National Mathematicians meeting 2024

The book of abstracts

Plenary sections

Gunnar Fløystad (UiB).

Title: Ordensstrukturer i matematikk

Abstract: Orden og symmetri er fundamentale strukturelle prinsipper i matematikk. Orden i sin mest basale form gir at for to objekter a og b så er det ene "foran" det andre, eller der er en "retning" fra det ene til det andre. Vi kan angi det som ab eller ba. Disse to kan også være urelatert, eller vi kan tillate at både ab og ba forekommer. Med to ytterligere enkle krav får vi det grunnleggende begrepet *preorden*. Andre enkle ordensbegreper kan også defineres.

Marie Rognes (Simula).

Title: Computational mathematics and brain multiphysics.

Abstract: TBA

Geir Dahl (UiO).

Title: Combinatorial Matrix Theory - an invitation.

Abstract: Combinatorial Matrix Theory (CMT) is an area dealing with the interplay between combinatorial structures and matrices.

The goal in this talk is to illustrate a few topics and results in CMT that are of a rather general nature. The presentation will be non-technical, so I will try to live up to the term "invitation" in the title. A couple of interesting combinatorial matrix classes, and related polytopes, are discussed and some connections to graph theory, graph spectra, computations and applications are given. The talk will also show how early work by Frobenius and others can be seen as the starting point of CMT. Some recent developments are briefly mentioned. Welcome!

Kristian Seip (NTNU).

Title: Fourier interpolation and the uncertainty principle

Abstract: Research on Fourier interpolation emerged from a celebrated 2019 paper of Radchenko and Viazovska which was an outgrowth of the solution to the sphere packing problem in dimension 8. I will give an overview of this and subsequent developments, with emphasis on a connection with the nontrivial zeros of the Riemann zeta function. I will comment briefly on Freeman Dyson's idea to approach the Riemann hypothesis by describing all one-dimensional quasicrystals.

Lecture by the Viggo Brun 2024 laureate

TBA.

Title: TBA

Abstract: TBA

Elisabeth Stephansen lecture

Giulia Di Nunno (UiO).

Title: TBA

Abstract: TBA

Section 1

Erlend Grong (UiB).

Title: Flows of vector fields on manifolds with boundary and corners.

Abstract: Lie groups are spaces where we have a smooth multiplication operation. Similar to how the matrix exponential map is a function from the linear space of all matrices to the non-linear space of invertible matrices, we can define a exponential map on each Lie group from an associated linear space.

The space of all diffeomorphisms of a compact manifold M can be considered as an infinite dimensional Lie group, where the image of its exponential map is given by all flows of vector fields. Although this exponential map is not locally surjective close to the identity, we can still generate any diffeomorphism by a finite composition of vector field flows. This result follows from Thurston's observation that the diffeomorphism group of a compact manifold M is simple, as long as M does not have a boundary.

Our results show that for compact manifolds with a smooth boundary, and possibly corners, we are still able to recover all diffeomorphisms by flows, even though the group will no longer be simple. We will in fact give a more detailed result for when a smaller subspace of vector fields is sufficient.

We will also show a discretized version of the previous problem, where we are able to transport an arbitrary number of landmarks to their desired destination using only flows of two vector fields.

The results are part of joint works with Alexander Schmeding (NTNU, Trondheim), Helge Glöckner (Paderborn) and Sylvie Vega-Molino (Bergen).

Franz Luef (NTNU).

Title: From time-series data to quantum harmonic analysis

Abstract: Time-series data are of relevance in a wide range of applications and thus processing this data appropriately is one of the main goals of data science. Since most time series data are assumed to be time-dependent, functional data analysis can be an attractive framework for this task.

However for infinite dimensional function spaces, finding appropriate representations of data is non-trivial. In one possible approach, one can use wavelet or time-frequency methods for functions, although these are for the most part not tuned to specific data sets. In another approach, using a method such as (functional) PCA gives a data set-specific decomposition, while one loses certain advantages of time-frequency or wavelet analysis, in particular stability from redundancy. By leveraging tools from Quantum Harmonic Analysis, we are able to combine the advantages from both approaches, giving a data set-specific, stable representation of functional data.

We try to describe briefly the main ideas of this approach, which is based on joint work with Monika Doerfler, Henry McNulty and Eirik Skrettingland.

Kurusch Ebrahimi-Fard (NTNU).

Title: Surface Signatures: Addressing Algebraic and Analytic Challenges

Abstract: The concept of path signatures has proven highly successful in data science applications by providing features that describe paths. This is partly due to the ability to compute the signature of a path in linear time, leveraging a dynamic programming principle based on Chen's identity. The path signature, an example of a product or time-ordered integral, is a 1-parameter object built on iterated integrals over a path. Unlike time series data, image data naturally corresponds to two-parameter surfaces in multidimensional space. Recent research has focused on extending the concept of signatures to multi-parameter surfaces. However, increasing the number of parameters, which involves iterated integrals over surfaces, introduces additional complexity—a challenge familiar in higher gauge theory where multi-parameter iterated integrals are significant. Based on work from the 2023/2024 project "Signatures for Images" at the Centre for Advanced Study (CAS) in Oslo, I will present two approaches to defining surface signatures. The algebraic and analytic structures involved are intricate, and I will discuss our current understanding and interesting observations. Based on joint work with Ilya Chevyrev, Joscha Diehl, Fabian Harang, Samy Tindel, and Nikolas Tapia (arXiv:2403.00130 and arXiv:2406.16856).

Sigmund Selberg (UiB).

Title: Singularity removal and global regularity for the $1+3$-dimensional Yang-Mills equations in the Lorenz gauge

Abstract: The Yang-Mills equations (YM) arise as an analogue of the Maxwell equations for a non-abelian gauge group. In the Lorenz gauge, YM takes the form of a non-linear system of wave equations. In three space dimensions, despite the fact that there is a conserved energy, blow-up may occur in finite time starting from smooth, compactly supported initial data. This follows from an example due to Shatah for the Wave Maps equation into the 3-sphere, which corresponds to the gauge group $SU(2)$. I will discuss how it is possible to avoid such singularities, for any smooth initial data, by making a local gauge change, within the confines of the Lorenz gauge, which allows the smooth continuation of the solution in a cone containing the original singularity, where the size of the cone depends only on the size of the total initial energy. We also prove that the solution can be continued globally in space-time, in the sense of gauge-equivalent solutions on overlapping cones covering the space-time. In the proof we apply the Uhlenbeck Lemma to make a local gauge change of the data in the base of the cone, and use a local existence theorem for low-regularity (finite-energy) data. The finite-energy local existence theorem for YM in the Lorenz gauge was proved in earlier work with A. Tesfahun (JEMS 2016) and relies crucially on showing that the main bilinear terms have a so-called "null structure", which counteracts non-linear wave interactions and improves the regularity. A slight complication is that this null structure relies on non-local transformations globally in space (Riesz transforms), so localising to cones is not trivial. To get around this problem we use an extension theorem for YM data, where the main issue is to preserve the Gauss law constraint.

Section 2

Katrin Grunert (NTNU).

Title: The Camassa–Holm equation - Uniqueness

Abstract: Solutions of the Camassa–Holm (CH) equation might enjoy wave breaking in finite time. This means that even classical solutions, in general, do not exist globally, but only locally in time, since their spatial derivative might become unbounded from below pointwise while the solution itself remains bounded and continuous. Furthermore, energy concentrates on sets of measure zero when wave breaking occurs. Thus the prolongation of solutions beyond wave breaking is non-unique and depends heavily on how the concentrated energy is manipulated. The two most prominent continuations are called dissipative, *i.e.*, the concentrated energy is taken out, and conservative, *i.e.*, the energy is given back into the system. The existence of both classes of solutions has been established with the help of a generalized method of characteristics, which allows to rewrite the Camassa–Holm equation as a system of ODEs in Lagrangian coordinates [3].

Here we will focus on the uniqueness of conservative [1] and dissipative solutions [2]. After motivating why weak solutions are not unique, we will discuss the overall idea behind proving uniqueness: Establishing a bijection between the properties satisfied by each class of solutions and the corresponding solution operator. The main ingredients are measure transport equations, ideas from sub- and supersolutions as well as a good understanding of the characterisation of equivalence classes in Lagrangian coordinates. Furthermore, we will compare the necessary properties guaranteeing uniqueness for dissipative solutions to those for the conservative ones.

References

[1] A. Bressan, G. Chen, and Q. Zhang, *Uniqueness of conservative solutions to the Camassa–Holm equation*, Discrete Contin. Dyn. Syst. **35** (2015), 25–42.
[2] K. Grunert, *Uniqueness of dissipative solutions for the Camassa–Holm equation*, arXiv:2311.15344.
[3] K. Grunert, H. Holden, and X. Raynaud, *A continuous interpolation between conservative and dissipative solution for the two-component Camassa–Holm system*, Forum Math. Sigma **3** (2015), Paper No. e1, 73pp.

Fred Espen Benth (UiO).

Title: TBA

Abstract: TBA

Kristina Rognlien Dahl (BI).

Title: Stochastics and stochastic optimal control

Abstract: In this talk we will give an introduction to the theory of stochastic optimal control as well as some recent applications.

Stochastic optimal control is a field within stochastic analysis which deals with the following problem: We aim to maximize a performance function depending on the state of a system. The system state is given by a stochastic differential equation which we can

control. There are two main approaches to solving this problem: The dynamic programming approach and the maximum principle approach. In this presentation, we will discuss the similarities and differences between these two approaches.

When using the maximum principle approach for solving the stochastic optimal control problem, we end up with a system of backward stochastic differential equations. We will discuss these kinds of systems and also describe some recent applications of stochastic optimal control to infectious diseases and default processes.

Torstein Kastberg Nilsen (UiA).

Title: Yudovich theory for rough perturbations of Euler's equation

Abstract: In the talk we will begin with some insights from Vladimir Arnold about the geometric structure of Euler's equation and how it can be used to introduce noise terms which preserve physical properties of the equation. We will then consider a purely Lagrangian formulation of the equation and see how to obtain well-posedness in the class of L^∞-solutions when $d = 2$, sometimes referred to as Yudovich theory.

Section 3

John Rognes (UiO).

Title: The even filtration

Abstract: I will motivate the search for a motivic cohomology theory defined for the brave new rings of stable homotopy theory, and explain how the even filtration of Hahn-Raksit-Wilson provides a useful solution to this problem.

Cordian Riener (UiT).

Title: TBA

Abstract: TBA

Andreas L. Knutsen (UiB).

Title: TBA

Abstract: TBA

Kris Shaw (UiO).

Title: TBA

Abstract: TBA

Section 4

André Massing (NTNU).

Title: TBA

Abstract: TBA

Susanne Solem (NMBU).

Title: TBA

Abstract: TBA

Thordis Thorarinsdottir (UiO).

Title: TBA

Abstract: TBA

Yan Li (UiB).

Title: Mathematical models and statistical properties for ocean hazards in the form of rogue waves

Abstract: Rogue waves are referred to as the ocean surface waves which appear out of nowhere with an amplitude much larger than their ambient environment. In this talk, I will present a few mathematical models which are widely used to model and understand rogue wave events, e.g., these based on Nonlinear Schrödinger equations and Zakharov equations. Statistical properties, which are used to quantify and indicate the rogue wave events compared with a Gaussian random process, will be addressed, including skewness, kurtosis, and the occurrence probability of rogue waves.