A GALOIS EXTENSION THAT IS NOT FAITHFUL

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Ben Wieland wrote to me on August 14th 2008, pointing out that not all G-Galois extensions $A \to B$ of commutative S-algebras are faithful, thus answering Question 4.3.6 of [1] in the negative. In fact, a counterexample already appears in Proposition 5.6.3(b) of that memoir, as the case $G = \mathbb{Z}/p$ where p is prime, $A = F(BG_+, H\mathbb{Z}/p)$ and $B = F(EG_+, H\mathbb{Z}/p) \simeq H\mathbb{Z}/p$. Then $A \to B$ is a G-Galois extension by the cited proposition. However, this extension is not faithful by the criterion given in Proposition 6.3.3 of the memoir, since the Tate construction B^{tG} has homotopy $\pi_*(B^{tG}) = \hat{H}^{-*}(G; \mathbb{Z}/p)$, which is not zero. Hence $A \to B$ cannot be faithful, by the latter proposition.

Reference

[1] John Rognes, *Galois extensions of structured ring spectra*, Mem. Amer. Math. Soc. **192** (2008), no. 898, 1–97.

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