Obligatory problem set no. 1 for MA 001 fall 2000

Each student who (for the first time) has registered for the exam in MA 001 for the fall of 2000, should hand in an individual solution to these problems to his or her ‘‘group’’ teacher, who then corrects and grades the solutions. Those qualifying for a passing grade (4.0 or better) are considered accepted. Remember to write your full name and which MA 001 group you are following on your solution. Solutions that are not accepted may be improved and handed in again, provided this is done in time for them to be graded by two weeks before the exam.

The first deadline for submitting solutions is Friday October 6th, 2000.

Problem 1

By absolute temperature we mean temperature measured in Kelvin. The temperature x °C equals the temperature (x + 273.15) K. A black body is an object that absorbs all the radiation energy hitting it. By Stefan's law, the heat radiation (P) from a black body is proportional to the 4th power of its absolute temperature (T):

$$P = a\,T^4$$

where a is a constant of proportionality.

(a) A black body is heated from 20.0 °C to 40.0 °C. Find the percentile increase in absolute temperature. Thereafter find the percentile increase in heat radiation. Give both answers to a meaningful precision.

(b) Another black body is heated from 20.0 °C until the heat radiation is doubled. Find the new temperature of the body, measured in °C. Give the answer with a meaningful precision.

Oppgave 2

Santa Claus' workshop is to produce x hard and y soft gift parcels this year, which later are to be delivered by raindeer sleigh. Each hard parcel takes 2 days to make, weights 2 kgs and has volume 1 liter. Each soft parcel also takes 2 days to make, weighs 1 kg and has volume 3 liters. The production is subject to the following conditions:

1. the workshop is open 360 days this year,
2. the sleigh can transport 300 kgs of parcels, and
3. the sleigh has room for parcels with a total volume of 400 liters.

(a) Display the 5 inequalities that x and y must satisfy for it to be possible to produce and deliver x hard and y soft parcels. Explain which inequalities correspond to which conditions.

(b) Shade the region in the xy-plane where x and y satisfy these 5 inequalities. Find the x- and y-coordinates of the corners of the region that lie in the first quadrant. (Hint: The x-coordinates are 70 and 120, respectively.)

(c) An old Santa Claussian rule of thumb estimates that the recipients appreciate soft parcels twice as much as hard parcels. If x hard and y soft parcels are delivered, the recipients' joy is estimated to be given by the function:

$$f(x, y) = x + 2y$$

Find the number of hard and soft parcels that can be produced and which makes the estimated recipients' joy f the greatest.

(d) A more recent market survey (of submitted wish lists) shows that the recipients' joy is better approximated by the function:

$$g(x, y) = 1.4x + 1.2y$$

Find the number of hard and soft parcels that can be produced and which makes the adjusted recipients' joy g the greatest.

Oppgave 3

A wave in the xy-plane is at the time t given as the graph of the function

$$f_t(x) = 4\cos(x) + 3\cos(x-t) .$$

For each value of t, the function $f_t(x)$ describes a harmonic oscillation in the xy-plane. As t increases, this wave moves in the xy-plane.

(a) Find the mean value, the amplitude, the period and the ''acrophase'' of $f_t(x)$ when $t=0$.

(b) Find the mean value, the amplitude, the period and the acrophase of $f_t(x)$ when $t=\pi/2$.

(c) Find the mean value, the amplitude, the period and the acrophase of $f_t(x)$ when $t=\pi$.

(d) Sketch the graphs of $f_t(x)$ for $t=0$, $t=\pi/2$ and $t=\pi$ in the same xy-coordinate system. The sketch should illustrate the values you computed in (a), (b) and (c).

(e) Show that for t between 0 and π the acrophase of $f_t(x)$ is the greatest when t has the value that makes the expression

$$3\sin(t) / (4 + 3\cos(t))$$

the greatest.

John Rognes / 19. september 2000