Macaulay 2 script to see that intersection of Pi_6 and Omega is a scroll over a Veronese surface (in two ways).

```
R=QQ[x,y,z,t,u,v] --the coordinates of W
M=matrix{{-x,0,x,0,0,0,0,0,-x,0,u,t+v,u-y,0,t},{-y,x,0,0,0,0,-u,-v,0,0,t,
0,-v,0,0},{-y,0,0,0,x,0,-u,-v,0,0,0,-z,0,u,0},{-z,0,x+z,u,t+v,u,
0,0,0,0,0,t,u,0,0},{-t,-u,-v,0,-y,x,0,-z,0,u,0,0,-z,-t,0},{x,0,0,-y,0,0,-z,
0,0,-v,-u,-v,0,0,0},{0,-y,0,x,0,x,t,x,-v,0,-z,y,-z,-v-t,u}} --the
transposed matrix of the system of equation v\wedge l=0
I=minors(7,M)-- the set of points over which there is a nontrivial solution
of the system
I=saturate(I)
I=radical(I)
degree(I)
dim(singularLocus(I))
generators(I)

J=minors(6,M) -- the set of points over which there is a projective line of
solutions of the system
J=saturate(J)
degree(J)
generators(J)
saturate(J,I) -- I is contained in J
saturate(I,J) -- J is contained in I

N=matrix{{0,0,0,0,0,-t+v,z,0,-z,-y,-v,y+u,0,0,-z},{0,t,-z,0,0,u,0,-z,0,x,y,
0,0,0,0},{0,t,-z,0,0,0,v,0,-t,x,0,0,0,y,0},{-t+v,0,y,0,-y,v,-x,0,x-z,0,0,y,
0,0,0},{z,-y,v,0,-t,x,0,v,-u,0,0,0,0,x,y},{0,0,u,-t,0,t,-z,0,0,y,0,0,x,
0,0},{0,u,0,-z,0,0,u,-t+v,-u,v,x,-t,y,z,y}}-- the transposed matrix of the
dual system
I=minors(7,N)-- the set of points over which there is a nontrivial solution
of the dual system
I=saturate(I)
I=radical(I)
degree(I)
dim(singularLocus(I))
generators(I)

J=minors(6,N) -- the set of points over which there is a projective line of
solutions of the dual system
J=saturate(J)
degree(J)
generators(J)
saturate(J,I) -- I is contained in J
saturate(I,J) -- J is contained in I

--We shall now describe the set $\Pi_6 \cap \Omega$
R=QQ[A,B,C,D,E,F,G] -- the coordinates of $\Pi_6$
P=matrix{{-A+F,B,A+D,G,C,E+G,0,G,-A,0,0,0,0,0,0},{-B-C,-G,0,-F,-E,
```

```
0,0,0,0,0,0,G,-A,0,0},{-D,0,D,0,0,0,-F,-E,0,0,-G,-C,-E-G,0,0},{-E,0,0,0,D,
0,G,0,0,0,B,A+D,0,-E-G,A},{0,-E,0,D,0,D,-B-C,0,0,E,A-F,0,A+D,C,G},{0,0,-E,
0,D,0,0,-B-C,-G,-F,0,A-F,-B,-G,0}}--the matrix of the system with
indeterminates v and parameter l
S=minors(6,P)--the set of points in $\Pi_6 \cap \Omega$ it is a threefold
double scroll over two Veronese surfaces
S=saturate(S)
degree(S)
dim(S)
generators(S)
```

&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&

Computing the discriminant/Killing form

```
R=QQ[a,b,c,d,e,f,g,h,i,j,k,l,m,n,A,B,C,D,E,F,G,H,I,J,K,L,M,N];
MM=matrix{{g,h,i,0,f,-e,a},{j,k,l,-f,0,d,b},{m,n,-g-k,e,-d,0,c},{0,-c,b,-
g,-j,-m,d},{c,0,-a,-h,-k,-n,e},{-b,a,0,-i,-l,g+k,f},{2*d,2*e,2*f,2*a,2*b,
2*c,0}};--element of g_2 according to Sato and Kimura. \cite {SK}

NN=matrix{{G,H,I,0,F,-E,A},{J,K,L,-F,0,D,B},{M,N,-G-K,E,-D,0,C},{0,-C,B,-
G,-J,-M,D},{C,0,-A,-H,-K,-N,E},{-B,A,0,-I,-L,G+K,F},{2*D,2*E,2*F,2*A,2*B,
2*C,0}};-- another element of g_2

P=MM*NN-NN*MM; --The Lie bracket

TT=matrix{{P_(0,6)},{P_(1,6)},{P_(2,6)},{P_(3,6)},{P_(4,6)},{P_(5,6)},
{P_(0,0)},{P_(0,1)},{P_(0,2)},{P_(1,0)},{P_(1,1)},{P_(1,2)},{P_(2,0)},
{P_(2,1)}}--The action of the Lie bracket on (A,..,N)


T=matrix{{g,h,i,0,2*f,-2*e,-a,-b,-c,0,0,0,0,0},{j,k,l,-2*f,0,2*d,0,0,0,-a,-
b,-c,0,0},{m,n,-g-k,2*e,-2*d,0,c,0,0,0,c,0,-a,-b},{0,-2*c,2*b,-g,-j,-m,d,
0,0,e,0,0,f,0},{2*c,0,-2*a,-h,-k,-n,0,d,0,0,e,0,0,f},{-2*b,2*a,0,-i,-l,g
+k,-f,0,d,0,-f,e,0,0},{-2*d,e,f,2*a,-b,-c,0,-j,-m,h,0,0,i,0},{-3*e,
0,0,0,3*a,0,-h,g-k,-n,0,h,0,0,i},{-3*f,0,0,0,0,3*a,-2*i,-l,2*g+k,0,-i,h,
0,0},{0,-3*d,0,3*b,0,0,j,0,0,-g+k,-j,-m,l,0},{d,-2*e,f,-a,2*b,-c,0,j,0,-h,
0,-n,0,l},{0,-3*f,0,0,0,3*b,-l,0,j,-i,-2*l,g+2*k,0,0},{0,0,-3*d,3*c,
0,0,2*m,0,0,n,m,0,-2*g-k,-j},{0,0,-3*e,0,3*c,0,n,m,0,0,2*n,0,-h,-g-2*k}}--
The matrix of the action.

T*transpose(matrix{{A,B,C,D,E,F,G,H,I,J,K,L,M,N}})-TT -- CHECKPOINT should
be zero


S=sub(T,{a=>a,b=>b,c=>b+13*e-12*a,d=>0,e=>e,f=>0,g=>e+2*a+b,h=>e
+15*b-7*a,i=>0,j=>e+8*b-4*a,k=>0,l=>b+7*a+8*e,m=>21*e+a-3*b,n=>0})--
```

Choosing a plane section

```
U=S-diagonalMatrix{B,B,B,B,B,B,B,B,B,B,B,B,B,B}
X=sub(saturate(ideal(det(U)),B),B=>0) --the ideal of the section of D_2

Y=decompose(X)-- should get two components

dim(Y_0)-dim(singularLocus(Y_0)) -- smoothness of the first component of
the plane section

dim(Y_1)-dim(singularLocus(Y_1)) -- smoothness of the second component of
the plane section

--We now check the intersection with the line from the remark
--The Killing form is BB(MM,NN)=24(aD+Ad+bE+Be+cF+Cf)+8(gG+kK+(g+k)(G+K)+jH
+Hj+iM+Im+nL+Nl))
--The line is then given as follows
P=sub(T,
{a=>0,b=>b,c=>0,d=>-1,e=>0,f=>0,g=>0,h=>3,i=>0,j=>0,k=>0,l=>0,m=>0,n=>0})
U=P-diagonalMatrix{B,B,B,B,B,B,B,B,B,B,B,B,B,B}
X=sub(saturate(ideal(det(U)),B),B=>0) --the line from the remark meets the
discriminant in only two points: 0 and infinity
```