Norwegian-Ukrainian Winter School 2018 on Stochastic Analysis, Probability Theory and Related Topics

Abstracts of the presentations

Monday, 22. January:

Professor Bernt Øksendal, University of Oslo, Norway

Title: An introduction to stochastic control, with applications in mathematical finance

Abstract

We give a short introduction to the stochastic calculus for Itô-Lévy processes, and review briefly the two main methods of optimal control of stochastic systems described by such processes, namely:

(i) Dynamic programming and the Hamilton-Jacobi-Bellman (HJB) equation

(ii) The stochastic maximum principle and its associated adjoint backward stochastic differential equation (BSDE).

The two methods are illustrated by application to the classical portfolio optimization problem in finance. A second application is the problem of risk minimization in a financial market. Using a dual representation of risk, we arrive at a stochastic differential game. This is solved by using the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation, which is an extension of the HJB equation to stochastic differential games.

Professor Andriy Pylypenko, Institute of Mathematics, National Academy of Science of Ukraine, Kiev, Ukraine

Title: "The limit behavior of perturbed random walks"

Abstract

We study the limit behavior of a perturbed random walk, i.e., a random walk (RW) whose transition probabilities are modified depending on the current state, number of times the walk has visited some states, etc. For example, let $\{S(k),\ k \geq 0\}$ be a symmetric random walk with the unit jump. It is well known that a sequence of processes $\{\frac{S([nt])}{\sqrt{n}},\ n \geq 1\}$ converges in distribution to a Wiener process $W(t), t \geq 0$. Consider a perturbed RW $\{\tilde{S}(k)\}$ whose transition probabilities differ from the ones of $\{S(k)\}$ only at 0:

$$P(\tilde{S}(k+1) = i \pm 1 | \tilde{S}(k) = i) = \frac{1}{2} \text{ for } i \neq 0$$

and

$$P(\tilde{S}(k+1) = 1 | \tilde{S}(k) = 0) = p,\ P(\tilde{S}(k+1) = 1 | \tilde{S}(k) = 0) = q.$$

It was proved by Harrison and Shepp [1] that

$$\frac{\tilde{S}([nt])}{\sqrt{n}} \Rightarrow X,\ n \to \infty,$$

where X is a skew Brownian motion, i.e., a continuous Markov process with transition probability density function

$$p_t(x, y) = \varphi_t(x - y) + \gamma sign(y)\varphi_t(|x| + |y|),\ x, y \in \mathbb{R},$$

where $\gamma = 2p - 1$, $\varphi_t(x) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}$ is a density of the normal distribution $N(0, t)$. It can be also shown [1] that X is a unique strong solution to the SDE

$$dX(t) = \gamma dL^0_X(t) + dW(t), t \geq 0,$$

where $L^0_X(t)$ is a local time of the process X at 0. We discuss different approaches to investigation of perturbed RW. In particular, the result of Harrison and Shepp is generalized to the case when random walks are perturbed at a finite number of points. The corresponding limit process may be a skew Brownian motion [2, 4, 7], reflected Brownian motion with the Feller-Wentzell boundary condition [6], Brownian motion with a hard membrane [3]; the last process is even non-Markov. If perturbations of transition probabilities depend on a number of times the walk has visited some states, then the limit process may be an excited Brownian motion [8, 9] and satisfies SDE of the form

$$dX(t) = f(t, X(t), L^{\cdot}_X(t))dt + dW(t), t \geq 0,$$

where f is a measurable function.

References

[1] Harrison, J.M., Shepp, L.A. (1981). On skew Brownian motion. *Ann. Probab.* **9(2)**, 309-313.

[2] Iksanov, A. M. and Pilipenko, A. Yu. (2016). A functional limit theorem for locally perturbed random walks. *Probab. Math. Statist.*, **36**, no.2, 353-368.

[3] Mandrekar, V. and Pilipenko, A. (2016). On a Brownian motion with a hard membrane. *Statistics and Probability Letters*, **113**, p. 62-70.

[4] Pilipenko, A. Yu.; Prykhodko, Yu. E. On the limit behavior of symmetric random walks with membranes. (Ukrainian) *Teor. Imovir. Mat. Stat.* No. 85 (2011), 84–94; translation in *Theory Probab. Math. Statist.* No. 85 (2012), 93-105.

[5] Pilipenko, A. and Prykhodko, Yu. (2015). On a limit behavior of a sequence of Markov processes perturbed in a neighborhood of a singular point. *Ukrainian Math. Journal*, **67**, no. 4, 499–516.

[6] Pilipenko, A. Yu. and Prykhodko, Yu. E. (2014). Limit behavior of a simple random walk with non-integrable jump from a barrier. *Theor. Stoch. Proc.* **19(35)**, 52–61.

[7] Pilipenko, A. and Sakhanenko, L. (2015). On a limit behavior of one-dimensional random walk with non-integrable impurity. *Theory of Stochastic Processes*, **20(36)**, no.2, 97 - 104.

[8] Pilipenko, A. (2017). A functional limit theorem for excited random walks. *Electronic Communications in Probability*, **22**, paper no. 39, 1-9.

[9] Pilipenko, A., Khomenko, V. (2017). On a limit behavior of a random walk with modifications upon each visit to zero. *Theory of Stochastic Processes*, **22(38)**, no.1, 71-80.

Tuesday, 23. January:

Professor Tom Lindstrøm, University of Oslo, Norway
Title: Ultrafilter convergence of stochastic processes

Abstract

Convergence of discrete processes to continuous processes is quite common in probability theory, just think of the convergence of random walks to diffusions. The traditional way of dealing with such problems is through the theory of weak convergence. Ultrafilter convergence offers an alternative approach with some advantages and some disadvantages. Ultrafilter convergence can also be thought of as a reworking of the nonstandard approach to stochastic analysis that is closer to the intuition of standard probabilists.

Professor Andrii Ilienko, National Technical University of Ukraine, "Igor Sikorski Kyiv Polytechnic Institute", Kiev, Ukraine
Title: Pigeonholes and around

Abstract

We will make several mathematical excursions united by a common meeting point — the pigeonhole principle (a.k.a. Dirichlet's box principle). Starting with some introductory problems, we then will focus on more intriguing and important applications — the Erdős-Szekeres theorem on monotonic subsequences, rational approximation, and transcendental numbers.

Professor Igor Orlovskyi, National Technical University of Ukraine, "Igor Sikorski Kyiv Polytechnic Institute", Kiev, Ukraine
Title: Introduction to Functional Equations

Abstract

Functional equations are one of the most interesting and beautiful mathematical topics. They appear in almost all areas of mathematics. The main characterization of such equations is that they define function implicitly by some relation, and a way to find their explicit form could be quite a challenge. There no universal methods to deal with such equations due to their vast varieties.

The aim of the lecture is to introduce the main ideas in functional equations on example of most known so called Cauchy equation and its applications in solving some of the functional problems.

Thursday, 25. January:

Dr. Achref Bachouch, University of Oslo, Norway

Title: Longstaff-Schwarz algorithm for the valuation of American-Bermuda options

Abstract

In this talk, I will present the Longstaff-Schwarz algorithm for the valuation of American-Bermuda options. This algorithm is based on a Least-Squares Monte Carlo method and was first proposed in the paper of Longstaff and Schwarz (2001): Valuing American Options by Simulation: A Simple Least-Squares Approach. I will also discuss implementation details and present some numerical examples.

Professor Artem Baiev, Vasyl'Stus Donetsk National University, Vinnitsa, Ukraine

Title: Stochastic methods for machine learning

Abstract

Recently, artificial intelligence and, in particular, machine learning have attracted the attention of a wide range of researchers. This is due to the significant potential of these technologies to solve various problems. The lecture will introduce the students to the methods of deep learning, in particular, by the method of stochastic gradient descent. The lecture will present the presentation of an important problem of archival documents research for the Ukrainian society.

Joint presentation by Professor Yurii Mlavets, Professor Yurii Kozachenko and Dr. Olga Syniavska, Uzhgorod National University, Uzhgorod, Ukraine

Title: Sub-Gaussian random variables and stochastic processes

Abstract

We show that the space of sub-Gaussian random variables has the Banach structure, examples and some properties of these variables. We consider equivalent norms in this space, exponential estimates for the distribution of a sum of independent sub-Gaussian random variables and strictly sub-Gaussian random variables. We consider sub-Gaussian random variables, as a generalization sub-Gaussian random variables.

We consider stochastic processes taking values in a space of sub-Gaussian random variables and study the properties of sample paths of these processes. In particular, pseudometrics generated by stochastic processes are introduced. Entropy characteristics of a pseudometric space are considered. Entropy conditions for the almost sure boundedness of a sub-Gaussian process are established. Estimates for the distribution of the supremum of a sub-Gaussian process are obtained as a corollary. We consider conditions for sample continuity. The application of the theory of sub-Gaussian random variables in the simulation of Gaussian stochastic stationary processes is considered.

Friday, 26. January:

Professor Andrii Dorohovtsev, Institute of Mathematics, National Academy of Science of Ukraine, Kiev, Ukraine

Title: Stochastic anticipating equations

Abstract

Lecture series on

1. Stochastic calculus for Gaussian processes.
2. Equations with the extended integral.
3. Filtration problem for nonsemimartingale noise.

The aim of the lectures is to present the theory of linear anticipating equations with the wide class of Gaussian processes. Since it was created in the middle of 70-th of previous century the extended or Skorokhod stochastic integral became to be widely used in the mathematical models where the noise has no independent increments but remains to be Gaussian. Probably the most known cases are related with the fractional Brownian motion as the noise. It occurs that the extended stochastic integral can be defined for a large class of Gaussian processes and, more over, can be considered as a partial case of general construction where the application of the strong random operator to the random elements in Hilbert space is treated. The developing of the theory of equations with the strong random operators requires new technical tools and leads to the interesting new effects comparatively the case of Ito stochastic equations. For example, the question of locality of extended stochastic integral leads to the problem of approximation of the set by Ornstein-Uhlenbeck semi-group. For the linear Skorokhod equation one can apply the characteristic method. This leads to the shift of initial condition on the probability space. Anticipating stochastic equations naturally arise in filtration theory where the noise and the useful signal are not adapted to same flow of sigma-fields.

Correspondingly to the above mentioned description the lectures are organized as follows. The first lecture is devoted to the stochastic calculus for the Gaussian integrators. Here we present general notions from the stochastic calculus and introduce the class of Gaussian integrators. We consider Ito formula for such processes and problem of locality of extended integral and stochastic derivative on the special subsets of probability space. The second lecture contains results about the untic ipating stochastic equations with the extended integral and Gaussian strong random operators. Here we obtain some representation for solutions which is more general then in one-dimensional case and can be applied to the anticipating SPDE. In the last lecture we consider the filtration problem for the case.