SAMPLE K-THEORY QUESTIONS

JOHN ROGNES

We will start the exam with one of these. [M] = Master, [P] = PhD.

1. Equivalences of categories [M]

Define the classifying space $|\mathscr{C}|$ of a small category $\mathscr{C}$. When is a functor $F\colon \mathscr{C} \to \mathscr{D}$ an equivalence? A homotopy equivalence?

Which weak homotopy types of simplicial sets are realized by nerves of categories, and which are realized by nerves of groupoids?

2. Bisimplicial sets [M,P]

What is a bisimplicial set $X_{\bullet,\bullet}$?

State the realization lemma, for a map $f_{\bullet,\bullet}\colon X_{\bullet,\bullet} \to Y_{\bullet,\bullet}$ of bisimplicial sets.

3. Theorem A [M,P]

Define the left fiber of a functor $F\colon \mathscr{C} \to \mathscr{D}$. Formulate Quillen's theorem A.

Let $(P, \le)$ and $(Q, \le)$ be preordered sets, and $f\colon P \to Q$ an order-preserving function. Suppose that for each $y \in Q$ there is a greatest $x \in P$ with $f(x) \le y$. Show that $N_\bullet f\colon N_\bullet P \to N_\bullet Q$ is a weak homotopy equivalence.

4. Waldhausen categories [M,P]

Define a category with cofibrations $(\mathscr{C}, co\mathscr{C})$, and a category with cofibrations and weak equivalences (= a Waldhausen category) $(\mathscr{C}, co\mathscr{C}, w\mathscr{C})$.

For a ring A, let $\mathscr{P}(A)$ be the category of finitely generated projective left A-modules. What is the minimal categories of cofibrations and weak equivalences that make $\mathscr{P}(A)$ a Waldhausen category?

5. K-theory spectra [M,P]

Define the symmetric K-theory spectrum of a Waldhausen category.

Explain when a functor $\mathscr{C} \times \mathscr{D} \to \mathscr{E}$ induces a pairing of symmetric spectra $\mathbf{K}(\mathscr{C}) \wedge \mathbf{K}(\mathscr{D}) \to \mathbf{K}(\mathscr{E})$.

6. Abelian and exact categories [P]

Define an abelian category, and give a description of exact categories.

Give an example of an exact category that is not abelian.

7. The Q-construction [P]

Define Quillen's Q-construction for an exact category $\mathscr{P}$.

Show that $Q\mathscr{P}$ is homotopy equivalent to $s_\bullet\mathscr{P} \simeq iS_\bullet\mathscr{P}$.

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8. Resolution [P]

State and prove the resolution theorem.

Department of Mathematics, University of Oslo, Norway
E-mail address: rognes@math.uio.no
URL: http://folk.uio.no/rognes