \mathbf{x})$$

The non-linear function for a LIF neuron





Brunel et.al, 2021

The poisson spike generator

The spike train emitted from the neuron is created
by an inhomogeneous poisson process with rate $\Phi(K^T\mathbf{x})$

The poisson distribution has had a lot of different applications

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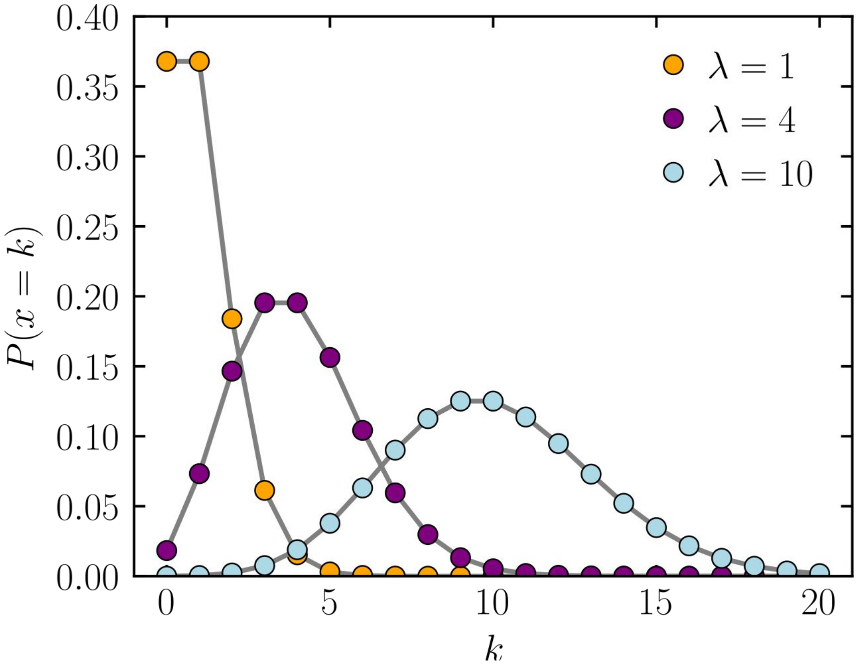
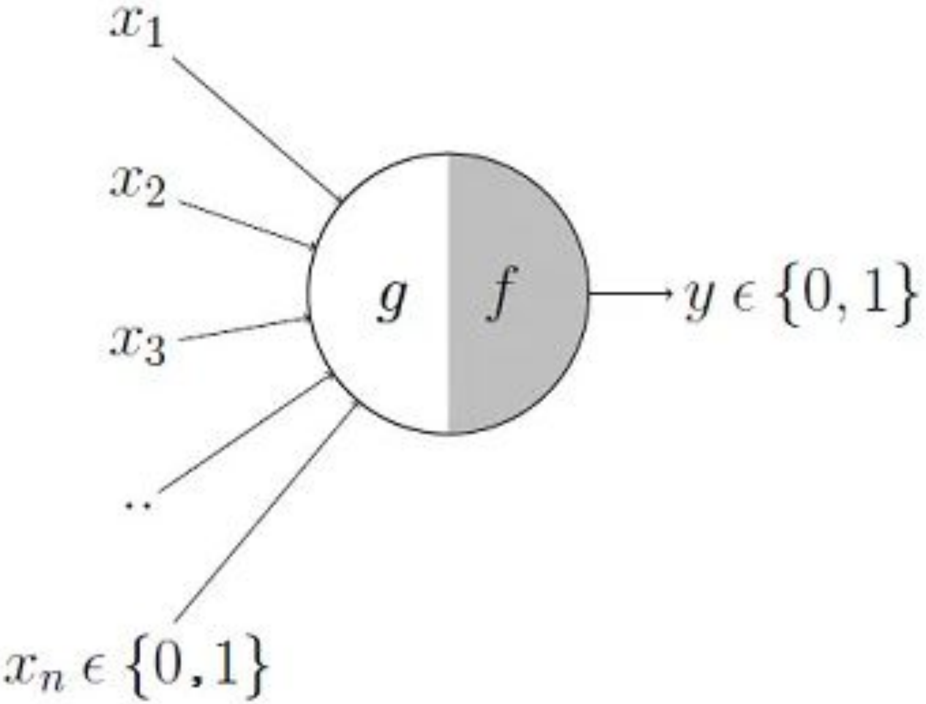
The poisson spike generator

The spike train emitted from the neuron is created by an inhomogeneous poisson process with rate $\Phi(K^T\mathbf{x})$

$$P(X = x) = \frac{\lambda^k e^{-\lambda}}{k!}$$

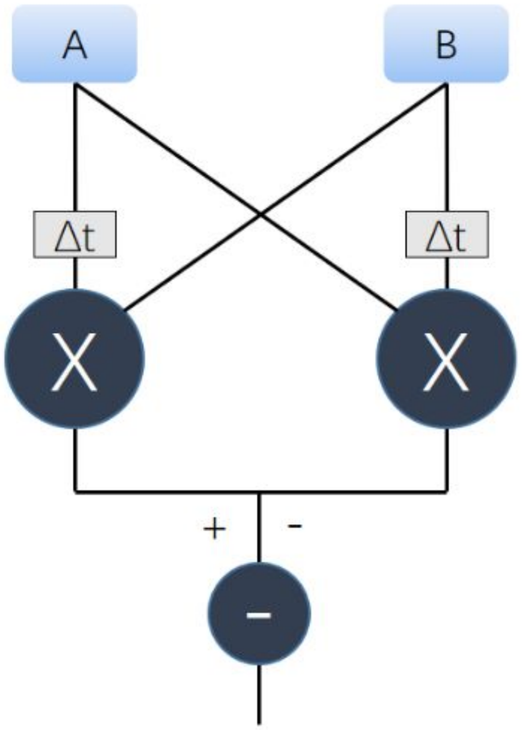


The traditional view with g being sum can be challenged



Neurologist has long been interested in neurons ability to multiply signals

Hassenstein-Reichardt-Detector



Qiu et.al, 2023

Recently, the multiplication of signals have been biologically proven and explained

Article

A biophysical account of multiplication by a single neuron

https://doi.org/10.1038/s41586-022-04428-3

Received: 21 June 2021

Accepted: 14 January 2022

Published online: 23 February 2022

Open access

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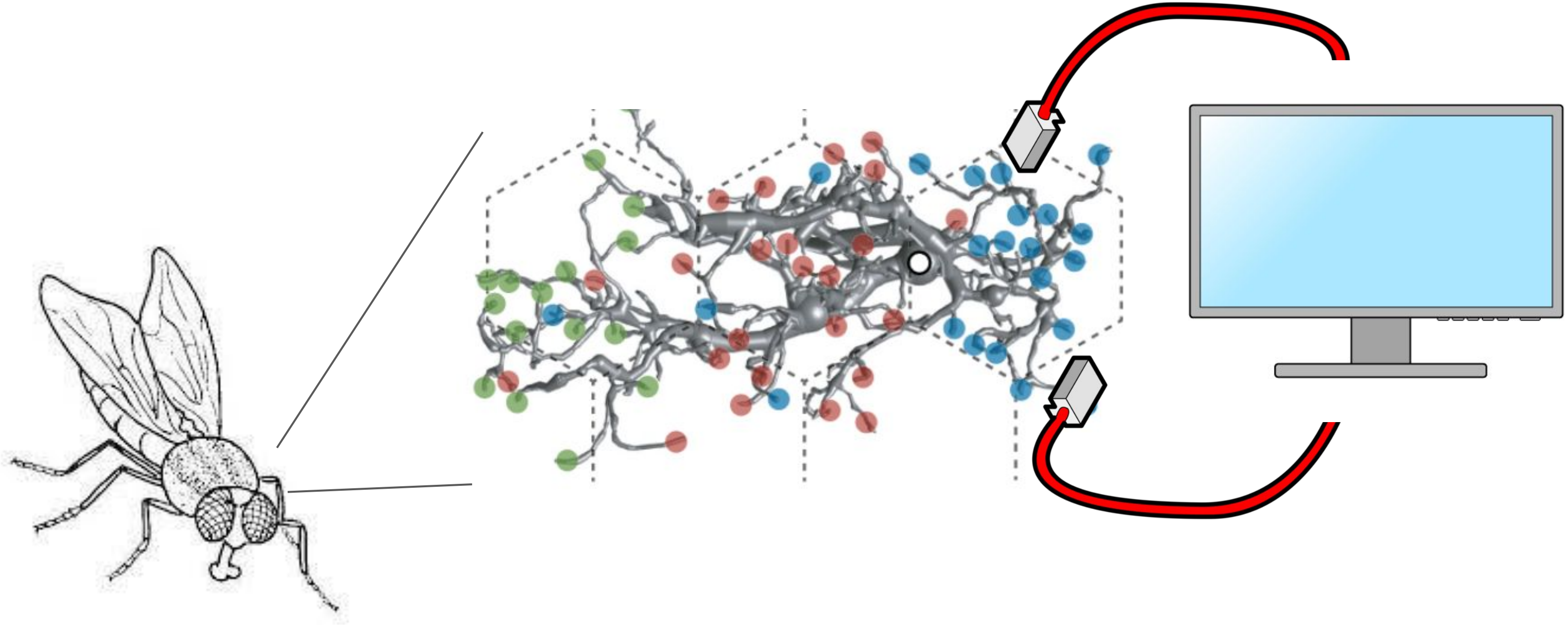
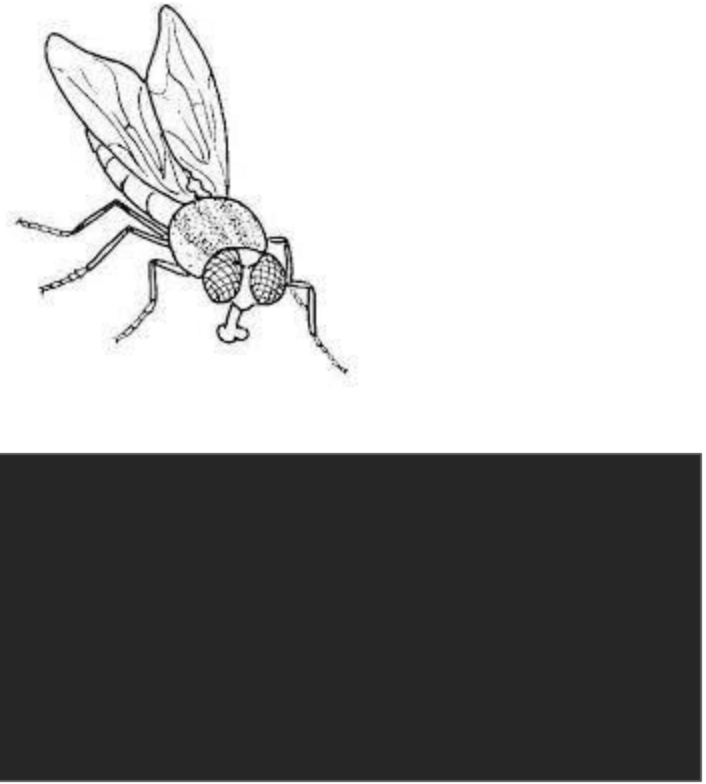
Lukas N. Groschner[1,2]✉, Jonatan G. Malis[1,2], Birte Zuidinga[1] & Alexander Borst[1]✉

Nonlinear, multiplication-like operations carried out by individual nerve cells greatly enhance the computational power of a neural system[1–3], but our understanding of their biophysical implementation is scant. Here we pursue this problem in the *Drosophila melanogaster* ON motion vision circuit[4,5], in which we record the membrane potentials of direction-selective T4 neurons and of their columnar input elements[6,7] in response to visual and pharmacological stimuli in vivo. Our electrophysiological measurements and conductance-based simulations provide evidence for a passive supralinear interaction between two distinct types of synapse on T4 dendrites. We show that this multiplication-like nonlinearity arises from the coincidence of cholinergic excitation and release from glutamatergic inhibition. The latter depends on the expression of the glutamate-gated chloride channel GluClα[8,9] in T4 neurons, which sharpens the directional tuning of the cells and shapes the optomotor behaviour of the animals. Interacting pairs of shunting inhibitory and excitatory synapses have long been postulated as an analogue approximation of a multiplication, which is integral to theories of motion detection[10,11], sound localization[12] and sensorimotor control[13].

The T4 neuron of the fly is monitored



The fly is placed in front of a monitor



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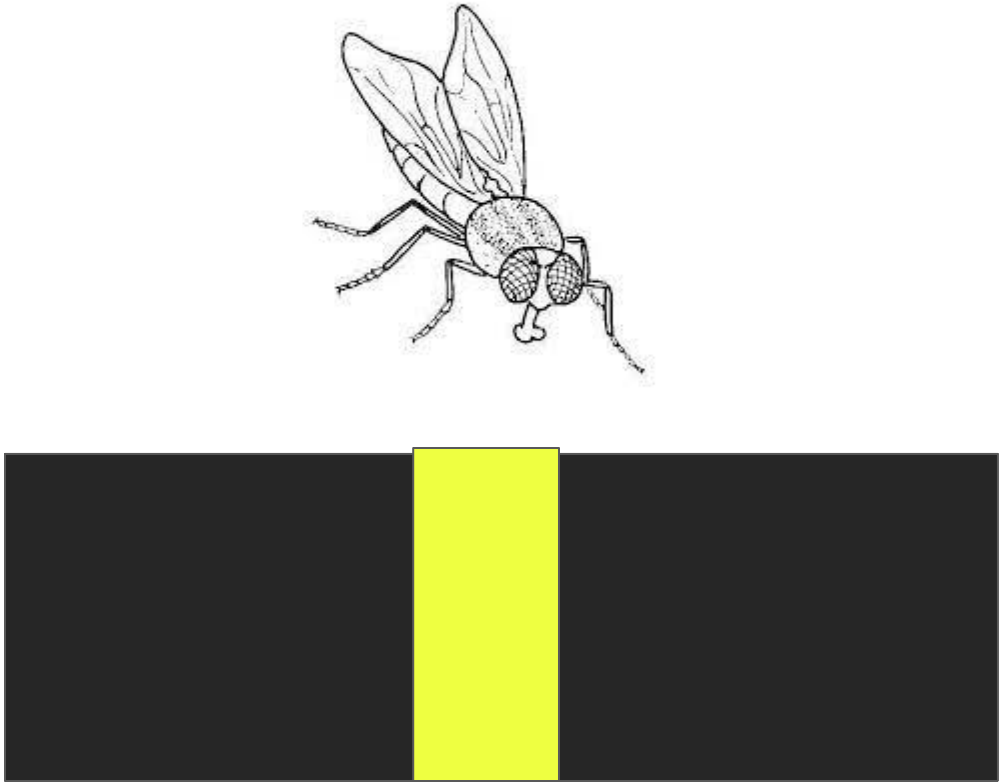

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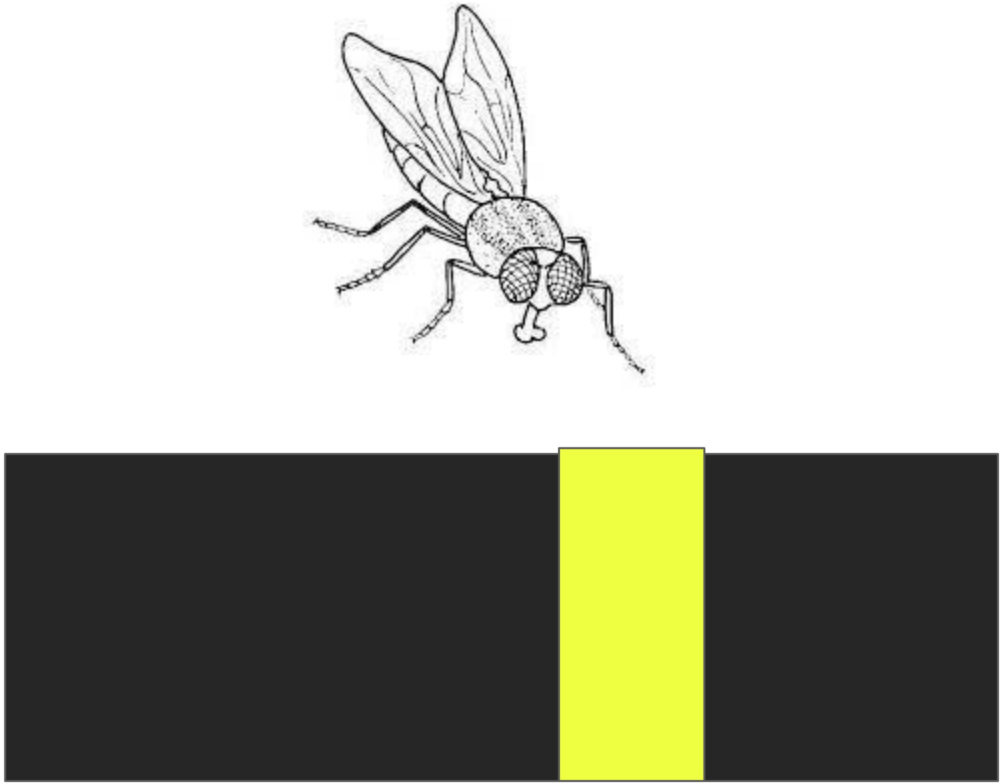

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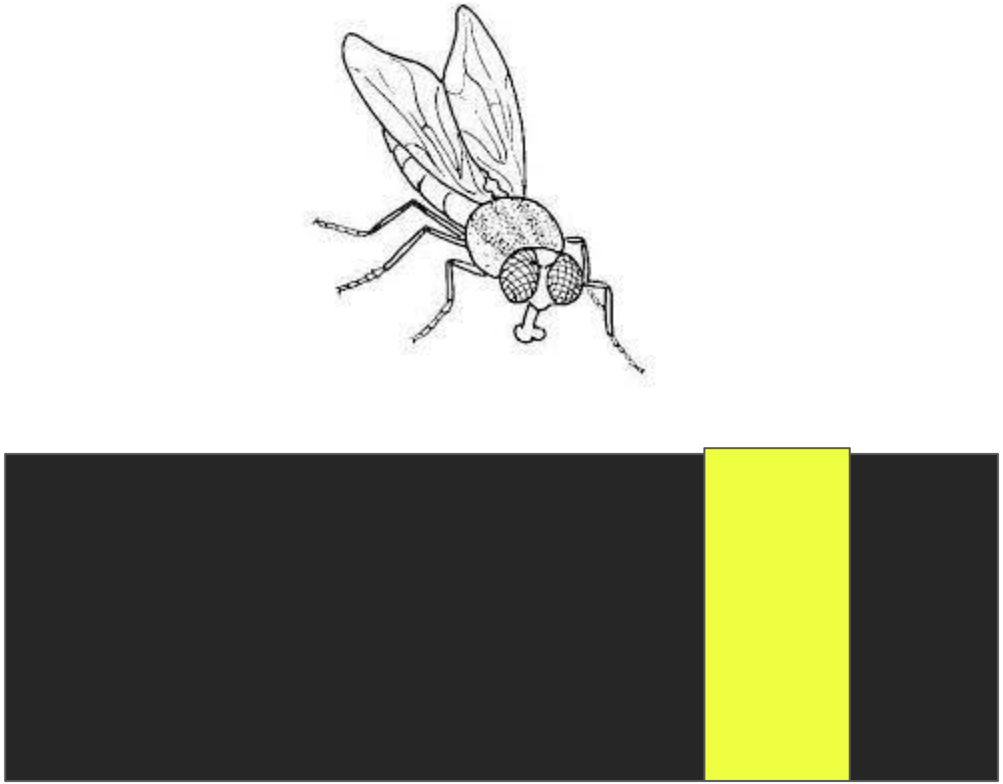
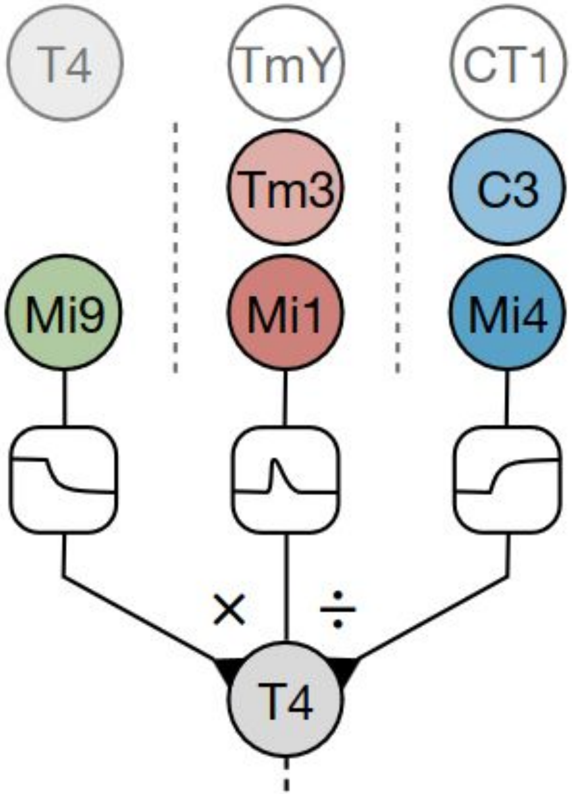
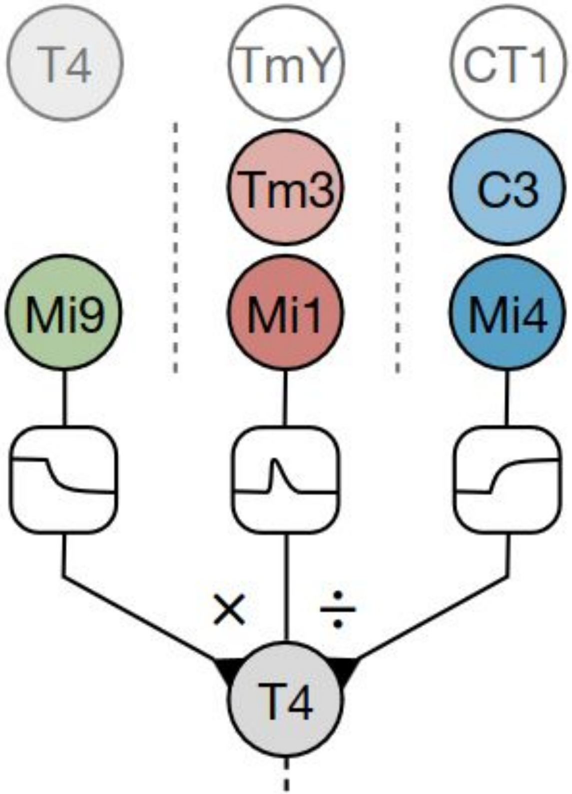


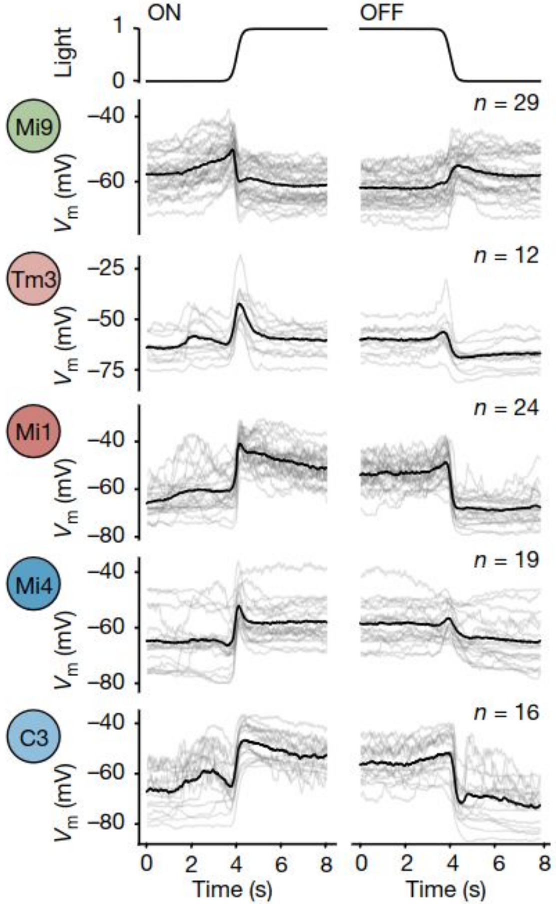
The T4 neuron has several incoming dendrites



Groschner et.al, 2021

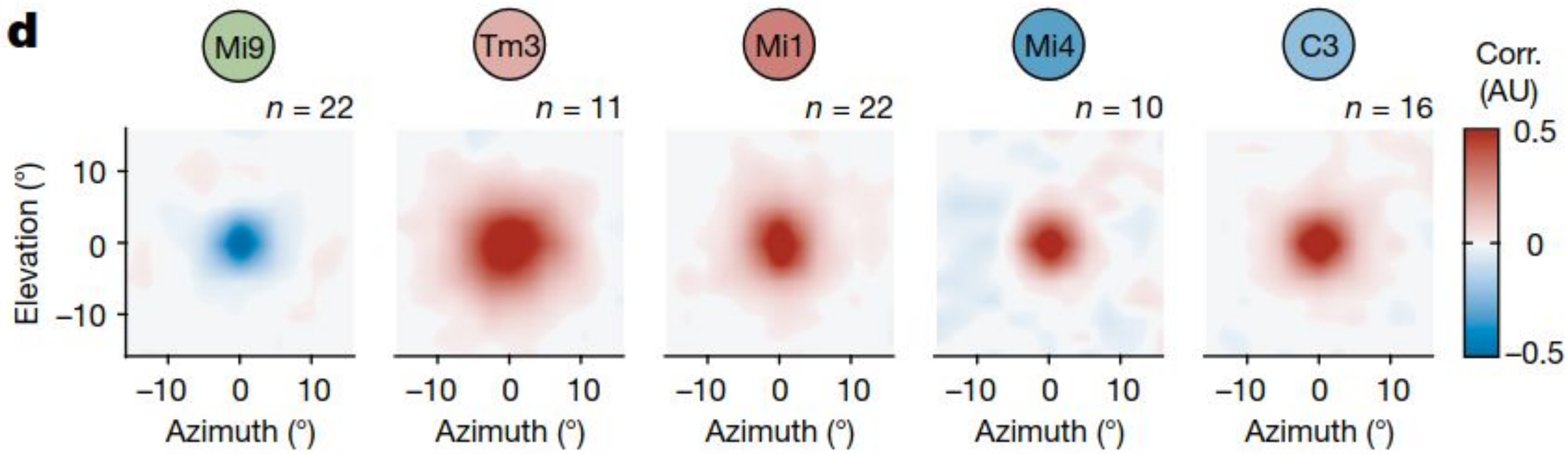
The voltage response of neurons component parts are measured





Groschner et.al, 2021

The correlation between the incoming synapses and the T4 neuron



Groschner et.al, 2021

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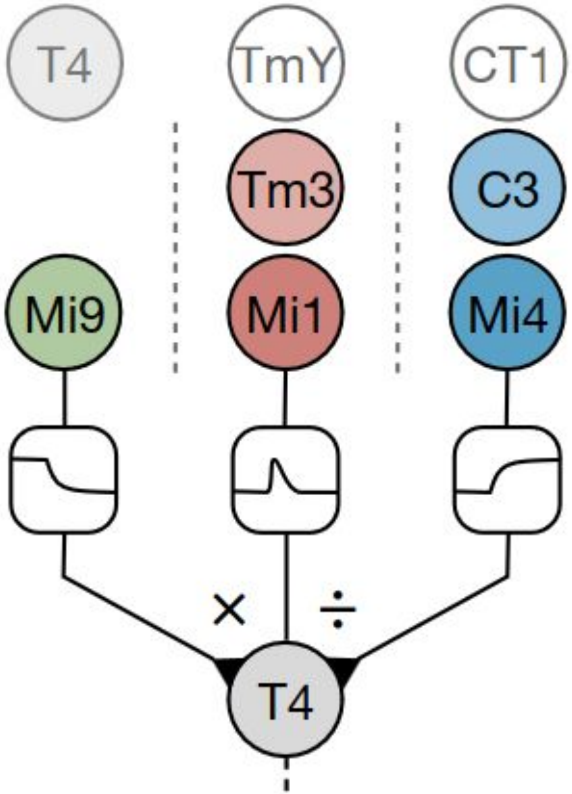

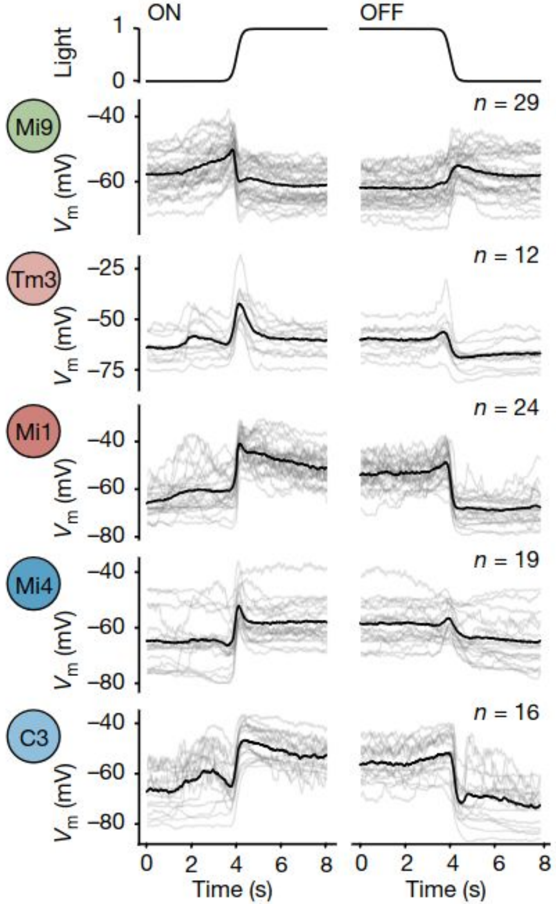

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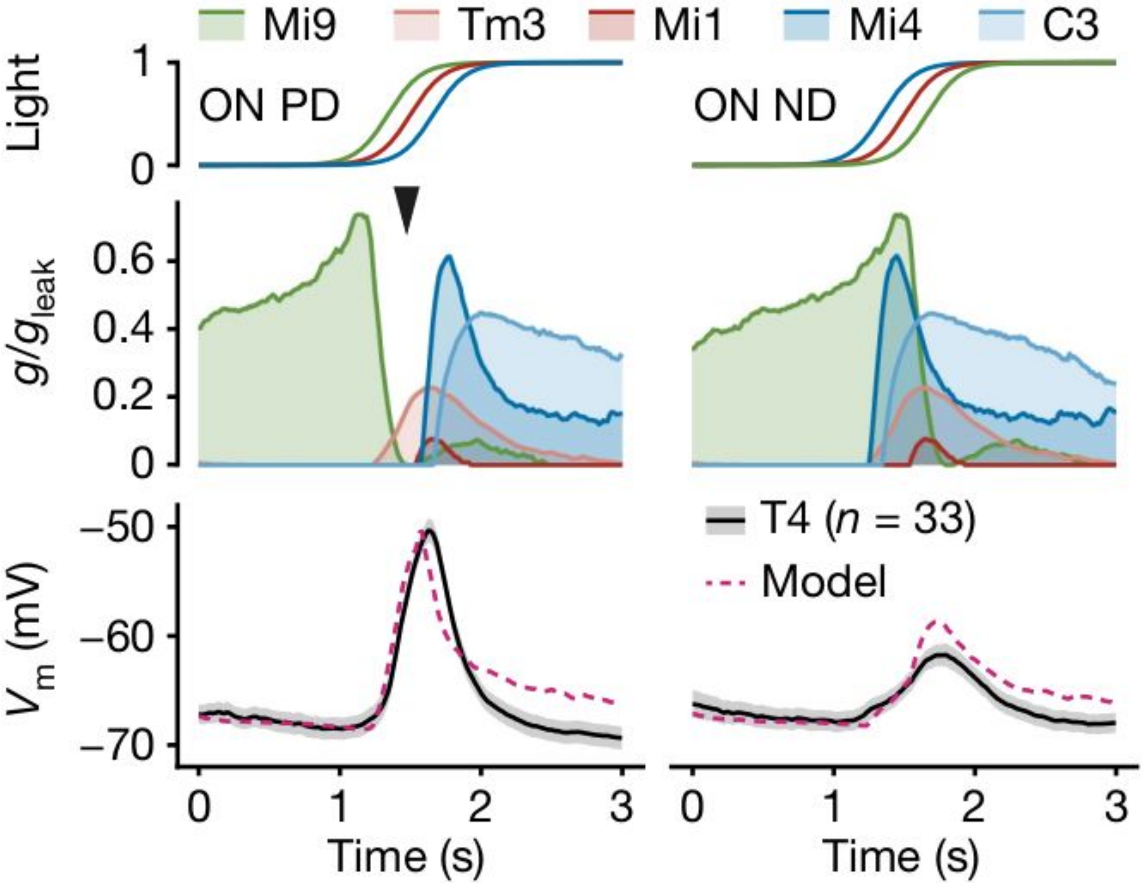
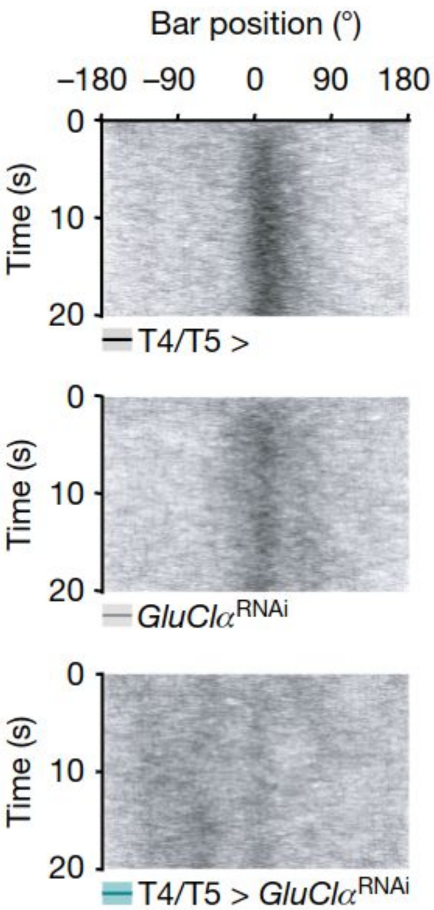
The neuron responds when the light moves in the primary direction



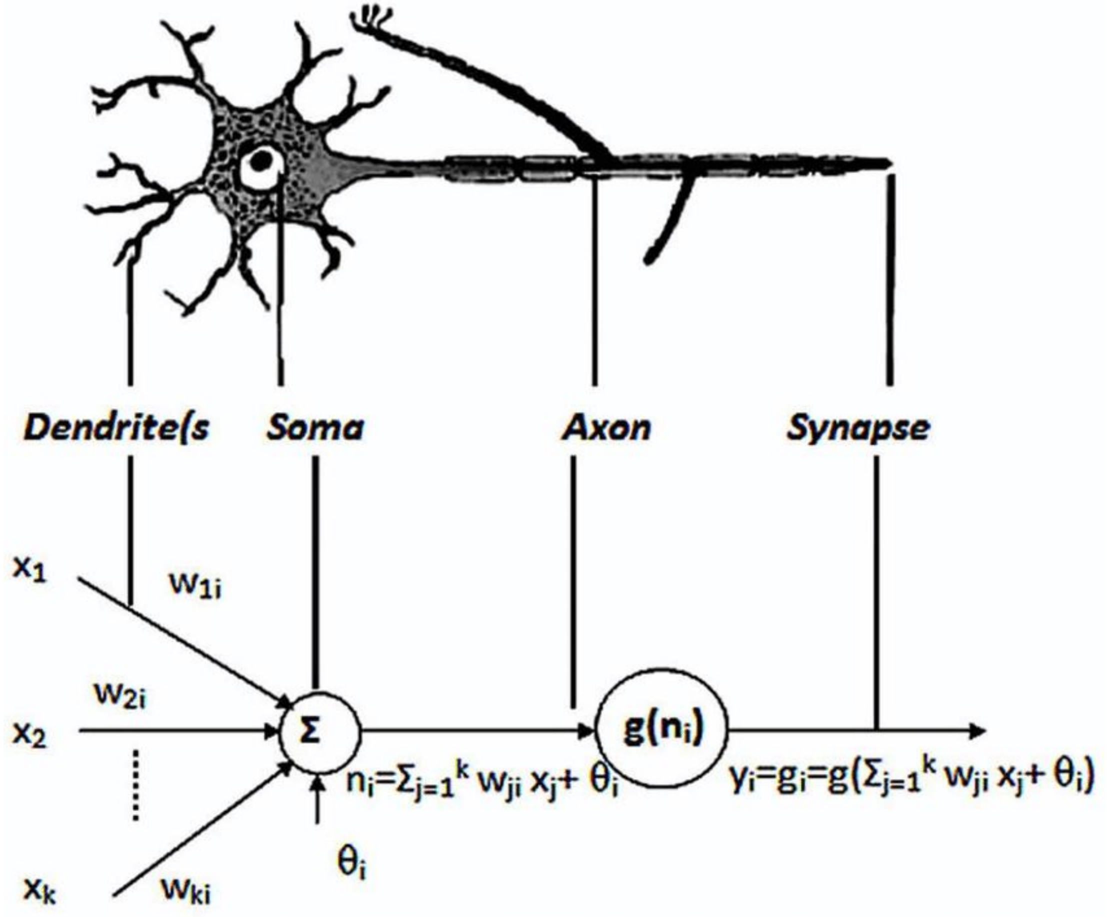
The flys ability to detect direction of moving light could be chemically disabled



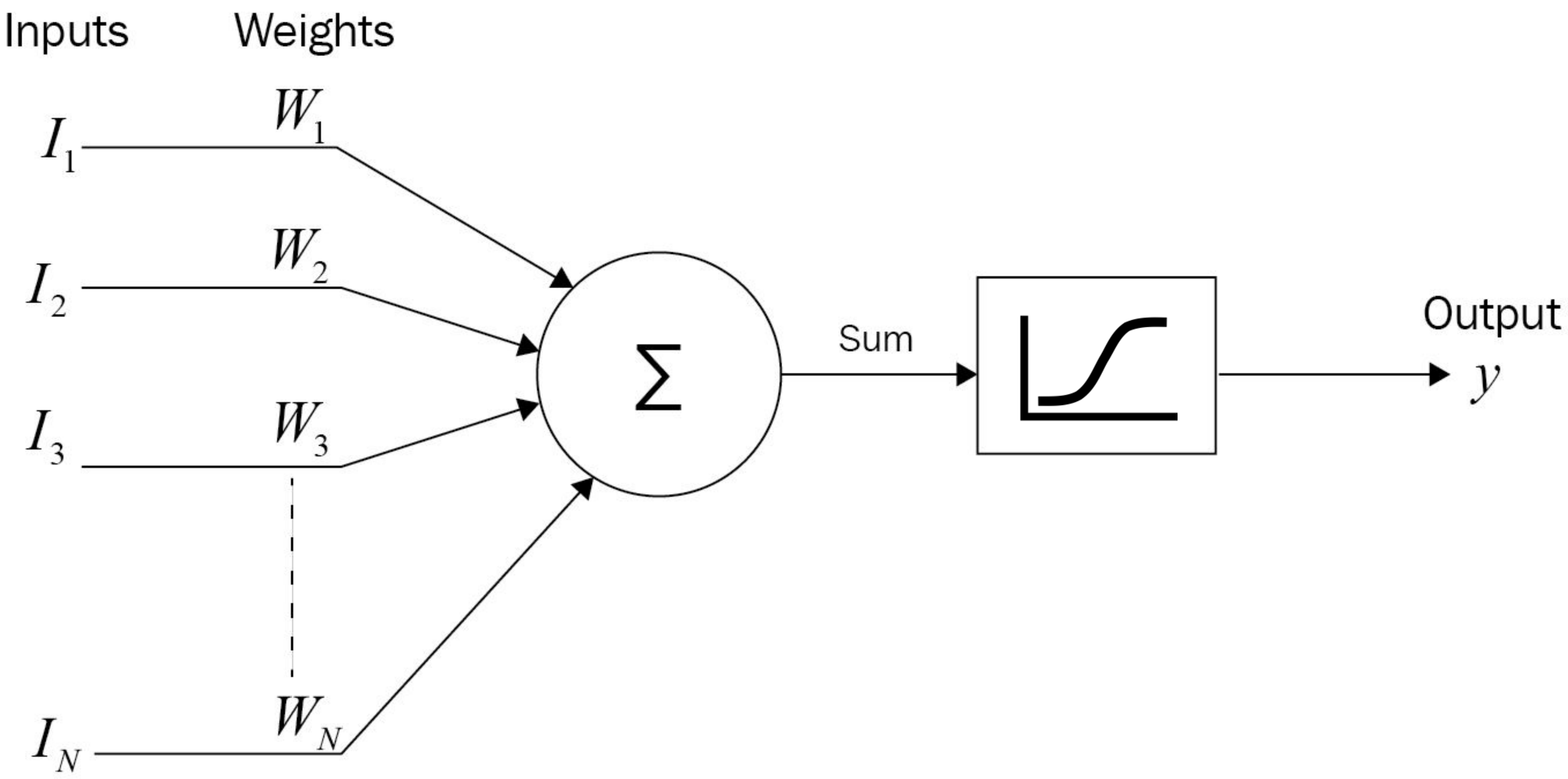
Groschner et.al, 2021

So why are we talking about neurons?

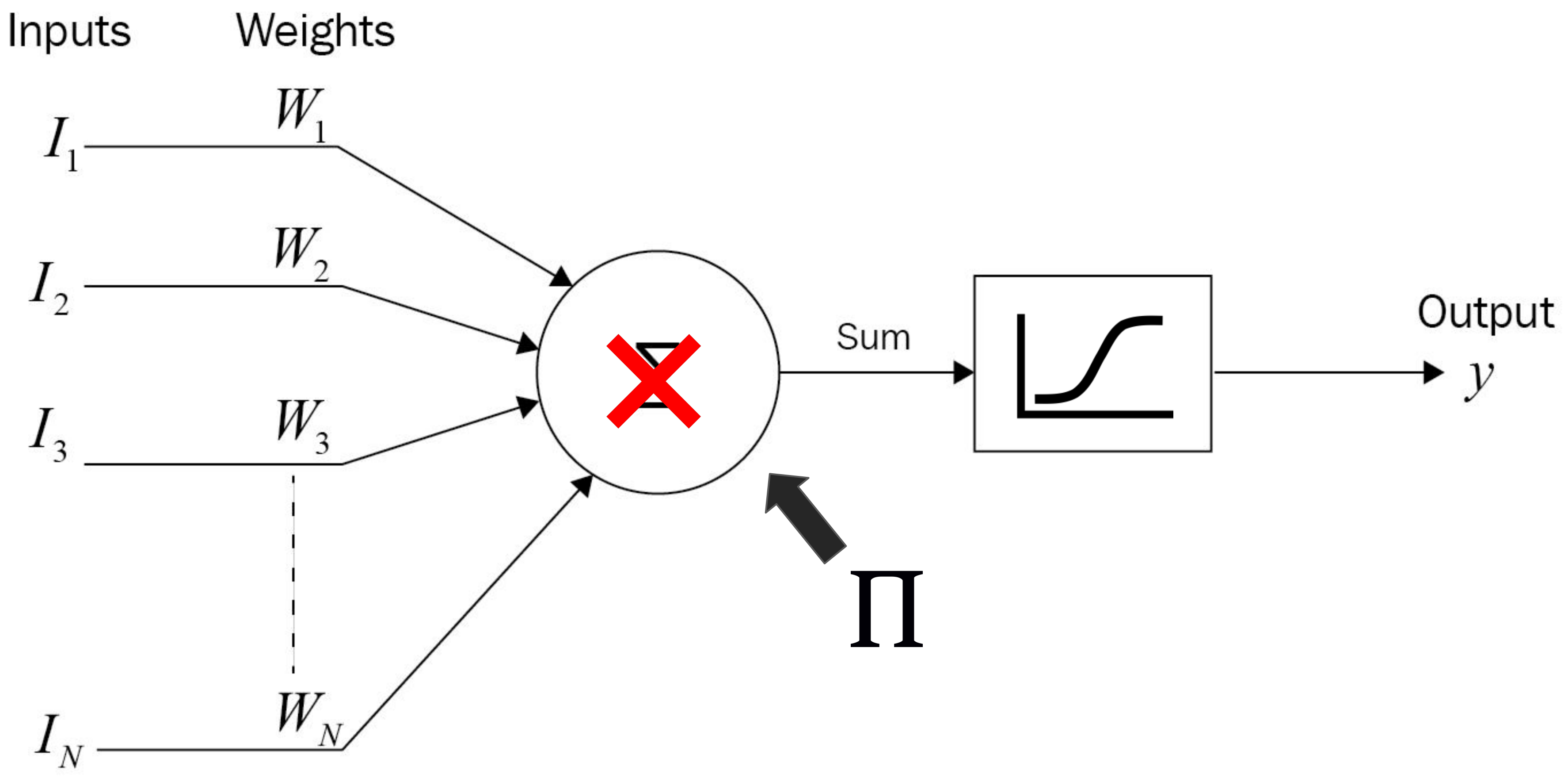
The neurons are the inspiration for neural networks



The standard view of a neural network neuron



Following biology we can consider other neuron models for NN



Multiplication neural networks have been explored

Product Units: A Computationally Powerful and Biologically Plausible Extension to Backpropagation Networks

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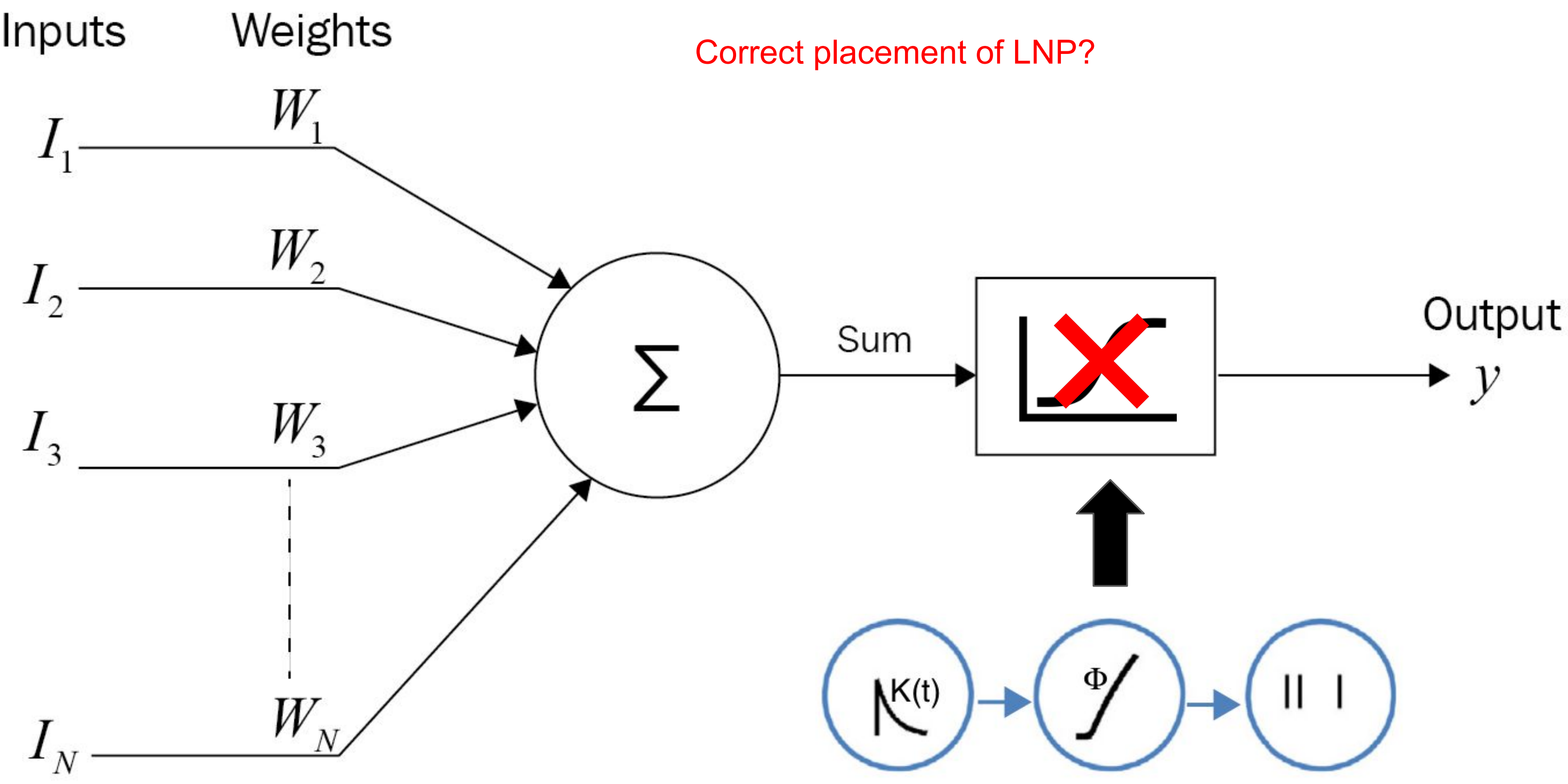
We introduce a new form of computational unit for feedforward learning networks of the backpropagation type. Instead of calculating a weighted sum this unit calculates a weighted product, where each input is raised to a power determined by a variable weight. Such a unit can learn an arbitrary polynomial term, which would then feed into higher level standard summing units. We show how learning operates with product units, provide examples to show their efficiency for various types of problems, and argue that they naturally extend the family of theoretical feedforward net structures. There is a plausible neurobiological interpretation for one interesting configuration of product and summing units.

The multiplicative sum is actually equivalent to the additive sum

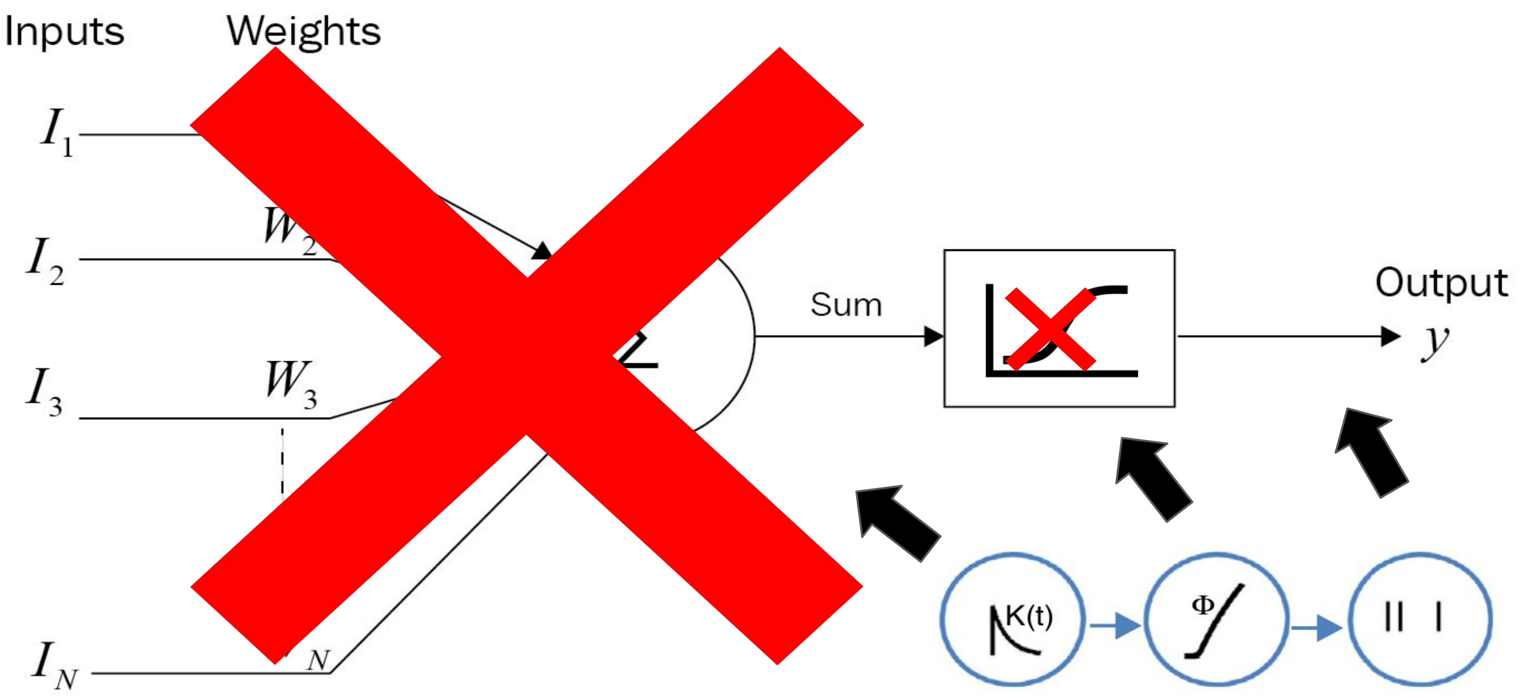
$$\Pi_j x_j^{w_{ij}} = \exp(\Sigma_j w_{ij} \ln(x_j))$$

$$\sigma' = \exp \circ \sigma \circ \ln$$

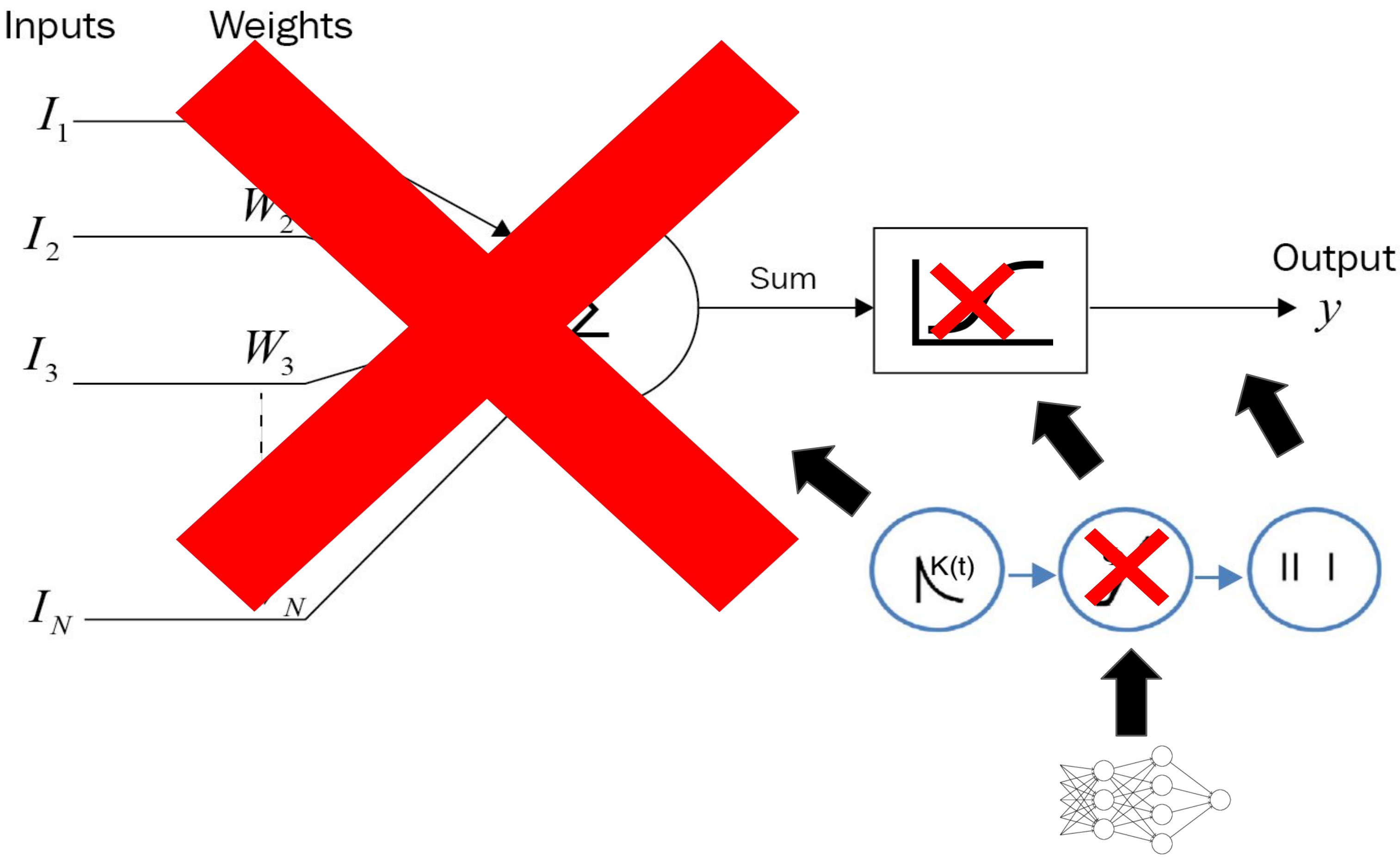
The object function can potentially be replaced by a LNP process



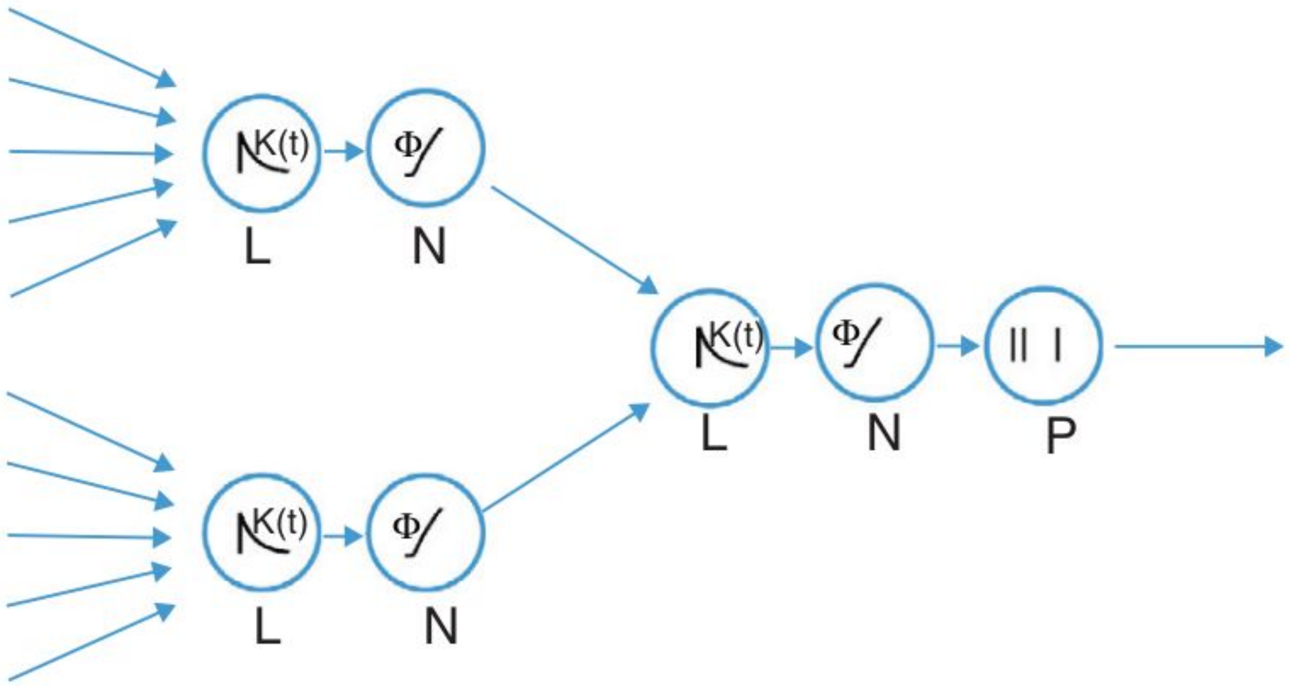
The object function can potentially be replaced by a LNP process



The object function can potentially be replaced by a LNP process



A more reasonable approach for a DNN might be to use a multilayer neuron model



Other possibilities

- **Probability distribution**

frontiers in Psychology

ORIGINAL RESEARCH
published: 25 March 2021
doi: 10.3389/fpsyg.2021.596231

Check for updates

Can the Brain Build Probability Distributions?

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How humans efficiently operate in a world with massive amounts of data that need to be processed, stored, and recalled has long been an unsettled question. Our physical and social environment needs to be represented in a structured way, which could be achieved by reducing input to latent variables in the form of probability distributions, as proposed by influential, probabilistic accounts of cognition and perception. However, few studies have investigated the neural processes underlying the brain's potential ability to represent a probability distribution's complex, global features. Here, we presented

Thank you for your attention