WEBINAR
RECENT DEVELOPMENTS IN STOCHASTICS WITH APPLICATIONS IN MATHEMATICAL PHYSICS AND FINANCE

LIST OF SPEAKERS AND ABSTRACTS

- **Heni Abidi**
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Numerical approximation of Hilbert space valued BSDE with jump

Abstract: We focus on numerical approximation for BSDE with jump in Hilbert space. Firstly, we give a spatial approximation. Then, we finish with a time discretization method. Some examples will be studied.

- **Wolfgang Bock**
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Recent Results on Weakly Self-Avoiding fractional Brownian Motion

Abstract: In this talk we use the combination of Dirichlet form methods and white noise analysis to construct a Markov process, which has the Edwards density w.r.t. the fractional white noise as invariant measure. We derive properties of this process in a special case and investigate a coarse-grained version. Numerical simulations show special properties of waekly self-avoiding fractional random walks which were unknown up to the speakers knowledge.

- **Francesco De Vecchi**
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Stochastic quantization of exponential quantum field theory

Abstract: We give a review of the results on the elliptic and parabolic Eucledian stochastic quantization of the two dimensional scalar field with exponential interaction (known also as Høegh-Krohn or Liouville model in the literature). This problem can be reduced to the study of a random PDE with a multiplicative noise given by the Wick exponential $:e^{\beta\Psi}:$ of the free Gaussian field Ψ. The distribution $:e^{\beta\Psi}:$, differently of the Wick powers $:\Psi^k:$ and the Wick trigonometric functions $:e^{i\beta\Psi}:$, is a positive Radon measure and it has a Besov regularity decreasing with the increasing of the integrability. The talk is mainly based on the joint work [1] with Sergio Albeverio and Massimiliano Gubinelli.

[1] Albeverio, Sergio, Francesco C. De Vecchi, and Massimiliano Gubinelli. "The elliptic stochastic quantization of some two dimensional Euclidean QFTs." *arXiv preprint arXiv:1906.11187* (2019).

• **Mao-Fabrice Djete**
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Mean Field Games of Controls: on the convergence of Nash equilibria

Abstract: We will talk about a class of mean field games where interactions are achieved through state and control processes. Using Fokker-Planck equations and a notion of measure-valued solution of MFG, we present a relationship between measure-valued MFG solutions and (approximate) Nash equilibria. More precisely, we show that the Nash equilibria of N player games have limits as N goes infinity, and each limit is a measure valued solution of the mean field games of controls. Conversely, any measure valued solution can be obtained as the limit of a sequence of N-Nash equilibria in N player games. The case of open loop controls will first be discussed, and in a second step with more restrictive assumptions, we will deal with the case of closed loop controls where some additional difficulties appear.

• **Jasmina Djordjevic**
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Perturbation effects on Backward Doubly Stochastic Differential Equations & their applications

Abstract: Problems of perturbations the field of backward stochastic differential equations (BSDEs) in general, are very useful and lead to many interesting results in theory and applications. Using a type of additive perturbation, a class of backward doubly stochastic differential equations (BDSDEs) is extended on so called nonhomogeneous type which in diffusion contains also a function dependent of a state process. The extensions is done under several conditions, Lipschitz, non-lipschitz, linear growth condition. In the same framework, L^p estimates are determent, comparison theorems and extension, Feyman-Kac formula, as well as Kneser problem for nonhomogeneous BDSDE.

Furthermore, problem of linearly perturbed nonhomogeneous BDSDE is observed and usual estimates are performed; the L^p-difference of the solutions of perturbed and unpertubed equations, L^p-stability, and for an arbitrary $\eta > 0$ an interval $[t(\eta), T] \subset [0, T]$ on which the L^p-difference between the solutions of both the perturbed and unperturbed equations is less than η could be established.

• **Olfa Draouil**
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White Noise calculus for time change processes

Abstract: In this talk, we investigate the time change process $\Lambda(t)$ in the framework of infinite dimensional analysis and especially in the theory of White Noise calculus. Using the Bochner- Minlos theorem, we introduce on the dual of the Schwartz space a conditional measure associated to the process of the time change $\Lambda(t)$. This conditional measure is a generalization of the classical Gaussian measure in the white Noise distribution theory when $\Lambda(t) = t$. Next we provide a generalization of Hida and Potthof distributions spaces associated to this conditional measure. Then we prove that the Brownian motion at the process $\Lambda(t)$ is a martingale with respect to an associated enlarged filtration.

• **Martin Keller-Ressel**
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Sparse and Semistatic Variance-Optimal Hedging

Abstract: We consider the problem of hedging a contingent claim with a "semistatic" strategy composed of a dynamic position in one asset and static (buy-and-hold) positions in other assets. We give general representations of the optimal strategy and the hedging error under the criterion of variance optimality and provide tractable formulas using Fourier integration in case of the Heston model. We also consider the problem of optimally selecting a sparse semistatic hedging strategy, i.e., a strategy that only uses a small subset of available hedging assets and discuss parallels to the variable-selection problem in linear regression. The methods developed are illustrated in an extended numerical example where we compute a sparse semistatic hedge for a variance swap using European options as static hedging assets.

• **Peter Kuchling**
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Anomalous Flocking in the Fractional Cucker-Smale Model

Abstract: The Cucker-Smale model is used to model a system of self-aligning agents such as groups of birds. By introducing the Caputo fractional time derivative, memory effects may be incorporated into the system. In this talk, we discuss sufficient conditions for the emergence of flocking in this model. We show that the fractional time-derivative causes the alignment to occur at an algebraic rate as opposed to the exponential rate of the classical model.

(Joint work with Seung-Yeal Ha and Jinwook Jung)

• **Achref Lemjid**
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Functional central limit theorems and $P(\phi)_1$-processes for the relativistic and non-relativistic Nelson models

Abstract: We construct $P(\phi)_1$-processes indexed by the full time-line, separately derived from the functional integral representations of the relativistic and non-relativistic Nelson models in quantum field theory. These two cases differ essentially by sample path regularity. Associated with these processes we define a martingale which, under an appropriate scaling, allows to obtain a central limit theorem for additive functionals of these processes. We discuss a number of examples by choosing specific functionals related to particle-field operators.

• **Bernt Øksendal**
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A financial market with singular drift and no arbitrage

Abstract: What happens if a large investor in a financial market intervenes to prevent the price of a certain asset from going below a certain barrier? Or when the central bank of a country tries to maintain the country's currency exchange rate above a certain level by intervening when it goes below that level?

According to a folklore based on historical examples, this type of intervention creates arbitrage in the market, i.e. it makes it possible for agents to obtain a profit without taking any risk. To understand this mathematically, one models this kind of intervention as a reflection upwards of the price process representing the given asset/exchange rate. If the price process a priori is a modelled as the solution of a stochastic differential equation driven by Brownian motion, then by the Tanaka formula, introducing reflection of the process is essentially the same as introducing a singular (e.g. a local time) term in the drift. And it has been known for more than 20 years, that if we have a financial market with a singular term in the drift of one of the risky assets, then one can construct arbitrages in this market.

(i) However, the situation is not clear if there are jumps in such a singular drift market. Do we still. have arbitrages then?

(ii) Moreover, the above result is based on the assumption that the agent can act immediately on changes in the prices. What happens if there is an information delay in the system?

In this talk we consider a jump diffusion market model with a singular drift term modelled as the local time of a given process, and with a delay $\theta > 0$ in the information flow available for the trader. We allow the stock price dynamics to depend on both a continuous process (Brownian motion) and a jump (Lévy) process. We believe that jumps and delays are essential in order to get more realistic financial market models. Using white noise calculus we compute explicitly the optimal consumption rate and portfolio in this case and we show that the maximal value is finite as long as the delay $\theta > 0$. This implies that there is no arbitrage in the market in that case. However, when theta goes to 0, the value goes to infinity. This is in agreement with the above result that there is an arbitrage when there is no delay.

Our model is also relevant for high frequency trading issues. This is because high frequency trading often leads to intensive trading taking place on close to infinitesimal lengths of time, which in the limit corresponds to trading on time sets of measure 0. This may in turn lead to a singular drift in the pricing dynamics.

The talk is based on joint work with Nacira Agram, Linnaeus University (LNU), Sweden.

• **Andreas Petersson**
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Approximating the covariance operator of the solution to the stochastic wave equation

Abstract: The problem of approximating the covariance operator of the mild solution to the stochastic wave equation on bounded domains in Euclidean space is considered. The equation is linear (but inhomogeneous) and driven by Gaussian noise, which ensures that this operator determines the law of the solution. Existence and uniqueness for an operator-valued integral equation for the covariance based on the semigroup representation of the mild solution is proven, along with an accompanying error decomposition formula for general approximations of the covariance. As an example, a temporal semidiscretization based on a rational approximation of the semigroup is considered. The presentation concludes with a discussion on how this discretization can be used to derive error bounds for a fully discrete method based on finite elements.

(Joint work with Annika Lang and Mihály Kovács)

• **Nizar Touzi**
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Continuous time optimal contracting and nonlinear representation of random variables

Abstract: We review recent results on cpntinuous time optimal contracting. As an application we provide a rigorous revisit of the continuous-time optimal contracting problem introduced by Sannikov, in the extended context allowing for possibly different discount rates of both parties. The agent's problem is to seek for optimal effort, given the compensation scheme proposed by the principal over a random horizon. Then, given the optimal agent's response, the principal determines the best compensation scheme in terms of running payment, retirement, and lump-sum payment at retirement.