NCGQG 2012 - HiOA, OSLO - Titles and abstracts

Marat Aukhadiev

On compact quantum semigroups and reduced semigroup C*-algebras

Let S be a non-trivial abelian cancellative semigroup having a zero. The C*-algebra generated by the regular representation of S on $\ell^2(S)$ may be considered as a compact quantum semigroup. In the talk, we will show how to define a non-trivial comultiplication such that this compact quantum semigroup admits a bounded counit, a Haar measure and contains a dense weak Hopf algebra (in the sense of Fang Li). This comultiplication induces a comultiplication on the quotient of this quantum semigroup by its commutator ideal, with respect to which the quotient becomes a compact quantum group, isomorphic as a quantum group to $C(G)$, the algebra of continuous functions on the dual G of the Grothendieck group of S, equipped with its standard comultiplication. We will also point out some interesting properties of the Haar measure.

Abhishek Banerjee

Hecke operators on line bundles over modular curves

Connes and Moscovici have introduced, for a principal congruence subgroup Γ of $SL_2(\mathbb{Z})$, a modular Hecke algebra $\mathcal{A}(\Gamma)$ that incorporates both the pointwise multiplicative structure of modular forms and the action of the classical Hecke operators. It is well known that a Γ-modular form g of weight k may be described as a global section of the k-th tensor power of a certain line bundle $p(\Gamma) : \mathscr{L}(\Gamma) \longrightarrow \Gamma\backslash\mathbb{H}$. The purpose of this talk is to develop a theory of modular Hecke algebras for Hecke correspondences between the line bundles $\mathscr{L}(\Gamma)$ that lift the classical Hecke correspondences between modular curves $\Gamma\backslash\mathbb{H}$.

Teodor Banica

Quantum permutations and Hadamard matrices

A well-known example of subfactors of integer index, still waiting to be understood, are those associated to the complex Hadamard matrices. The invariants of these subfactors, studied by Jones, are known to appear as well as multiplicities in the representation theory of a certain quantum permutation group. The challenging question is to put this quantum group, and the whole Hadamard problematics in general, into the context of Voiculescu's free probability and/or Connes' noncommutative geometry. I will describe in my talk several questions and results in this direction, which are partly conjectural. (Joint work with Bichon, Collins, Schlenker and Skalski.)

Julien Bichon

Hochschild homology of Hopf algebras and free Yetter-Drinfeld modules

In this talk, based on arXiv:1204.687, I will explain how one can relate the Hochschild homologies of Hopf algebras having equivalent tensor categories of comodules, in case the counit of one of the Hopf algebras admits a resolution by free Yetter-Drinfeld modules. This general procedure is applied to the quantum group of a bilinear form, for which generalizations of results by Collins, Hartel and Thom in the orthogonal case are obtained.

Martijn Caspers

The operator algebra approach to quantum Gelfand pairs

Consider pairs of operator algebraic quantum groups $\mathbb{G}$ and $\mathbb{H}$ such that $\mathbb{H}$ is compact and is identified as a quantum subgroup of $\mathbb{G}$. In this talk we focus on the corepresentations of $\mathbb{G}$ that admit vectors that are invariant under the action of $\mathbb{H}$. In particular, we look at quantum Gelfand pairs. Using von Neumann algebraic techniques we show how a quantum version of the Plancherel-Godement theorem can be established. This decomposition theorem is made precise for extended

$SU_q(1,1)$ with the circle as subgroup. Peculiar fact is that though that the 'extension' of $SU_q(1,1)$ excludes this pair from being a Gelfand pair, this example still incorporates all the Gelfand pair properties one could wish for.

Ludwik Dabrowski

Gauss-Bonnet formula on curved noncommutative torus

Perturbations of the flat geometry of the noncommutative two-dimensional torus will be presented. They are described by spectral triples with the Dirac operator, which is a differential operator with (matrix) coefficients in the commutant of the algebra of smooth functions. The zeta-function at 0 vanishes and so the Gauss-Bonnet formula holds, up to the second order in perturbation. The first two terms of the perturbative expansion of the corresponding local scalar curvature are calculated.

Kenny De Commer

Non-compact quantum groupoids and reflection equation algebras

We will discuss our approach to construct quantizations of a certain family of non-compact semi-simple Lie groups. In general, these quantizations will not be quantum groups, but quantum groupoids with a finite, classical set of objects. The above construction is closely connected with the representation theory of reflection equation algebras, endowed with a particular real structure. Concerning the latter we will present some open problems and provide some possible lines of attack. We also consider the example of quantum $SU(2)$ to show how the above abstract theory works out in practice.

Piotr M. Hajac

Free actions of compact quantum groups on unital C*-algebras

Let F be a field, Γ a finite group, and $\mathrm{Map}(\Gamma, F)$ the Hopf algebra of all set-theoretic maps $\Gamma \to F$. If E is a finite field extension of F and Γ is its Galois group, the extension is Galois if and only if the canonical map resulting from viewing E as a $\mathrm{Map}(\Gamma, F)$-comodule is an isomorphism. Similarly, a finite covering space is regular if and only if the analogous canonical map is an isomorphism. In this talk, we extend this point of view to actions of compact quantum groups on unital C*-algebras. We prove that such an action is free if and only if the canonical map (obtained using the underlying Hopf algebra of the compact quantum group) is an isomorphism. In particular, we are able to express the freeness of a compact Hausdorff group action on a compact Hausdorff space in algebraic terms. (Joint work with P. F. Baum and K. De Commer).

Nikolay Ivankov

On a category of unbounded correspondences

We develop the categorical framework for the Baaj-Julg picture of KK-theory. We show the possibility to construct a weak 2-category with finitely generated C*-algebras as objects and certain kind of unbounded KK-cycles with additional data as morphisms. In this picture, the 1-morphisms from a C^*-algebra A to a C*-algebra B may be considered as a maps from the a midly restricted set of unbounded A,C-KK-cycles to analogous set of unbounded B,C-KK-cycles, where C is an arbitrary separable C*-algebra.

Paweł Kasprzak

Closed quantum subgroups

The notion of a closed quantum subgroup of a given locally compact quantum group was introduced by S. Vaes. His definition uses a mixture of the C*-reduced, C*-universal, and von Neumann algebraic version of a given quantum group and its dual. Recently, S.L. Woronowicz proposed a definition based on the notion of a bicharacter and the concept of the C*-algebra generated by a quantum family of elements. In my talk, I shall give a number of equivalent characterizations of

Woronowicz's definition and make a link between the Vaes and Woronowicz approaches. (Joint work with Matthew Daws, Adam Skalski and Piotr M. Soltan.)

Ulrich Krähmer

Batalin-Vilkovisky structure on Ext and Tor

This talk discusses the algebraic structure of homology theories defined by a left Hopf algebroid U over a possibly noncommutative base algebra A, such as for example Hochschild or Poisson (co)homology. The main result is that under suitable conditions, cohomology and homology form what Nest, Tamarkin, Tsygan and others call a noncommutative differential calculus. As an application, Ginzburg's result that the cohomology ring of a Calabi-Yau algebra is a Batalin-Vilkovisky algebra is generalised to twisted Calabi-Yau algebras such as the standard quantum groups, quantised universal enveloping algebras, the standard quantum 2-sphere or quantum vector spaces.

Yulia Kuznetsova

A duality of locally compact groups which does not involve the Haar measure

We present a duality construction for locally compacts groups that is simpler than the theory of Kac algebras and does not involve the Haar measure in the definition of the duality functor. On the category of Hopf C^*-algebras (with no Haar weight given), we define a functor $A \mapsto \hat{A}$ such that for every locally compact group G, the algebra $C_0(G)$ is reflexive, and its dual is $C^*(G)$. Moreover, any commutative Hopf C^*-algebra isomorphic to its second dual is equal to $C_0(G)$ for some locally compact group G.

David Kyed

Amenability and dimension-flatness

I will describe a notion of amenability for subalgebras in a finite von Neumann algebra and explain how it leads to a certain dimension-flatness result, and how it connects with the existing operator algebraic notions of amenability. As an application, I will show how this unifies the existing vanishing theorems for L^2-Betti numbers in the presence of amenability and how it leads to a positive answer to a conjecture due to Lück in a group-measure space theoretic setting. (This is based on joint works with Vadim Alekseev and Henrik D. Petersen.)

Ralf Meyer

C*-Quantum groups with projection

C*-Quantum groups with projection are the quantum group analogue of semidirect products of groups. In parallel to a theorem by Radford for Hopf algebras with projection, we describe a C*-quantum group with projection using a quantum group in a braided tensor category of coactions of a certain quasitriangular C*-quantum group. (Joint work in progress with Sutanu Roy and Stanislaw Lech Woronowicz.)

Henri Moscovici

Modular curvature for conformal metrics on noncommutative 2-tori

This talk will present the main results of my joint work with A. Connes, arXiv:1110.3500, on the conformal geometry of noncommutative 2-tori in the framework of modular spectral triples. We obtained explicit expressions for the curvature functionals determined by the value at zero of the zeta functions of Laplacians, in terms of the generating function for the Bernoulli numbers applied to the modular operator. Computer aided calculations based on Connes' adaptation of the pseudodifferential calculus lead to explicit but intricate curvature formulas, which besides the above generating function involve functions of two variables in the modular operator. The dependence of the latter on the Bernoulli generating function is elucidated by computing in two different ways the gradient of the analytic torsion on the space of conformal factors. We also proved the analogue of the classical result asserting that the maximum value of the determinant of the Laplacian for metrics of fixed area is attained only at the constant curvature metric.

Colin Mrozinski

Quantum groups of $GL(2)$ representation type

We classify the cosemisimple Hopf algebras whose corepresentation semi-ring is isomorphic to that of $GL(2)$. This leads us to define a new family of Hopf algebras which generalize the quantum similitude group of a non-degenerate bilinear form. A detailed study of these Hopf algebras gives us an isomorphic classification and the description of their corepresentation categories.

Michael Mueger

Isocategoricity of compact groups

Ryszard Nest

On quantization of Hamiltonian actions of Poisson Lie groups

This is a report on work in progress, joint with Boris Tsygan. We will give a general scheme for formal quantization of Hamiltonian actions of Poisson Lie groups on Poisson manifolds and construct some explicit examples.

Réamonn Ó Buachalla

Noncommutative Kähler geometry and the quantum flag manifolds

Despite over 25 years of intense research in noncommutative Riemannian geometry, noncommutative complex geometry is only now beginning to receive serious attention. The idea of a noncommutative version of Kähler geometry is even more unexplored. The most obvious candidate here for a family of examples is the family of quantum flag manifolds. However, even the noncommutative Riemannian geometry of these spaces is still quite poorly understood. What we do know, thanks to the work of Heckenberger and Kolb, is that associated to every quantum flag manifold is a differential complex deforming the classical de Rham complex. Moreover, this complex is unique up to certain natural assumptions. This talk will present a general framework for re-expressing these complexes as quantum associated bundles in the sense of Majid. When applied to quantum projective N-space, this reformulation naturally yields a noncommutative Dolbeault double complex, a Hodge operator, a Kähler metric, and a deformation of the Kähler identities. Moreover, it proposes natural candidates for Kähler spectral triples in the sense of Fröhlich, Grandjean, and Recknagel.

Claudia Pinzari

On the connected components of compact matrix quantum groups

S. Wang has recently introduced the notion of a connected compact quantum group using representation theory. Starting from this, we introduce the connected component and the notion of a totally disconnected cqg. Unlike the classical case, we note that, by connection with the classical Burnside problem, totally disconnected compact matrix quantum groups are not finite, not even profinite in general. Motivated by this, we give conditions implying normality of the connected component and finiteness of the quantum group of connected components. We finally discuss an example arising from Drinfeld's quantization of $SU(2)$ for negative values of the deformation parameter. (Work in progress with L. Cirio, A. D'Andrea and S. Rossi.)

Alexei Pirkovskii

Noncommutative analogues of Stein spaces of finite embedding dimension

We define and study Fréchet algebras which may be viewed as noncommutative analogues of algebras of holomorphic functions on Stein spaces. Our algebras are called holomorphically finitely generated (HFG) and are defined in terms of J. L. Taylor's free holomorphic functional calculus. We show that a commutative Fréchet algebra is HFG if and only if it is isomorphic to the algebra of holomorphic functions on a Stein space of finite embedding dimension. Combined with a theorem of O. Forster, this implies that the category of commutative HFG algebras is anti-equivalent to the category of Stein spaces of finite embedding dimension. Thus arbitrary (i.e., noncommutative) HFG algebras may be viewed as "noncommutative Stein spaces of finite embedding dimension".

We show that the category of HFG algebras is stable under quotients and under analytic free products. This fact implies that many natural Fréchet algebras are in fact HFG, and also yields a number of new examples. For instance, the Arens-Michael envelope (i.e., the completion w.r.t. the family of all submultiplicative seminorms) of a finitely generated algebra is HFG. Following the above "noncommutative" philosophy, we may view the algebras obtained in this way as "algebras of holomorphic functions on noncommutative affine varieties" (or, more exactly, on "noncommutative affine schemes of finite type"). Among concrete examples of such algebras are the algebras of holomorphic functions on the quantum affine space and on the quantum torus. Other examples of HFG algebras include the algebras of holomorphic functions on the free polydisk, on the quantum polydisk, and on the quantum ball. Quite surprisingly, it turns out that the algebras of holomorphic functions on the quantum polydisk and on the quantum ball are isomorphic, in contrast to the classical case.

Georgy Sharygin

Deformation quantisation and integrable systems

In my talk I will address the question, if one can determine a deformation quantisation of a commutative Poisson algebra so that a given commutative subalgebra (of Poisson-commuting elements) inside it remains undeformed. I will describe an infinite series of Hochschild cohomology classes, which are obstructions to this process. These classes in a particular case of polynomial algebras coincide with the classes introduced by Garay and van Straten.

Adam Skalski

On spectral triples on crossed products by equicontinuous actions

A Kasparov-product inspired method of constructing spectral triples on crossed products by actions of discrete groups is discussed. A sufficient condition for the method to work, earlier introduced and applied by Jean Belissard, Mathilde Marcolli and Kamran Reihani for actions of $\mathbb{Z}$, is identified with the topological equicontinuity of the action, if only the original triple is Lipschitz regular (in the sense of Rieffel). We discuss certain examples and further related problems. (Joint work with Andrew Hawkins, Stuart White and Joachim Zacharias.)

Piotr Soltan

Generation of C*-algebras and the concept of a closed quantum subgroup

In the work of S.L. Woronowicz strong emphasis is placed on an extended notion of a C*-algebra generated by a quantum family of elements. I will describe this notion as well as some related concepts. Various examples will be presented and finally I will relate this topic to the recent work on closed quantum subgroups of locally compact quantum groups.

Vardan Tepoyan

Isometric representations of totally ordered semigroups

Let S be a subsemigroup of an abelian torsion-free group G satisfying $S \cap (-S) = \{0\}$. Generalizing well-known results of Coburn and Douglas, Murphy proved that if $G = S \cup (-S)$ (i.e. S induces a total order on G), then all C^*-algebras that are generated by a faithful isometric non-unitary representation of S are canonically isomorphic. In our talk, we will show that the converse statement holds. Moreover, considering $G = \mathbb{Z} \times \mathbb{Z}$, we will prove that if the subsemigroup S induces a total archimedean order on G, then there exist at least two irreducible isometric representations of S that are not unitarily equivalent. In contrast, if $G = \mathbb{Z} \times \mathbb{Z}$ is equipped with the lexicographical order and $S = G^+$, then all such representations are unitarily equivalent. (Joint work with M. Aukhadiev).

Leonid Vainerman

On $\mathbb{Z}/2\mathbb{Z}$-graded fusion categories

We classify $\mathbb{Z}/2\mathbb{Z}$-graded fusion categories whose 0-component is pointed. A number of concrete examples is presented. (Joint work with J.-M. Vallin.)

Alfons Van Daele

Morita equivalence of algebraic quantum groups

Consider a finite-dimensional Hopf algebra (A, Δ_A), acting from the right on a finite-dimensional space Y. Assume that also Y carries a coassociative map $\Delta_Y : Y \to Y \otimes Y$, compatible with the action in the sense that $\Delta_Y(ya) = \Delta_Y(y)\Delta_A(a)$ for all $y \in Y$ and $a \in A$. Then (Y, Δ_Y) is called a right *Morita module coalgebra* if the canonical maps T and T' from $Y \otimes A$ to $Y \otimes Y$, defined by

$$T(y \otimes a) = \Delta_Y(y)(1 \otimes a) \qquad \text{and} \qquad T'(y \otimes a) = \Delta_Y(y)(a \otimes 1)$$

are bijective. It can then be shown that there exist a reflection. It is another finite-dimensional Hopf algebra C, acting from the left on Y so that Y is a left Morita module coalgebra. The two Hopf algebras A and C are said to be (strongly) Morita equivalent. In this talk, I will consider the generalization of this result to multiplier Hopf algebras and in particular to algebraic quantum groups (i.e. multiplier Hopf algebras with integrals). Recall that all discrete and all compact quantum groups are algebraic quantum groups. (This is joint work in progress with Kenny De Commer.)

Christian Voigt

Proper actions of discrete quantum groups

We discuss the notion of torsion and proper actions in the context of discrete quantum groups. This is closely related with the problem of defining the correct analogue of the Baum-Connes assembly map for general discrete quantum groups.

Stanislaw Lech Woronowicz

Quasitriangular quantum groups and braided categories of C*-algebras

Let G be a quasitriangular locally compact quantum group and C^*_G be the category of C*-algebras with actions of G. We shall show C^*_G is equipped with a natural associative crossed product $\boxtimes$ that is a functor from $\mathrm{C}^*_G \boxtimes \mathrm{C}^*_G$ into C^*_G. The theory of locally compact quantum groups involved in the game is based on the concept of manageable multiplicative unitaries, in particular we do not assume the existence of Haar weights. (Joint work with Ralf Meyer and Sutanu Roy.)

Makoto Yamashita

Deformation of algebras associated to group cocycles

Fell bundles over a discrete group can be deformed using 2-cocycles on the base group. We give a K-theoretic isomorphism of such deformations, generalizing the previously known cases of the theta-deformations and the reduced twisted group algebras. We shall also discuss the pairing of invariant cyclic cocycles with the K-groups of deformed algebras.

Robert Yuncken

K-homology and harmonic analysis on the quantum flag variety of $SU_q(3)$

We describe a Dirac-type class in equivariant K-homology for the full flag variety of $SU_q(3)$. The construction is based upon the Bernstein-Gelfand-Gelfand complex, which uses the canonical fibrations of the flag variety. In the process, we develop the longitudinal harmonic analysis associated to these fibrations.